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PREFACE

In planning the reorganization of this Observatory in 1905, one of the objects sought was to provide means for modern astrophysical investigations. To this end a large reflecting telescope of the Cassegrain form, having an aperture of 37½ inches and an equivalent focal length of sixty feet, was designed and constructed at the Observatory under my supervision, the optical parts having been obtained from the John A. Brashear Company of Pittsburgh. This instrument was completed in May, 1911, and since then it has been used almost exclusively for photographing stellar spectra with a single-prism spectrograph. The telescope, spectrograph, and engines for measuring spectra have been briefly described in the first volume of these *Publications,* which also contains some of the first results secured by the aid of this telescope. More than thirty-seven hundred spectrograms have now been made, and with few exceptions they are suitable for exact measurement. Nearly all of the papers of the present volume are based upon data obtained from a small proportion of these spectrograms, that is, from those which have already been measured and discussed. Other investigations are in progress.

The spectrographic work has been under the immediate supervision of Professor Ralph H. Curtiss, who designed the spectrograph and measuring engines, and, with Dr. Merrill, planned the programs for work. Dr. Curtiss has also prepared the enlarged spectra, which are used as illustrations in this volume.

WILLIAM J. HUSSEY.

Ann Arbor, November, 1916.

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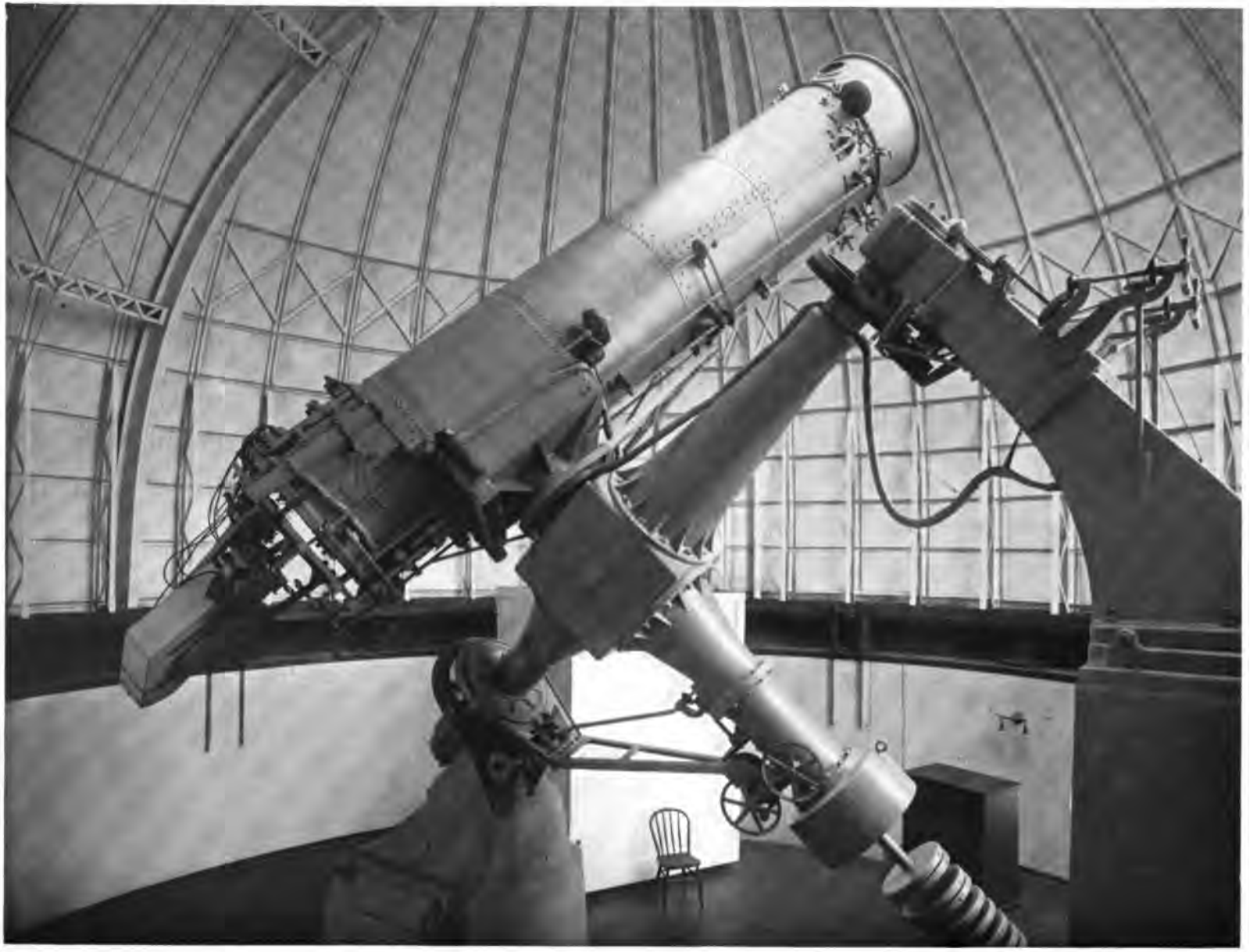

PLATE B. THE THIRTY-SEVEN AND ONE-HALF INCH REFLECTING TELESCOPE

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STUDIES OF CLASS B STELLAR SPECTRA CONTAINING EMISSION LINES

THE SPECTRUM OF γ CASSIOPEIAE

By RALPH H. CURTISS

INTRODUCTION.

Immediately after the 37½-Inch Reflector of this Observatory was completed in May, 1911, a program of observation of bright line stellar spectra of Class B was begun with a single-prism spectrograph attached to this telescope. This program was in continuation of a study of spectra of this particular group, inaugurated by the writer in 1906, in connection with a detailed investigation of the spectrum of β Lyræ. At the present time several hundred plates have been made on this program, including short lists of plates on a number of stars and long series on several typical objects.

The apparatus used in connection with this program has been described fully in earlier papers in this Publication.[1] It is interesting to note here that the dispersion of the spectrograph employed in this work is only a little greater than that of the Mellon Spectrograph with which the study of β Lyræ, mentioned above, was carried on. The spectra made with the two instruments are thus readily comparable.

The historical importance of the problem of γ Cassiopeiæ, as well as the relative brightness of the total light of this star, suggested the advisability of a detailed study of its spectrum. The choice of this spectrum for early investigation followed logically in view of the relatively simple character of its observed features. It was hoped that the analysis of this simpler spectrum would assist in the study of more complicated cases whose complex spectral features, sometimes associated with remarkable light variations, have made the problem of these stars a classic one.

GENERAL DATA.

The star, γ Cassiopeiæ (H. R. P. No. 264, α 1900.0 = 0 hr. 50.7 min., δ 1900.0 = + 60° 11′) is assigned a visual magnitude of 2.25, without certain evidence of light variation. The integrated light impresses the eye as pure white. This star lies in the Milky Way; it has no measurable parallax; it was found by Boss in his *General Catalogue of 6188 Stars* to have a proper motion of + 0ˢ.0040 in right ascension and of — 0″.002 in declination. Apparently then this star is very distant; and its volume and absolute magnitude must be relatively great.

THE SPECTRUM.

Classification. The spectra of the stars of the sub-group to which γ Cassiopeiæ belongs are assigned almost exclusively to "Class B and Class A peculiar," or to Class B and Class A with bright lines. The distinguishing characteristic of this sub-group seems to be the presence in the spectra of one or more hydrogen lines of the Huggins series in which emission is strong enough to be observed. The other lines in the spectrum may be bright, dark, or neutral, but the Orion lines outside of the Huggins series of hydrogen are nearly always dark.

Within this sub-group there are variations similar to those existing within the normal Class B and Class A divisions. The relative strength of critical lines of Class B spectra, such as λ 4686, the ζ Puppis series of hydrogen, and the silicon and nitrogen lines near Hδ, indicates affiliation with Class Bo in some cases, while the weakness or absence of these lines with the relative strengthening of the lines of the metals indicates development into Class A in other cases. Indeed emission lines of the type characteristic of these spectra are observed in two Class F spectra and perhaps in Class G.

A knowledge of the distribution of the known bright line spectra of this sub-group among the various divisions of the Draper classification may

[1] *Detroit Observatory Publications*, Vol. I, page 37.

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be gained by a count of the stars in Table IV, page 182, Volume 56, of the *Harvard Annals* and of eight additional stars in a table by Merrill.[2] Such a count leads to the following results:

TABLE I. DISTRIBUTION OF SPECTRA OF CLASSES B TO F CONTAINING BRIGHT LINES.

CLASS	NUMBER OF STARS
B to B2	19
B3 to B5	43
B6 to B8	6
B9 to A0	29
A1 to A4	1
A5 to A7	1
A8 to F0	2

Evidently these stars, so far as present discoveries go, tend strongly to group within the B division of the Draper classification and are rarely found outside of the interval, B to A0. Possibly the preponderance of Class B spectra among discoveries in this sub-group finds explanation in the fact brought out in the next paragraph.

It is evident also that the total light of these stars, so far as known, is stronger on the average the more nearly their spectra conform to that of Class B0. Thus, the average B.D. magnitude of fifty-six of these stars in the Harvard table, of spectral classes, B0 to B5, is 5.3, whereas this average magnitude for the twenty-eight stars of Classes B8 to A2 is 8.1.

Though the presence of emission lines is distinctive of this sub-group, the *strength* of the emission lines relatively to the continuous spectrum seems not to be clearly dependent upon the so-called effective age of the star. Doctor P. W. Merrill's study[2] of many of the spectra in this sub-group in Class B throws light on this point. Thus, in his γ Cassiopeiæ and ϕ Persei groups of seventeen stars, in which the hydrogen emission lines are strongest, the average spectrum is rated at Harvard as B3.1, whereas in his b^2 Cygni and Electra groups of twenty-one stars, in which this emission is weaker, the average spectrum has essentially the same rating, or B3.3. This point may be brought out in a more striking manner if we assign intensity 3 to the hydrogen lines in the γ Cassiopeiæ and ϕ Persei groups, intensity 2 to these lines in the b^2 Cygni group, and intensity 1 in the Electra group, determining on this basis the average strength of the hydrogen emission lines for spectra in the *subdivisions* of Class B. The results obtained in this way for thirty-eight stars are given in Table Ia. They do not bear evidence of a connection between strength of hydrogen emission and spectral class within the B division.

TABLE Ia. VARIATION IN INTENSITY AND NUMBER OF HYDROGEN EMISSION LINES WITH SPECTRAL CLASS.

CLASS	INTENSITY OF HYDROGEN EMISSION	NO. OF HYDROGEN EMISSION LINES	NUMBER OF STARS
B0 to B1	2.7	3.4	8
B2	2.2	2.9	11
B3	2.6	2.9	29
B4 to B5	2.0	2.6	14
B8	2.5	2.3	6
A0	—	3.2	29
A2 to F	—	2.2	4

It is interesting also to consider whether there is any connection between spectral class and the number of hydrogen emission lines visible in stellar spectra of this sub-group, especially in view of the fact that in general the total strength of these emission lines is found to increase with their number. Referring to the 101 stars of Table I, we find for those having hydrogen emission lines from Hα to Hϵ and Hζ, an average spectral class of B3.0; for those to Hδ, B7.4; for those to Hγ, B5.0; for those to Hβ, B5.9; and for one with Hα only, B5. Or, again, if it be permitted to take averages to tenths of numbers of emission lines, the data in column three of Table Ia may be obtained for the 101 stars of Table I, showing the average number of hydrogen emission lines found in the spectra of these stars within subdivisions of Classes B to F. The data for these averages are taken from Table IV, page 182, Volume 56, *Harvard Annals*, with ad-

[2] *Lick Observatory Bulletins*, Vol. VII, page 162.

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ditions based on recent results. So far as these data go, there appears to be little or no connection between *intensity or number of hydrogen emission lines* and *average spectral class;* nothing to indicate from this point of view that this sub-group of spectra terminates in reality with Class F. Indeed, on the basis of the facts so far known, the conclusion is suggested that the extension of spectral surveys to fainter objects may result in the discovery of many new members of this sub-group with spectra of Classes A and F and possibly of Class G.

The metallic lines in these spectra, if present, may be bright or dark, both types occurring in some cases in the same spectrum. Sherman[3] and Merrill[2] have pointed out that the metallic bright lines correspond to strong chromospheric emission in a number of cases, and Baxandall[23] called attention to the dominance of iron emission in them.

It is of interest to inquire whether the occurrence of these chromospheric lines is in any way associated with spectral class. In this connection we find that the mean Harvard classification of eleven spectra in which Merrill has found these bright metallic lines is B2.5, only one having a spectrum later than B3, whereas the mean classification of thirty-eight of these spectra examined by Merrill (six not classified by Merrill are excluded) is B3.2. That the spectra of these eleven stars average nearer to Class Bo than does the mean spectrum of Merrill's list, and that ten of these stars have spectra classed as B3 or earlier on the basis of the Orion lines, indicate that the metallic emission lines tend to appear in these spectra only under conditions thought to characterize stars in the earlier stages of development. Apparently, also, when these lines occur, they tend to be stronger when the hydrogen emission is stronger.

Characteristics Historically Considered. Secchi, at the very beginning of his work on stellar spectra, noticed emission lines in the light of γ Cassiopeiæ. In the *Astronomische Nachrichten* No. 1612, under date of 1866, August 23, he states that the spectrum of this star has a bright line at Hβ, and several others too faint to measure. In a report published February 8, 1867, Huggins announces his identification of Hα, as well as Hβ, as a bright line and also his observation of some dark lines of absorption in this spectrum.[5] Later both Secchi and Huggins saw D_3 emission in γ Cassiopeiæ in addition to that of Hβ and Hα.[4]

Vogel, on June 18, 1872, observed Hβ and D_3 as emission lines and saw also an absorption band in the red, but, though he examined carefully the red end of the spectrum, the Hα line was not visible.[6]

Von Konkoly examined the spectra of γ Cassiopeiæ and β Lyræ repeatedly, between 1874 and 1883, without seeing bright lines, and in 1882, von Kovesligethy was equally unsuccessful; von Gothard observed both of these stars frequently after the autumn of 1881, but saw no trace of bright lines until a night quite unfavorable for observing in 1883, when, according to his record, on August 13 of that year, he saw bright Hα and the absorption band at λ 633$\mu\mu$ in the spectrum of γ Cassiopeiæ.[7]

During this same period (from 1874 to 1883) Copeland records a bright line well seen at about the place of Hβ in this spectrum and another emission line at about λ 477 $\mu\mu$, on October 28, 1877, when there is no record of an examination of the Hα region; and, on December 20, 1879, Lord Crawford, J. G. Lohse and Copeland saw Hα "superbly visible" with a spectroscope separating the D lines.[8]

The Greenwich record of observations of the Hβ emission line during this period, made with a half-prism spectroscope attached to the 12.8-Inch Refractor, is as follows: 1880, October 1. Brilliant against background of continuous spectrum. Broad and diffuse at edges. Central condensation. November 21. Bright. 1881, December 7. Difficult to measure. Absence of mention of emission lines, D_3 and Hα, does not imply their extinction, for these observations were made on the Hβ line for the measurement

[3] *American Journal of Science*, Vol. 30, Dec., 1885. *Astronomische Nachrichten*, Vol. 113, page 311.

[4] *Sugli Spettri Prismatici delle Stelle Fisse*, Mem. I, page 10; Mem. II, page 62.

[5] *Monthly Notices*, Vol. 27, page 131.

[6] *Bothkamp Beobactungen*, Heft II, page 29.

[7] *Astronomische Nachrichten*, Vol. 106, page 293.

[8] *Monthly Notices*, Vol. 47, page 92.

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of radial velocity and D_3 and Hα were not looked for.[9]

On August 20, 1883, one week after his discovery of the possible revival of intensity of Hα in γ Cassiopeiæ, von Gothard renewed his observations, notwithstanding unfavorable atmospheric conditions, and, with a small slit spectroscope with half prism of low dispersion, obtained approximate wave-lengths and intensities of the bright lines, Hα, D_3, and Hβ, and of the dark band at λ 633 $\mu\mu$. Hα had the greatest intensity and appeared, with a small ocular spectroscope by Zöllner, as a sharp brilliant line. Hβ was more difficult to see and D_3 could be observed only a few times when the seeing was especially good. The relative intensities of Hα, D_3, and Hβ were expressed by the numbers, 5, 1, and 2 respectively. There was no question as to the reality of the observed phenomena on this date, but von Gothard seemed to have had misgivings as to the reliability of his observations of bright Hα on August 13, for in a letter received by von Konkoly, on August 22, he qualified his announcement of the discovery by stating that, on account of the unfavorable atmospheric conditions, the line might have been merely a subjective phenomenon, caused by contrast with a dark band more refrangible than Hα.

On the evening of the receipt of von Gothard's announcement, von Konkoly observed the star at O'Gyalla, first with the dispersion of a train of three prisms of the Zöllner eye-piece spectroscope with a cylindrical lens in the optical train. The Hα line appeared *very faint*. But, when two trains of prisms were used, without the cylindrical lens, Hα was seen as an *exceedingly bright* knot in the narrower thread of continuous spectrum and a much fainter Hβ was noticed. On the same evening, with a little slit spectroscope equipped with one 60° prism, Hα was observed again and compared with the Geissler tube spectrum of hydrogen, but Hβ was too faint to be seen under these circumstances. A strong absorption band was seen to border Hα sharply on its more refrangible side, and, on the less refrangible side, another broad absorption line was suspected.[10]

[9] *Monthly Notices*, Vol. 49. page 300.

On August 24, 25 and 26, at the Astrophysical Observatory in Hereny, von Gothard, von Konkoly, and von Than saw Hα and Hβ very well and D_3 less easily, with a Zöllner eye-piece spectroscope of relatively high dispersion attached to the 10¼-inch Browning reflector. Von Konkoly, in his own account of the observations of the twenty-sixth, when the air was very clear and steady, states that the red line was exceedingly bright; D_3 and Hβ emission were well defined; and bright Hγ was seen. The dark absorption band more refrangible than Hα was very distinctly defined and assurance was felt that the line somewhat less refrangible than Hα was not an effect of contrast.[10]

On the following night, August 27, von Konkoly availed himself of the superior power of the 27-Inch Refractor at Vienna, using his own Zöllner eye-piece spectroscope with two trains of three prisms each, without a cylindrical lens. Hα appeared as a knot of light of dazzling brilliance; D_3, Hβ and Hγ were fainter. The broad absorption band, a little more refrangible than Hα, was complex and exceedingly strong. The absorption band on the other side of Hα had an undoubtedly real existence. The presence of dark D and b lines could be positively asserted, though they were very faint. There was absorption also on the more refrangible side of Hγ. These characteristics are brought out in sketches of γ Cassiopeiæ's spectrum in Volume VI, of the *O'Gyalla Publications*.

On September 1, 1883, von Gothard saw intense Hα and Hβ emission, also Hγ and dark D but no D_3. Later neither D nor D_3 was visible. Between September 23 and November 23, the Hα, Hβ and Hγ emission lines were observed and also dark lines of the b group. D_3 continued invisible. Measures of the bright Hβ line yielded a wave-length of 4871Å.[11]

In this same period Hβ was easily seen at Greenwich. But during the following year there was evidence of continued or renewed variability. The Greenwich observations of Hβ emission in 1884 were: August 11 and 25, faint;

[10] *Astronomische Nachrichten*, Vol. 107, page 61.
Observatory, Vol. 6, page 332.
O Gyalla Beobachtungen, Vol. 8, page 5.

[11] *Astronomische Nachrichten*, Vol. 108, page 237.

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September 4, very faint; September 10, barely visible; September 11, measurable; September 18, very faint; September 20, faint, sharp, well defined at edges. These observations were made with a half-prism spectroscope attached to the 12.8-Inch Refractor.[12]

In November, 1885, O. T. Sherman announced the results of his studies of the spectrum of γ Cassiopeiæ with the 8-Inch Refractor of the Yale College Observatory. The spectroscope was a direct vision instrument with a collimator six inches long and two trains of prisms of three pieces each. In observing a star spectrum the slit was opened to 5 mm. making the star image and not the slit the source of light. Under these circumstances only rough wave-lengths could be determined and the difficulty of observation of lines at the ends of the visible spectrum must have been great. Nevertheless, Sherman gives us in addition to the hydrogen lines the wave-lengths of eleven emission and six absorption lines in this star (not including D_1 and D_2), fourteen or fifteen of which can be identified with spectral features now well substantiated. As shown in column one of Table IV, where his wave-lengths are given, Sherman observed bright Hα, D_3, Hβ, Hγ, and Hδ and also several emission lines of iron. He compared these emission lines with Young's chromospheric lines and reached the important conclusion that the number of coincidences rendered it extremely probable that the lines observed were those of the solar atmosphere.[13] The intensity of D_3 was not specified but since it was used in determination of wave-lengths it was probably clearly visible.

Observed at Dun Echt on September 3, 1885, with the 15-Inch Refractor, Hα was very bright, Hβ just measurable, while D_3 could not be made out with certainty. At the same observatory, on January 11, 1887, Hα was extremely bright.[14] Observed frequently at Kensington, between 1886 and 1894, the Hα and Hβ emission lines were always visible. On all but the third of four dates specifically mentioned (September 18, October 13 and 24, 1889; October 21, 1894) D_3 emission was also noted.[15]

In the summer of 1889, Keeler examined the spectrum of γ Cassiopeiæ frequently with a small spectroscope attached to the 36-Inch Telescope of the Lick Observatory and observed many details but no changes in its spectrum. Hα and Hβ were brilliant, narrow and sharp; Hγ was seen with some difficulty. Alternations of intensity in the green could be interpreted as bright or dark lines. Keeler seemed to favor the view that they were dark lines, the more prominent of which were identified with the b group. A fairly prominent dark band or group of lines was observed nearer to Hα than the estimated position of D, but no trace of bright or dark lines could be seen in the vicinity of D although bright D_3 was observed at Kensington on September 18 and October 13 of that year.[16] Later, at Allegheny, Keeler succeeded in photographing dark lines in the spectrum of γ Cassiopeiæ.[17]

In the meantime, in 1887 and 1888, at Greenwich, the bright lines, Hα, D_3 and Hβ, were seen apparently to continue their variations. Thus, in 1887, Hα was *not seen* while Hβ was *very distinct* on February 16, whereas, on December 5, Hα was *brilliant* and Hβ was *faint*. Eleven days later Hα was *not seen* with the cylindrical lens in place before the slit but was *distinctly seen* without the cylindrical lens. And on the same night, Hβ appeared *faint* with the cylindrical lens and was *not seen* without it. In 1888, September 19, with and without the cylindrical lens Hα was *very bright* and Hβ was rather faint. These observations were made with the single prism instrument mentioned above. On four dates in these two years there are records of observations with the half prism spectroscopes, giving much greater linear scale. In general these observations also indicate continued variation of the Hβ line. But care was necessary in the interpretation of results for on February 16, 1887, the Hβ line was *very distinct* with low dispersion, but was *very faint* and *narrow* with four fold greater power. On December 16 of the same year, when

[12] *Monthly Notices*, Vol. 49, page 300.

[13] See reference No. 3.

[14-15] *Proceedings, Royal Society of London*, Vol. 57, page 174.

[16] *Publications of the Astronomical Society of the Pacific*, Vol. 1, page 80.

[17] *Scheiner's Astronomical Spectroscopy*, Frost's translation, page 249.

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instruments of high and low linear dispersion were used again, the results were accordant. Only on September 19, 1888, was D_3 seen and then as a faint emission line. Undoubtedly with attempted allowance for instrumental differences, Maunder considered that the Greenwich observations from 1880 to 1888 appeared to show: that the bright lines, $H\alpha$, D_3, and $H\beta$, in γ Cassiopeiæ varied, but not simultaneously nor similarly; that *either* $H\alpha$ or $H\beta$ was the most conspicuous line in the spectrum.[18]

In the Draper Catalogue, issued in 1890, ten photographic records of the spectrum of this star are listed. Except in three cases where the image was too dense or too near the edge of the plate, $H\beta$ was registered as a bright line.

In connection with a study of β Lyræ, Belopolsky reported the absence of D_3 from three spectrograms of γ Cassiopeiæ made in 1892. But Belopolsky's plates were probably not sensitive enough in this region to show any but the brighter lines.[19]

On November 19, 1894, J. N. Lockyer reported the results of his preliminary studies of 53 spectrograms of γ Cassiopeiæ, distributed between the dates, 1888, November 20, and 1894, November 16. His deductions may be summarized as follows: (1) Bright $H\beta$, $H\gamma$, and $H\delta$ are *constantly* seen in the Kensington photographs; $H\epsilon$ and $H\zeta$ appear when the photographic conditions have been good. (2) Additional lines, for the most part ill-defined, appear on all good negatives. (3) *During the period covered by the photographs, there is no evidence of any change in the intensities of the principal bright lines.* (4) The bright lines of hydrogen are double on all of the photographs taken with sufficient dispersion. (5) There is no evidence of (great) orbital motion from May, 1892 to November, 1894. (6) Assuming the presence of two sources of bright hydrogen lines, the relative velocity in the line of sight is 115 miles per second. (7) The bright lines of hydrogen are superposed upon broad dark bands. (8) Besides the dark bands in the positions of the hydrogen lines there are other ill-defined dark lines. (9) The dark lines in the spectrum of γ Cassiopeiæ correspond very closely with the lines seen in the spectrum of ζ and γ Orionis.[15]

In a paper[20] dated June 28, 1894, appearing late in 1895, Campbell refers to certain striking facts attested by his spectrograms. He found the hydrogen emission lines to decrease rapidly in intensity in succession toward the violet and to be situated within broad dark lines. Partially dark lines were observed in other parts of the spectrum. Further, on copies of Harvard photographs he noted many dark lines and observed that the broad dark hydrogen lines increase in intensity as they decrease in wave-length.

In her detailed discussion of the spectra of bright stars, begun in 1888, and published in 1897 (*Harvard Annals,* Volume 28, Part I), Miss Maury has made a careful analysis of the spectrum of γ Cassiopeiæ. In describing a remarkable spectrogram, taken November 23, 1892, and reproduced near the end of Volume 28 of the *Harvard Annals,* she characterized the hydrogen lines as *doubly reversed,* calling attention in this connection to Jewell's observation of complex reversals in solar lines. She noted also, as did Campbell, the diminution of intensity in each succeeding hydrogen line of shorter wave-length, $H\epsilon$ being nearly neutral and the lines of this element beyond $H\zeta$ without visible bright components. At the same time the broad underlying dark band and the narrow dark reversal became more conspicuous so far as observed. Other bright lines beside those of hydrogen were seen in the spectrum. The strongest of these, at λ 5023, was erroneously identified as an Orion line. The wave-lengths of a number of dark Orion lines were determined and these, with similar data for a number of emission lines, are found in column two of Table IV of this paper. Miss Maury states further that $\lambda\lambda$ 5015.73 and 5047.82 of helium are clearly reversed in γ Cassiopeiæ and that $\lambda\lambda$ 4026.4, 4387.8 and 4144.0 and possibly $\lambda\lambda$ 4471.8 and 4009.5 are suspected of having central narrow reversals.[21]

In the *Monthly Notices* for January, 1896, H. F. Newall reports the doubleness of $H\gamma$ and $H\beta$ clearly seen with a single-prism spectrograph.

[18] See reference No. 9.

[19] *Astronomy and Astrophysics,* Vol. 12, page 259.

[20] *Astrophysical Journal,* Vol. 2, page 177.

[21] *Harvard Annals,* Vol. 28, page 100 et seq.

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In May, 1899, Sidgreaves published an extensive note on the spectrum of γ Cassiopeiæ, based on photographs distributed over a period of eight years. There were fifty-two plates, of which half were made with an eight-inch glass and half with the Perry Memorial Objective of fifteen inches aperture. Wave-lengths and intensities of a number of bright and dark lines between Hε and λ 5576, measured by Sidgreaves, are given in Table IV of this paper. Zero intensity for a radiation line indicates an intensity equal to that of the neighboring continuous spectrum. Sidgreaves measured a number of faint bright lines in addition to the hydrogen series and identified the group at λ 5170 with magnesium. The identity of the helium dark lines was also brought out. This investigator found no signs of variation of the hydrogen lines during the period of eight years covered by his observations, but he did suspect changes in the faint metallic emission lines, especially at λλ 4586 and 5020.[22]

In *Lick Observatory Bulletins,* Numbers 237 and 246, Merrill gave brief descriptions of results obtained from 28 plates of various parts of this spectrum, made between 1896 and 1913, with six different spectrographs attached to the 36-Inch Refractor of the Lick Observatory. On low dispersion plates of 1912, numerous faint emission lines and broad poor dark lines were recorded. Faint but unmistakable D_3 emission was noted. Hα under high dispersion did not appear clearly reversed. Photometric intensity curves of the Hα, Hβ, and Hγ lines brought out in detail the structure of these lines in harmony with and in extension of published descriptions. Polarization tests of Hβ led to no positive conclusions. The series contained no internal evidence of spectral variation.

In 1914, Baxandall reported the results of studies of the Kensington plates, including many which had been made since 1894. With the aid of comparisons with spectra of other stars, an extensive table of wave-lengths of dark lines in the spectrum of γ Cassiopeiæ was prepared. (See column five, Table IV.) There was distinct evidence that the chief bright lines other than those of hydrogen were identifiable with the enhanced lines of various metals, iron predominating. An intercomparison of the best of the plates obtained since 1894 showed that there were no definite changes in the spectrum on different dates.[23]

The Ann Arbor spectrograms are discussed below. It is appropriate to note here that they bring out the close qualitative correspondence of the features of this spectrum in the years, 1911 to 1915, with those of photographic observations reported by Lockyer, Campbell, Miss Maury, Sidgreaves, Merrill, and Baxandall, and further that they register no intensity variations in the spectral lines not accounted for by uncertainties of photographic contrast.

Spectral Variations. The above fairly comprehensive summary of the published observations of the spectrum of γ Cassiopeiæ is of interest especially in connection with its bearing upon the reported variations of some of the spectral features involved.

Referring first to the D_3 line of helium, the visual observations of this spectral feature would indicate that capricious changes have taken place in the intensity of this line, for it was seen distinctly by some observers and was invisible to others at or near the same epoch, and the same observer at different times, in some cases, had different impressions of its brightness. But the indications of uncertainty affecting the visual observations of the much brighter lines of hydrogen, discussed below, make very doubtful conclusions with reference to the physical reality of such observed changes. On the other hand the very fact that this line was seen so frequently by visual observers indicates that, in that period, from 1872 to 1888, it must have been stronger, at least spasmodically, than at present, for now, even photographically, it is a faint line. To be sure von Konkoly suspected and Sherman observed a number of faint emission lines in this spectrum which very possibly were then, as now, comparable in brightness with the present D_3 emission, but it seems doubtful whether so many visual observers would have recognized this line if it had been so faint as it is at the present time. In 1883, von Konkoly assigned to D_3 an intensity one-half that of Hβ and one-fifth that

[22] *Monthly Notices,* Vol. 59, page 505.

[23] *Publications of the Solar Physics Committee.* "The Spectrum of γ Cassiopeiæ," 1914.

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of Hα. On the Ann Arbor plates, discussed in this paper, the D_3 line is estimated to be one-fifth as bright as Hβ and one-eighth as bright as Hα. These intensity estimates may represent the extent of a possible decline in the strength of this line in a period of thirty years.

As for the faint metallic emission lines, there seems to be no evidence to show that they have undergone any secular changes like that suggested in the case of D_3, for they were seen clearly by only one visual observer, and are not recorded by others observing in the same year, and thus were very probably faint, as they are found to be at present. Even on the spectrogram their visibility is so dependent upon photographic conditions that valid conclusions as to their constancy or variation are difficult or impossible to reach. It is not strange that one photographic investigator, at least, has shifted suspicion of variability from the hydrogen to the faint metallic emission lines. But indicated variations of such lines are especially hard to establish.

For the Hα and Hβ lines, which were sometimes seen as brilliant and again as very faint or invisible with the same spectrograph, the case is clearly suggestive enough to warrant careful attention. Briefly: Secchi and Huggins saw bright Hα and Hβ in the late sixties, but Vogel, in 1872, did not find the Hα line. Then, from 1874 to 1883, no emission lines were seen by von Konkoly, notwithstanding frequent search, nor by von Gothard who looked for them often during the last two years. During this period, however, bright hydrogen emission was seen in England in 1877, 1879, 1880, and 1881. Thereafter, these emission lines were observed in Europe, at Kensington, and at the Lick Observatory, apparently without variation, but at Greenwich hydrogen emission was found to vary greatly as late as 1887, when, on December 5, Hα was brilliant, while eleven days later, with the same instrument, it was "not seen." At the same time, as seen from Greenwich, the Hβ emission continued to vary, though in no such pronounced manner. Thereafter the photographic method superseded the visual quite generally.

These visual observations were undoubtedly profoundly affected by instrumental conditions, particularly in the case of the Hα line, which is near the edge of the region for which visual objectives are corrected. Thus, on August 20, 1883, when von Konkoly did view the Hα emission after nine years of watching, it is significant that he found the line very faint when he observed it with one train of prisms, whereas with two such prism trains he found it exceedingly bright. Noteworthy, also, was the greater success which followed his observations as he employed in rapid turn telescopes of increasing aperture. Again, on February sixteenth, 1887, at Greenwich, Hβ emission was "very distinct," as seen with one spectroscope, whereas with another it was very faint; and on December sixteenth of the same year, Hα was "not seen" with a cylindrical lens before the slit, but was "distinctly seen" without such lens. Other similar cases might be cited, but without amplifying further, the conclusion seems to be indicated that the results of early visual observations of the spectrum of γ Cassiopeiæ, like those of β Lyræ, must be regarded with caution.

With the adoption of the photographic method evidence of variation of the hydrogen emission lines in the photographic region was not found and visual observations with equipment of greater average power as compared with the apparatus of the old visual observers, continued to record hydrogen emission. Later Hα was observed photographically without suspicion of variation on the following dates at least: 1911, June 21, 27, July 25, 26, 27, 28; 1912, July 30, August 1, 6, September 12, October 2; 1913, July 12, December 18; 1914, January 1, October 24; 1915, November 13; 1916, January 6. And from the hundreds of spectrograms of γ Cassiopeiæ in the photographic region, no variations in the other hydrogen lines have been announced.

In view of the absence of observed variations in the hydrogen lines after the photographic era began, it appears that an interesting comparison could be drawn if visual observations with a small refractor were available during this period. It seems that such data are reported, in 1906, by S. E. Percival of Somerset, England, as follows:

"My instrument is one of Hilger's Zöllner spectroscopes with three cylindrical lenses of different powers; the telescope is a Cooke 3¾-inch. I have been puzzled by the apparent behaviour of this (Hα) line (in γ Cassiopeiæ).

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Sometimes I have seen it with great ease and distinctness, at others I have barely glimpsed it; at others again I have totally failed to see it. Thus, I saw it splendidly when the star was high up in, I think, the early days of January. On May 2, I failed to see it, but, on May 4, I saw it quite distinctly. Last night, May 18, though the sky was clear and the spectrum steady, I barely glimpsed it; sometimes indeed a *dark line* seemed to replace it."[24]

It is regrettable that no photographic observations of the Hα region were reported at this time. However, Hartmann gives a list of six spectrograms taken between July 12 and October 5 of that year (1906), and, though he characterizes the spectrum in the photographic region in general, he does not refer to variations in the intensities of the lines.[25]

Reviewing the evidence relative to the variability of features in the spectrum of γ Cassiopeiæ, it seems that we may conclude at once that, during the period from 1874 to 1884, and possibly until 1888, the variations of the hydrogen emission lines, *if real,* were of short period, as Copeland has suggested. After 1883 (or 1888) there seems to be little chance that any appreciable variation existed. In general the visual observations upon which conclusions with reference to these variations are based, were profoundly affected by instrumental conditions and must be regarded with caution. On the other hand the constancy of the photographic record must be given great weight. Further it is significant that visual observations with apparatus of limited power yielded results indicating great variations in the Hα line as late as 1906, when observers in the photographic region mention no changes. However, notwithstanding the evidence tending to discredit the work of the early visual observers, it would seem that the time has not come to reject the results gathered by them in connection with the much discussed variations of the hydrogen lines in the spectrum of γ Cassiopeiæ. But astronomers at the present time will hesitate to regard the reality of these variations as established until confirmation is found through the photographic method.

[24] *Journal British Astronomical Association,* Vol. 16, page 319.

[25] *Astronomische Nachrichten,* Vol. 173, page 102.

FORMER RADIAL VELOCITIES.

Radial velocities of γ Cassiopeiæ have been announced by Vogel and Scheiner on page 99 of *Potsdam Publ.,* Vol. VII, by Hartmann on page 102 of *Astronomische Nachrichten,* Volume 173, and by Merrill on pages 163 and 164 of *Lick Observatory Bulletin,* Number 237.

The observations of Vogel and Scheiner were:

1888, October 6,	+0.8 km.
1889 January 9,	−7.8
Mean velocity,	−3.5

Hartmann found the lines in this spectrum hard to measure because of their diffuseness. Also he thought that there were relative shifts among the lines. The velocities correspond to measures of the middle of the lines, which were in general of symmetrical structure. These nine velocities with their mean are given in Table II. Considering the velocity of the star as constant, the probable error of a single observation of Hartmann proves to be ± 4.6 km., which is in close accord with his idea of the accuracy of the several velocities. Nevertheless, Hartmann considered that the apparent slow change in the radial velocity of this star was real and announced it as an object with variable radial velocity, adding the suggestion that more definite information with reference to the nature of the variation would be obtained through observations extending over a long time interval. Possibly the reader will have misgivings as to the safety of this announcement.

TABLE II. THE POTSDAM OBSERVATIONS.

SPECTROGRAPH	DATE	VELOCITY	RESID.
I	1900, July 2	+ 0.5 km	+ 9.0 km
I	Sept. 21	+ 3.1	+11.6
I	1901, Sept. 27	−19.3	−10.8
III	1906, July 12	−10.1	− 1.6
I	17	− 5.8	+ 2.7
I	Sept. 22	− 9.3	− 0.8
I	24	−14.4	− 5.9
I	30	−10.0	− 1.5
I	Oct. 5	−11.1	− 2.6
	MEAN	− 8.5	

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The sixteen observations made at the Lick Observatory are conveniently divided into four groups depending on the spectrograph used, the measurer and the features measured. The entire series extends over sixteen years and each set centers about a well defined epoch. The means of the four groups are all nearer to zero than Hartmann's mean, but the mean for the third and strongest group differs only two kilometers from that of Hartmann. In his *Second Catalogue of Spectroscopic Binary Stars*, containing data available up to March 15, 1910, Campbell noted that the Lick Observatory observations did not confirm Hartmann's announcement. Considering the twenty-seven velocities of γ Cassiopeiæ derived at Potsdam and the Lick Observatory, the conclusion is reached that more observations would be necessary to establish the reality of Hartmann's reported velocity variation.

TABLE III. THE LICK OBSERVATORY OBSERVATIONS.

SPECTROGRAPH	MEASURER	FEATURE MEASURED	G. M. T.	VELOCITY
			1896	
Original Mills.	Campbell.	Bright components Hγ Emission.	Aug. 9.99	— 1 km
			19.00	— 1½
			Sept. 23.86	— 4
			Nov. 11.78	— 4
			MEAN	— 2.6
			1903	
Remounted Mills.	Moore.	Bright components Hγ Emission and central absorption.	Nov. 1.96	— 1½
			1.98	— 4
			MEAN	— 2.8
			1910	
Same.	Merrill.	Same.	Nov. 14.68	— 2.4
			15.84	— 7.3
			17.59	— 6.2
			1911	
			Jan. 7.61	± 0.0
			Aug. 7.93	—10.5
			Nov. 23.76	— 9.6
			24.77	— 8.8
			MEAN	— 6.4
			1912	
Three-Prism, λ4900 central.	Merrill.	Hβ Line.	Aug. 21.94	— 2.9
			22.92	— 7.6
			29.90	+ 2.7
			MEAN	— 2.6

DETROIT OBSERVATORY STUDIES.

Material. The writer's studies of γ Cassiopeiæ are based upon seventy-four spectrograms made in the years, 1911 to 1914, with the single-prism spectrograph described in Volume I of these *Publications*. Seventy of these spectrograms are listed in Table VI and four more are referred to in the foot note following that table. With a

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few exceptions the negatives were made with Red Label Lantern Slide Plates. Three plates were sensitized to visual light. Two plates, made with a very long slit, were especially useful in the study of faint wide absorption lines.

Wave-Lengths and Intensities of Lines. The wave-lengths, intensities, number of measures and relative weights for the lines in the spectrum of γ Cassiopeiæ, observed at Ann Arbor, are found in the four columns next to the last in Table IV. The preceding columns contain parallel or additional results obtained by Sherman, Miss Maury, Sidgreaves, and Baxandall for the photographic region, and by Sherman, Sidgreaves, and Merrill for the visual region of the spectrum. The last column contains laboratory wave-lengths, identifications, symbols, and remarks for many of the lines. In this column the wave-lengths for the metallic emission lines, except those in the visual region observed by

TABLE IV. ASSEMBLED WAVE-LENGTHS OF LINES IN THE SPECTRUM OF GAMMA CASSIOPEIAE.

SHERMAN	MAURY	SIDGREAVES		BAXANDALL	CURTISS				IDENTIFICATIONS, ETC.
WAVE-LENGTH (1) Å	WAVE-LENGTH (2) Å	WAVE-LENGTH (3) Å	INT. (4) Å	WAVE-LENGTH (5) Å	WAVE-LENGTH (6) Å	INT. (7)	NO. (8)	WT. (9)	(10)
.....	.....	.....	....	.....	3888.81E	6	2	2	3889.20, Hζ.
.....	.....	.....	....	3889.1	3888.80	4n	2	2	3889.20, Hζ.
.....	3912.2	.....	....	.....	.....	...	..	..	.....
.....	.....	.....	....	3920	3920.0	5	1	1	Blend. N, O, C.
.....	3927.1	.....	....	3926.7	3927.4	4nn	1	1	3926.7, Helium.
.....	.....	.....	....	.....	3930.4	8	1	1	.....
.....	.....	.....	....	3933.8	3933.86	3.6	39	27	3933.83, K Calcium.
.....	.....	.....	....	.....	3935.7	5	1	1	3936.06, Helium.
.....	.....	3970E	0	.....	3970.19E	14	7	4	3970.18, Hε.
.....	.....	.....	....	3970.2	3970.26	3.5	30	22	3970.18, Hε.
.....	.....	3983	1	.....	.....	...	..	..	.....
3993	3994.9	3995	2	3995.2	3995.8	4	1	1	.....
.....	.....	.....	....	.....	3998.0	6	1	1	.....
.....	4009.5	4009	5	4009.4	4009.6	6	7	5	4009.42, Helium.
.....	.....	.....	....	.....	4010.7	8	3	1	.....
.....	.....	.....	....	.....	4011.4	5	2	2	.....
.....	4026.4	4025	6	4026.3	4026.56	12n	53	28	4026.37, Helium.
.....	.....	4037	1	.....	.....	...	..	..	.....
.....	.....	4043	1	.....	.....	...	..	..	.....
.....	.....	.....	....	.....	4065.4E	4	2	2	.....
.....	.....	.....	....	.....	4067.2	4	3	4	4067.22, β Lyræ.
.....	4069.4	4069	3	.....	4069.5	4	2	1	.....
.....	.....	.....	....	.....	4071.5	5	4	2	.....
.....	4072.0	4076	2	4073.–	4073.4	5	3	2	Oxygen Triplet.
.....	4089.2	4088	1	4089.2	4089.7	7n	1	1	4089.00, Si.
Obs.	Obs.	4101E	2	.....	4101.92E	20	48	42	4101.92, Hδ.
.....	.....	.....	....	4101.8	4101.90	3.1	60	68	4101.92, Hδ.
.....	4116.2	.....	....	4116.5	.....	...	..	..	Silicon.
.....	.....	4118	3	.....	4118.8	8n	2	2	Blend. Si and He.
.....	4120.5	.....	....	4121.0	.....	...	..	..	Helium.

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TABLE IV. ASSEMBLED WAVE-LENGTHS OF LINES IN THE SPECTRUM OF GAMMA CASSIOPEIAE—CONTINUED.

SHERMAN	MAURY	SIDGREAVES		BAXANDALL	CURTISS				IDENTIFICATIONS, ETC.
WAVE-LENGTH (1) Å	WAVE-LENGTH (2) Å	WAVE-LENGTH (3) Å	INT. (4) Å	WAVE-LENGTH (5) Å	WAVE-LENGTH (6) Å	INT. (7)	NO. (8)	WT. (9)	(10)
.....	.....	4131E	3	.....	.....	...	..	..	.....
.....	.....	.....	....	.....	4141.21	3	2	2	.....
.....	4144.0	4144	4	4143.9	4144.1	8n	8	5	4143.92, Helium.
.....	.....	.....	....	.....	4145.9	5	3	2	.....
.....	.....	4155	1	4155.0	.....	...	..	..	Blend, Oxygen.
.....	.....	4170	3	4169.1	4169.3	9n	2	2	4169, Helium.
4180E	.....	4177E	2	.....	4177.4E, A	3	2	2	4177.70, Fe-V 12.
.....	.....	4185	2	.....	.....	...	..	..	.....
.....	.....	.....	....	.....	4200.2	4	1	1	4200.3, Hδ'.
.....	.....	.....	....	.....	4207.4E	5	3	2	.....
.....	.....	.....	....	.....	4214.6E	3	1	1	4215.9, Sr 40.
.....	.....	4234E	2	.....	4233.60E	5.0	22	16	4233.33, Fe-Cr 20.
.....	.....	.....	....	.....	4233.41	3.0	15	13	4233.33, Fe-Cr 20.
.....	.....	4239	1d	.....	.....	...	..	..	.....
.....	4254.1	4253	1	4253.8	.....	...	..	..	.....
.....	.....	.....	....	.....	4260.11E	4.0	7	4	4260.23, Fe 2. 4260.64, Fe 8.
.....	4267.4	4266	1	4267.4	4267.5	7n	1	1	4267.15, Carbon.
.....	.....	.....	....	.....	4271.8E	5	2	2	4271.32, Fe 6. 4271.93, Fe 15.
.....	.....	4281	2w	.....	4282.1	4	2	1	.....
.....	4285.1	.....	2	4285.1	.....	...	..	..	Sulphur.
.....	.....	4295	2	.....	4295.4	8	1	1	.....
.....	.....	4302E	2	.....	.....	...	..	..	.....
.....	.....	4306	2	.....	4307.8	4	4	4	.....
.....	.....	4317	1	4318.–	4317.8	10n	1	1	Oxygen pair.
.....	.....	4326	1	.....	.....	...	..	..	.....
Obs.	Obs.	4340E	8	.....	4340.64	24	69	77	4340.63, Hγ.
.....	Obs.	Obs.	—	.....	4340.66	2.5	55	57	4340.63, Hγ.
.....	.....	.....	....	.....	4353.3E	4	4	2	4352.9,? Fe 4.?
.....	.....	.....	....	4367.0	4367.4	7n	1	1	4367.—, Oxygen.
.....	.....	4382E	1	.....	4384.25E	5.0	21	14	4383.72, Fe 15. 4385.55, Fe 5.
.....	.....	.....	....	.....	4384.61	3	7	5	4383.72, Fe. 15. 4385.55, Fe 5.
.....	4387.8	4388	4	4388.1	4388.3	5	8	5	4388.10, Helium.
.....	.....	.....	....	.....	4390.9	4	5	4	.....
.....	.....	4395E	2	.....	.....	...	..	..	.....
.....	.....	.....	....	.....	4400.5	3	2	1	.....
.....	.....	.....	....	.....	4403.07	4	4	3	.....

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TABLE IV. ASSEMBLED WAVE-LENGTHS OF LINES IN THE SPECTRUM OF GAMMA CASSIOPEIAE—CONTINUED.

SHERMAN	MAURY	SIDGREAVES		BAXANDALL	CURTISS				IDENTIFICATIONS, ETC.
WAVE-LENGTH (1) Å	WAVE-LENGTH (2) Å	WAVE-LENGTH (3) Å	INT. (4) Å	WAVE-LENGTH (5) Å	WAVE-LENGTH (6) Å	INT. (7)	NO. (8)	WT. (9)	(10)
.....	.....	.....	....	4416	.....	...	..	..	Oxygen.
.....	.....	.....	....	.....	4418.8E	4	1	1	.....
.....	.....	.....	....	.....	4418.8	2	1	1	.....
.....	.....	4431	3	.....	.....	...	..	..	.....
.....	.....	.....	....	4437.7	.....	...	..	..	Helium.
.....	.....	4451	1	.....	.....	...	..	..	.....
.....	.....	4462E	1	.....	4462.22E	3	2	2	.....
.....	.....	.....	....	.....	4469.39	7	6	3	.....
.....	4471.8	4471	6	4471.7	4471.8	5	6	5	4471.68, Helium.
.....	.....	4481E	2	.....	4481.6	3	8	8	4481.40, Magnesium.
.....	.....	4485	1	.....	4484.21	3	2	2	.....
.....	.....	.....	....	.....	4491.46E	4	4	3	4491.57, Fe 6.
.....	.....	.....	....	.....	4491.5	2	1	1	4491.57, Fe 6.
.....	.....	4518E	3	.....	4521.8E	3	3	3	4520.40, Fe 8. 4522.83, Fe-Ti-Eu 12
.....	.....	.....	....	.....	4557.5E	4	6	4	.....
.....	4568.6	.....	....	.....	.....	...	..	..	.....
.....	.....	4573	4w	.....	4571.5	5	1	1	.....
.....	.....	4586E	6n	.....	4583.76E	5.7	34	26	4584.02, Fe-V 15.
.....	.....	.....	....	.....	4583.93	2.8	27	19	4584.02, Fe-V 15.
.....	.....	4596	3	.....	.....	...	..	..	.....
.....	.....	4612	3	.....	.....	...	..	..	.....
4623E	.....	4628E	3	.....	4629.3E	6	7	4	4629.52, Fe-Co 12.
.....	4641.0	.....	....	.....	4640.7	5	3	3	.....
.....	4649.2	4647	6w	.....	4649.9	5	3	2	4649.5, Carbon.
.....	.....	.....	....	.....	4651.0	11	22	12	Blend.
.....	.....	.....	....	.....	4653.1	4	3	2	.....
.....	.....	.....	....	.....	4657.0E	7	12	6	.....
.....	4661.7	.....	....	4661.8	4662.3	6	10	6	Oxygen.
.....	.....	4664E	4n	.....	4668.4E	6	8	4	.....
4673.5	4675.3	.....	....	4676.3	.....	...	..	..	Oxygen.
.....	4685.4	4681	3n	4686.0	4685.8	6	1	1	4685.98, Hydrogen.
.....	4712.8	4711	1	4713.3	4713.4	5	2	2	4713.31, Helium.
Obs.	Obs.	4861E	10	4861.5	4861.51E	24	71	80	4861.53, Hβ.
.....	.....	.....	....	.....	4861.58	1.3	8	7	4861.53, Hβ.
4920E	.....	.....	....	.....	4925.7E	5	3	2	4924.11, Fe 20.
4990	.....	5006	3	.....	.....	...	..	..	.....
5020	5023E	5020E	4d	.....	5018.3E	4	5	4	5018.63, Fe 15.
.....	.....	.....	....	.....	5018.1	4	4	4	5018.63, Fe 15.

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TABLE IV. ASSEMBLED WAVE-LENGTHS OF LINES IN THE SPECTRUM OF GAMMA CASSIOPEIAE—CONTINUED.

THE VISUAL REGION.

SHERMAN	SIDGREAVES		MERRILL	CURTISS				IDENTIFICATION, ETC.
WAVE-LENGTH (1) Å	WAVE-LENGTH (2) Å	INT. (3)	WAVE-LENGTH (4) Å	WAVE-LENGTH (5) Å	INT. (6)	NO. (7)	WT. (8)	(9)
5020	5020E	4d	5018.4E	5018.3E	4	2	2	5018.63, Fe 20
.....	5048	2	.....	5048	7	2	2	.....
.....	5104	2	.....	5106	8n	1	1	.....
.....	5160	3	.....	5162	4	2	2	.....
5167.5E	5170E	4	5169.0E	5169E	6	3	3	5167.68, Fe —, b_4.?
.....	.....	..	.....	.....	..	..	..	5169 16, Fe 15, b_3.?
.....	.....	..	.....	5168	2	1	1	Reversal.
.....	.....	..	.....	5174	4	1	1	5172.9 ?, b_2.?
.....	.....	..	.....	5180	5	2	2	.....
.....	.....	..	.....	5186E	3	1	1	5183.8 ?, b_1.?
.....	5214	3n	.....	5214E	3	2	1	.....
.....	.....	..	.....	5234E	4	2	1	5234.8 — 10, Young.
.....	5256	3n	.....	.....	..	..	..	.....
.....	.....	..	.....	5278E	3	3	2	5276.15, Cr—Fe 10.
.....	.....	..	.....	5281	5	2	2	.....
.....	.....	..	.....	5286E	3	2	1	.....
.....	5295	4n	.....	5294	6	2	1	.....
5309.8E	5316E	2	5316.4E	5317E	4	3	3	5316.86, Fe—Co 12.
.....	.....	..	.....	5327E	2	1	1	Blend of Fe lines.
.....	5350	4n	.....	5355	10	1	1	.....
.....	5376	3	.....	5376	7	2	2	.....
.....	5411	3	.....	.....	..	..	..	.....
5422E	.....	..	.....	5426E	3	2	1	5429.91, Fe 10.
.....	5524	3	.....	5523	5	2	2	.....
5557.5E	5540E	2	.....	5537E	4	1	1	5535.07, Fe 12, Young.
.....	5576E	2	.....	.....	..	..	..	....
5760	.....	..	.....	5760	5	1	1	.....
.....	.....	..	.....	5852	6	2	1	.....
Obs.	.....	..	Obs.	5876E	5	3	3	5875.87, Helium D_3.
.....	.....	..	.....	5882	6	2	1	.....
6160E	.....	..	.....	6149E	4	1	1	Blend, Fe 5, Young.
6280	.....	..	.....	6278	12	3	2	.....
.....	.....	..	.....	6307	6	3	3	.....
.....	.....	..	.....	6320E	4	2	2	6318.2, Fe—Ca 3, Young.
.....	.....	..	.....	6340	9	1	1	.....
6356E	.....	..	.....	6350E	4	1	1	....
.....	.....	..	.....	6367	6	2	2	.....
Hα	.....	..	Hα	Hα	40	4	—	6563.

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Young, are due to Rowland, the intensities to S. A. Mitchell, as observed by him in the flash spectrum. The other wave-lengths were obtained from various sources. The letter E, following a wave-length, marks an emission line; the letter A, an absorption line. Wave-lengths with no letters following correspond also to absorption lines.

The wave-lengths in Table IV are based on the assumed values for the lines, Hδ, Hγ, and Hβ, given in the last column of this table. For the better determined lines the probable errors resulting from comparison among the values from the different plates are given in Table V. Uncertainties in the interpolation curve employed and in the assumed wave-lengths would increase these probable errors by a few hundredths of an angstrom.

TABLE V. WAVE-LENGTHS AND PROBABLE ERRORS OF SELECTED LINES.

LINE	CORRECTED WAVE-LENGTH Å	PROBABLE ERROR Å
K	3933.86 A	± 0.028
Hε	3970.19 E	± 0.060
Hε	3970.26 A	± 0.036
He	4026.56 A	± 0.027
Hδ	4101.92 E	± 0.017
Hδ	4101.90 A	± 0.014
Hγ	4340.64 E	± 0.009
Hγ	4340.66 A	± 0.013
Fe	4583.76 E	± 0.058
Fe	4583.93 A	± 0.065
Hβ	4861.51 E	± 0.013
Hβ	4861.58 A	± 0.058

The identifications suggested in Table IV for the emission lines seem well established. That these emission lines, probably without exception, correspond to strong chromospheric lines is well brought out. The marked prominence of iron emission in these lines is also evident.

In view of the references of early observers to the presence of the b group of magnesium in the spectrum of γ Cassiopeiæ, the data in Table IV referring to this region are of some interest. There seems to be no question as to the presence of an iron emission line at λ 5169.16 Å; and since this line is normally about six angstroms wide, b_4, b_3, and possibly b_2, if present, would be blended or lost in it. Probably also the emission line, b_1, is included in the emission line measured at λ 5186 in such a way that it can not be distinguished if it exists. It is possible then that magnesium emission does contribute to the lines which have been observed at λ 5169 and λ 5186. But in that case we would expect to find at least the first of these lines exceptionally wide, whereas the measured width of 6.6 Å exceeds the normal width of 6.1 Å, taken from Plate II, by a relatively small amount. Sidgreaves considered that the identification of λ 5169 Å as a magnesium group was confirmed by the existence of λ 4481 Å of this element as an emission line in the spectrum of this star. However I have not been able to find this emission line on my plates. Keeler inclined to the belief that the b group was dark in this spectrum. Aside from the occurrence of an absorption line at λ 5174 Å, which might involve b_2, there seems to be no evidence gained from the plates of this Observatory in support of this idea. It seems probable that the b group of magnesium is not present in this spectrum.

The line-intensities assigned by Sidgreaves and the writer are based on scales so different that intercomparison is not readily made. The writer's intensity numbers are based on the scale used in his previous papers. In general these numbers for absorption lines are several times those assigned by Sidgreaves, but discrepancies occur which suggest differences in interpretation of the observed features. For the emission lines Sidgreaves has based his intensities on the excess of density over that of the neighboring continuous spectrum. Thus the wide band at Hε is assigned intensity zero. The writer has estimated his intensities of emission lines on the basis of their extent and density, assuming that they are superposed upon absorption as the line structures indicate. The two systems lead to widely different results making numerical comparison difficult.

The wide absorption lines in the spectrum of γ Cassiopeiæ are too weak and ill-defined to permit of discriminating studies of intensity

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TABLE VI. THE ANN ARBOR OBSERVATIONS.

NO. OF PLATE	DATE, G. M. T.		Hδ, Hγ, AND Hβ LINES				E—A		EMISS. INT.			EMISSION WIDTH		
			VEL.	RESID.	NO.	WT.	VEL.	WT.	Hδ	Hγ	Hβ	Hδ	Hγ	Hβ
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	1911	d	km.	km.			km.					Å	Å	Å
134	July	17.895	+ 1.1	+ 8.4	4	4	— 3	1	15	22	25	4.01n	4.31S	5.56s
197	Aug.	30.812	— 9.7	— 2.4	5	8	± 0	4	..	..	25	3.92W	4.31w	5.83w
198		30.830	— 5.4	+ 1.9	5	5	+ 2	2	..	25	20	3.99n	4.46s	5.48n
199		30.841	— 9.0	— 1.7	5	8	± 0	3	..	25	20	.....	4.84n	5.18w
200		30.855	— 9.6	— 2.3	5	8	— 1	3	20	25	25	3.89n	4.01s	5.03w
210	Sept.	19.797	— 6.0	+ 1.3	2	2	....	..	..	20	25	.....	4.29s	5.12n
211		19.803	—13.7	— 6.4	5	5	+ 3	2	15	25	25	3.95w	4.05n	5.24w
212		19.815	— 1.8	+ 5.5	5	5	+ 3	2	20	25	20	3.71W	4.18w	4.77W
214		22.836	—14.5	— 7.2	5	7	+ 6	3	25	25	20	3.86w	4.10n	4.29W
215		22.848	— 6.0	+ 1.3	5	6	— 9	2	18	25	25	3.80s	4.24S	4.77w
221		23.824	—10.9	— 3.6	5	6	+ 7	2	25	25	25	4.07w	4.31n	4.85w
222		23.835	—10.6	— 3.3	5	5	— 1	2	15	20	25	4.31s	4.41S	5.51n
229		27.809	— 2.4	+ 4.9	4	4	+ 4	1	..	20	25	.....	4.07s	4.71n
230		27.820	—12.2	— 4.9	4	4	+ 16	1	..	25	25	.....	4.31s	4.71n
235	Oct.	2.757	—10.6	— 3.3	3	2	— 3	1	..	20	22	.....	4.39n	4.94w
236		2.771	— 1.7	+ 5.6	3	3	— 7	1	..	20	20	.....	4.14w	4.65W
241		7.784	—11.6	— 4.3	4	4	— 3	1	..	20	20	.....	4.01w	4.94W
242		7.792	— 6.6	+ 0.7	5	7	+ 6	2	25	20	20	4.01w	4.31w	4.50W
246		11.762	— 6.6	+ 0.7	4	4	— 6	1	18	25	25	4.25n	4.14n	5.36w
247		11.772	—11.5	— 4.2	4	4	+14	1	16	25	25	3.96n	4.45s	5.27w
254		13.657	—10.3	— 3.0	3	3	— 2	2	..	20	25	.....	4.48s	5.21w
255		13.679	—15.–	— 7.–	1	1	....	..	..	..	25	.....	.....	5.62s
264		18.759	— 3.2	+ 4.1	5	5	— 7	2	20	20	25	3.38n	4.37s	5.00w
265		18.765	— 0.7	+ 6.6	4	6	—12	2	20	25	18	4.38w	4.64n	5.36w
282		27.727	—10.9	— 3.6	4	3	—16	1	15	20	25	3.98w	4.28w	5.48w
283		27.732	— 7.3	± 0.0	5	5	— 5	2	16	25	25	4.02n	4.31n	5.15w
	1912													
1108	Sept.	26.785	+ 0.4	+ 7.7	4	4	± 0	1	..	20	25	.....	4.31n	5.33n
1447	Nov.	10.726	—16.6	— 9.3	2	1	....	..	..	20	25	.....	4.62S	5.56s
1448		10.747	—12.0	— 4.7	4	4	+ 9	1	..	25	25	3.99n	4.33s	5.33w
1507*		27.638	— 9.0	— 1.7	4	3	+ 3	1	..	20	25	3.80s	4.18S	5.51S
1508*		27.646	—11.–	— 4.–	1	1	....	..	..	..	25	.....	.....	5.62S
1530		30.620	— 7.1	+ 0.2	3	3	....	..	16	25	25	3.89n	4.69n	4.71w
1531		30.632	— 5.9	+ 1.4	5	8	+ 3	3	20	..	20	4.51w	4.45w	4.47w
1537*	Dec.	4.662	— 3.6	+ 3.7	5	4	— 1	3	20	20	25	3.81n	4.79n	4.88w
1538*		4.667	—16.0	— 8.7	5	5	+12	2	20	20	22	4.01n	4.48n	4.88w
1540	Dec	7.552	— 3.2	+ 4.1	4	4	± 0	2	20	20	25	3.78n	4.24s	4.97n
1541†		7.560	—14.7	— 7.4	4	3	— 2	2	..	30	20	.....	4.65n	5.36w
1544		8.592	—10.9	— 3.6	5	5	— 1	2	20	20	18	4.05w	4.37w	5.06w
1545		8.615	— 4.8	+ 2.5	3	4	— 9	1	..	30	25	.....	4.07w	5.39W
1550*		11.660	— 2.0	+ 5.3	5	7	+ 2	3	..	25	20	3.87w	4.05n	4.68W

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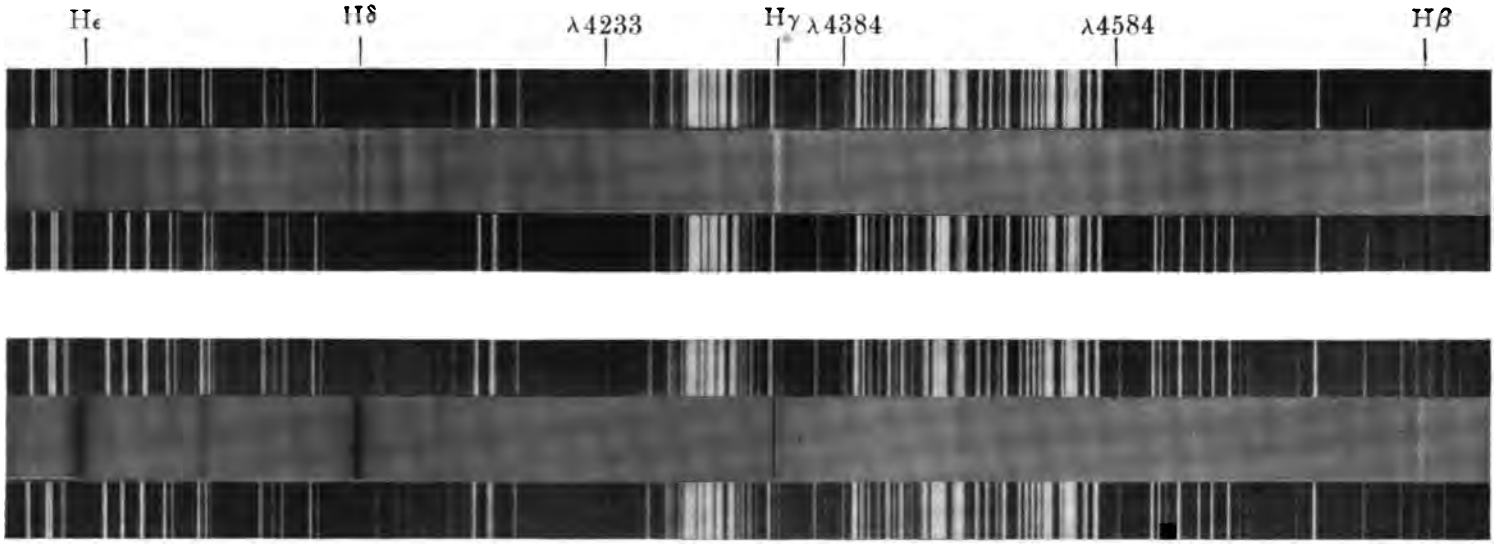

Plate C. Spectra of the Stars, Gamma Cassiopeiae, 1914, Feb. 19, and Beta Monocerotis, 1914, Nov. 22, with Titanium Spark Comparison

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TABLE VI. THE ANN ARBOR OBSERVATIONS—CONTINUED.

NO. OF PLATE	DATE, G. M. T.		Hδ, Hγ, AND Hβ LINES				E—A		EMISS. INT.			EMISSION WIDTH		
			VEL.	RESID.	NO.	WT.	VEL.	WT.	Hδ	Hγ	Hβ	Hδ	Hγ	Hβ
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	1912	d	km.	km.			km.					Å	Å	Å
1552		14.610	—10.6	— 3.3	4	4	....	..	25	25	25	3.93n	4.56s	5.12n
1553†		14.615	—15.2	— 7.3	5	4	+ 3	1	20	25	25	3.95s	4.75S	4.88n
1564		22.602	— 5.7	+ 1.4	5	8	+ 1	3	25	25	20	.....	4.65w	4.65W
1565		22.606	—10.2	— 2.9	4	4	+ 5	1	..	30	20	.....	4.16n	5.00w
	1913													
1572	Jan.	12.600	— 9.5	— 2.2	4	4	— 8	1	..	25	20	.....	4.46w	4.85W
1573		12.621	— 8.5	— 1.2	5	6	— 4	3	30	25	20	4.08w	4.71n	4.56w
1574		12.626	— 3.3	+ 4.0	4	4	+ 6	1	..	20	25	.....	4.20w	5.12w
1600	Feb.	8.587	—16.0	— 8.7	5	6	+13	2	20	25	25	4.51w	4.39n	5.62w
1601		8.603	—13.1	— 5.8	4	4	—22	1	..	20	20	.....	4.16w	4.44W
2244	Aug.	24.833	— 6.4	+ 0.9	5	5	+ 2	2	20	25	28	3.53s	4.64s	5.42n
2245		24.843	— 6.4	+ 0.9	4	4	—11	1	..	25	25	.....	4.41s	4.53w
2351*	Oct.	4.703	—13.3	— 6.0	2	2	....	..	..	22	25	.....	4.52n	5.24w
2352‡		4.715	— 5.8	+ 1.5	5	4	+ 3	2	20	25	25	4.02n	4.41n	4.82w
2362		9.672	— 1.4	+ 5.9	4	3	— 9	1	..	20	20	.....	4.26n	4.77w
2363		9.687	— 9.0	— 1.7	5	7	± 0	3	25	30	25	3.68n	4.43n	5.33w
2378		12.720	— 2.9	+ 4.4	5	6	+ 1	2	16	25	25	4.01n	4.39s	4.85n
2379		12.739	— 4.6	+ 2.7	5	5	+ 4	2	18	20	25	4.19n	4.50s	4.71n
2394		18.679	— 6.3	+ 1.0	4	4	....	..	..	30	20	.....	4.62n	4.56w
2395		18.698	+ 6.7	+14.0	5	5	—17	1	25	25	25	.....	4.90s	5.56n
2396		18.758	— 6.5	+ 0.8	4	3	— 6	1	..	25	15	.....	4.62w	4.71w
2402		25.720	—15.9	— 8.6	4	2	+19	1	25	30	30	.....	.....	5.27s
2403		25.740	+ 0.5	+ 7.8	5	6	+ 2	2	25	25	25	3.86w	4.16n	5.42w
2414	Nov.	1.677	— 5.1	+ 2.2	5	6	+ 4	3	15	40	30	3.56n	4.64n	5.74w
2415		1.695	— 0.5	+ 6.8	4	5	± 0	3	∴	25	25	.....	4.09n	4.56w
2524	Dec.	27.598	— 9.7	— 2.4	5	7	± 0	3	20	30	20	4.02w	4.48n	4.68W
2525		27.621	—14.4	— 7.1	5	5	+10	2	25	25	25	3.62n	4.12s	5.49w
	1914													
2530	Jan.	1.555	— 3.4	+ 3.9	5	7	—12	3	16	30	25	3.68w	4.62n	4.94n
2564	Feb.	5.604	— 3.0	+ 4.3	2	2	....	..	..	20	..	.....	4.35n	5.03n
2583		19.570	— 9.7	— 2.4	3	3	—16	2	25	..	25	3.75s	.....	5.42w
2584		19.583	+ 1.0	+ 8.3	5	8	+ 5	4	20	25	20	3.64w	4.58w	4.82W

In addition to the spectrograms listed above, there were used in wave-length determination: Plates 2511, of 1913, December 18; 3037, of October 24, 1914; 3077 and 3078, of December 31, 1914. Plates 2511, 2564 and 3037 were sensitized to visual light. Plates 3077 and 3078 were made with a long slit giving a wide spectrum similar to that usually made with the objective prism. Plates with numbers followed by asterisks (*) in column one were made by Mellor, single daggers (†) by Lindsay, double dagger (‡) by Merrill. All the remaining plates were made by Curtiss.

variations. Indeed in a long series of plates a relatively small number will be found to have the nice combination of photographic conditions necessary to bring these lines out. The metallic emission lines are also not well adapted to intensity studies, since their edges are often not clearly marked and in density they exceed the continuous spectrum but little. On the other

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hand the emission and central absorption components of Hδ, Hγ, and Hβ are fairly suitable for studies of intensity changes.

In columns nine, ten and eleven of Table VI the writer's estimates of the intensities of the emission lines of Hδ, Hγ, and Hβ are given for each spectrogram. A glance will show that some range exists among the estimates of any one line, but such variations appear between plates made in rapid succession on the same night. Evidently the variations found here are ascribable to uncertainties of photographic registration and of intensity estimates. On the other hand these estimates do indicate that no great variations, such as those reported by visual observers, are recorded on these plates. Further, in Table VII,

TABLE VII. MEAN INTENSITY OF HYDROGEN EMISSION LINES.

YEARS AND MONTHS	INTENSITIES					
	Hζ	Hε	Hδ	Hγ	Hβ	Hα
1911, July–Oct.	..	..	19	23	23	..
1912, Sept.–1913, Feb.	..	..	21	24	23	..
1913, Aug.–1914, Feb.	..	..	21	26	24	..
1911–1914	6	14	20	24	23	40

where the intensity estimates are grouped into means by seasons, there appears no evidence of appreciable change in the average intensities of these emission lines from year to year.

the case of Hβ being due to the decreased sensitivity of lantern slide plates in this region. This increase in intensity of hydrogen emission with wave-length would be still more marked here if the excess of brightness over that of the neighboring continuous spectrum had formed the basis of intensity estimates as in the case of former observations.

The individual intensity estimates for the central hydrogen absorption and for the absorption lines, K and λ 4026, which were measured many times on these plates, are not reproduced here. It is sufficient to say that they do not support any hypothesis of variation. The means by years and the final means are found in Table VIII. The decrease in intensity of the central hydrogen absorption with increasing wave-length is well brought out though to some extent the effect of varying dispersion is involved here. There seems to be no evidence of marked intensity variations in the same line from year to year.

In view of the suspicion, expressed by Sidgreaves, of variation in the intensities of metallic emission lines in the spectrum of γ Cassiopeiæ, interest will be heightened in a tabulation of the data of the Ann Arbor spectrograms bearing on this point. In Table IX, have been collected the estimated intensities, on the scale employed for the hydrogen lines, of the emission and central absorption lines of λλ 4233, 4384 and 4584, the latter of which was mentioned specifically by Sidgreaves. Evidently there is no indication of

TABLE VIII. MEAN INTENSITIES OF CENTRAL HYDROGEN ABSORPTION, K AND λ 4026.

YEARS AND MONTHS	INTENSITIES						
	Hζ	Hε	Hδ	Hγ	Hβ	K	λ4026
1911, July–Oct.	...	3.3	3.1	2.4	1.–	3.7	12.2
1912, Sept.–1913, Feb.	...	3.8	3.4	2.8	1.3	3.8	11.4
1913, Aug.–1914, Feb.	...	3.5	2.9	2.5	1.4	3.4	12.2
1911–1914	4.–	3.5	3.1	2.5	1.3	3.6	11.9

The mean intensities for all hydrogen emission lines, in the last line of Table VII, bring out the well known increase of intensity of these lines from Hζ to Hα, the apparent exception in

real variation here from plate to plate and no certain indication of change from year to year. Apparently if these lines do vary, the range must be small or the period long; or possibly the fluctu-

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TABLE IX. INTENSITIES OF METALLIC EMISSION AND ABSORPTION LINES.

PLATE	DATE G. M. T.	λ 4233 INTENSITY		λ 4384 INTENSITY		λ 4584 INTENSITY	
		EMISS.	ABS.	EMISS.	ABS.	EMISS.	ABS.
	1911						
198	August 30.83	5	2.5	5	...	5	3.5
199	30.84	6	...	4	...	5	3
200	30.85	5	2	4	...	7	2
211	September 19.80	...	...	...	...	...	5
212	19.81	...	...	5	3	8	4
214	22.84	4	...	...	...	6	...
221	23.82	5	...	7	2	6	...
229	27.81	...	...	...	...	5	2
230	27.82	...	3	...	...	5	2.5
236	October 2.77	...	3.5	...	...	...	...
242	7.79	6	3	5	3	5	...
264	18.76	...	...	...	...	6	2
265	18.77	...	...	8	...	...	...
283	27.73	...	...	...	...	...	2
	1911 MEAN	5.2	2.8	5.4	2.7	5.8	2.9
	1912						
1108	September 26.78	...	...	...	...	5	2
1530	November 30.62	...	...	...	...	5	3
1531	30.63	5	3.5	...	...	5	2
1538	December 4.67	...	3.5	...	...	...	...
1540	7.55	...	...	...	...	5	2
1541	7.56	...	...	...	...	5	...
1544	8.59	7	2.5	4	...	5	...
1545	8.61	5	...	...	...	5	...
1550	11.66	5	...	...	...	8	3
1552	14.61	...	...	...	...	5	...
1564	22.60	5	5	4	...	6	3
1565	22.61	5	2	...	...	...	...
	1913						
1572	January 12.60	...	...	5	...	5	2
1573	12.62	5	...	5	...	8	...
1574	12.63	4	2	4	3	5	3
1600	February 8.59	...	2.5	5	...	8	4.5
1601	8.60	4	4	4	4	4	3
	1912-1913 MEAN	5.0	3.1	4.4	3.5	5.6	2.8
2244	August 24.83	...	2.5	...	...	...	...
2245	24.84	...	...	...	...	6	3
2351	October 4.70	...	3.5	...	...	...	...
2363	9.69	5	...	5	...	5	2
2394	18.68	...	...	5	...	...	...

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TABLE IX. INTENSITIES OF METALLIC EMISSION AND ABSORPTION LINES. CONTINUED.

PLATE	DATE G. M. T.		λ 4233 INTENSITY		λ 4384 INTENSITY		λ 4584 INTENSITY	
			EMISS.	ABS.	EMISS.	ABS.	EMISS.	ABS.
2390	October	18.76	...	...	...	...	5	...
2414	November	1.68	6	3	...	...	...	...
2524	December	27.60	6	...	5	...	6	...
2525		27.62	...	...	6	...	6	3
	1914							
2530	January	1.56	5	3	5	2.5	5	3
2584	February	19.58	6	...	5	...	5	2.5
	1913-1914	MEAN	5.6	3.0	5.2	2.5	5.4	2.7
	1914							
3037	October	24.69	5	2.5	5	...	6	3
3038		24.71	5	2	5	3	6	2.5
3039		24.73	6	2	5	...	...	...
3077	December	31.60	4	...	6	2	6	3
3078		31.64	5	2.5	5	2	6	...
	1914, Oct.-Dec.	MEAN	5.0	2.2	5.2	2.3	6.0	2.8

ations are spasmodic. As for the early evidence of change, Sherman's observation of a dark λ 5020 may easily have been an error of record or he may have observed an absorption border of this line. Further, in view of the relative faintness of these metallic emission lines and in view of the structure found in them, it seems reasonable that the apparent variations of λ 4584, observed by Sidgreaves, are ascribable to the uncertainties of photographic registration. That Miss Maury referred to λ 5023 and not to λ 4584 as one of the strongest of the emission lines in the spectrum of γ Cassiopeiæ, aside from those of hydrogen, is explained if we note that this observer did not identify λ 4584 as an emission line but did consider it erroneously as a bright region between two Orion lines. Apparently the probability of the reality of the suspected marked changes in the metallic emission lines in this spectrum is small.

There remains the possibility that changes may be taking place in the relative brightness of the two components into which the emission lines are divided by the central dark reversal. Such changes are described on a later page of this volume in connection with the bright line star, H. R. 985. Changes, possibly of this character, are well known phenomena of the spectrum of β Lyræ. The Ann Arbor spectrgrams of γ Cassiopeiæ have been examined with a view of bringing out such changes if they exist. To this end the intensity of each component of the emission lines of Hζ, Hε, Hδ, Hγ, λ 4584 and Hβ, referred to the neighboring continuous spectrum, has been estimated on each plate, when available. These individual estimates are not reproduced here. It is sufficient to say that they do not support any hypothesis of variation. The means by years and the final means for the set are found in Table IXa. Variations of the means for any component and of the relative brightness of two components of the same line from year to year are not too great to be accounted for entirely by uncertainties of estimation and of photographic registration. Apparently this type of variation is not present in the spectrum of γ Cassiopeiæ, so far as these direct relative estimates are concerned.

Line Structures. Several observers have pointed out that the hydrogen lines in the spec-

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trum of γ Cassiopeiæ are in general doubly reversed. Narrow central absorption divides a wide emission line and this in turn is centrally superposed upon an extremely wide absorption background. The Ann Arbor spectrograms bring out the fact that this structure extends also to the other emission lines. In Table IV are found several measures of central absorption in metallic emission lines, and in Table X, measures of the absorption border of λ 4584.

TABLE IXA. INTENSITY OF EMISSION COMPONENTS RELATIVE TO THE CONTINUOUS SPECTRUM.

YEARS AND MONTHS	Hζ		He		Hδ		Hγ		Hβ		λ 4584	
	V	R	V	R	V	R	V	R	V	R	V	R
1911, July–Oct.	1.00	1.00	1.05	1.04	1.16	1.15	1.47	1.47	1.89	1.88	1.17	1.17
1912, Sept.–1913, Feb.	1.00	1.00	1.06	1.05	1.22	1.19	1.46	1.44	1.88	1.91	1.19	1.20
1913, Aug.–1914, Feb.	1.02	1.04	1.08	1.07	1.23	1.23	1.43	1.43	1.87	1.90	1.15	1.16
1914, Oct.–1916, Jan.	1.02	1.02	1.03	1.03	1.20	1.22	1.40	1.36	1.83	1.83	1.15	1.15
1911–1916	1.01	1.01	1.05	1.05	1.20	1.20	1.44	1.43	1.87	1.88	1.17	1.17

The wide absorption borders on either side of the hydrogen emission lines are well known features of spectra of this class. In the case of β Lyræ a study of these borders has led the writer to conclude that they are made up of a group of narrow lines. In the case of γ Cassiopeiæ the same conclusion is suggested; and the less complex character of the hydrogen lines in this star has tended to facilitate the study of this line structure.

For all of the emission lines on all of the plates of Table VI the writer has measured the positions of the components of the absorption borders where settings were possible. At best these lines are difficult. Frequently they are blended, making identification uncertain. In general, when forming means, it is not possible to pick out the single lines from the blends. After some experiment the writer has adopted the expedient of plotting the frequency curves of the measures of such lines, using for abscissæ distances from the centers of the emission lines, and for ordinates the number of measures made within selected narrow limits. The positions of the maxima of such curves were assumed to correspond to the positions of components of the group, thus making possible the determination of the wave-lengths of these lines.

A frequency curve of this character for the Hδ line is shown in Plate I of this paper. The abscissæ represent distances from the center of the emission line in terms of micrometer turns. The ordinates are proportional to the number of lines measured on all plates between limits of five microns on either side of the plotted points.

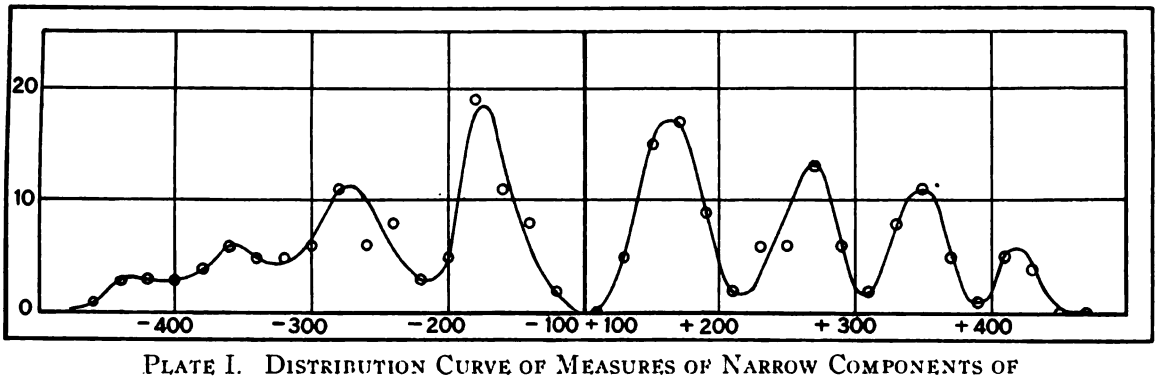

PLATE I. DISTRIBUTION CURVE OF MEASURES OF NARROW COMPONENTS OF ABSORPTION BORDERS OF Hδ

The dispersion at the Hβ line was too small on these plates to permit of consistent resolution of the component lines in the absorption

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borders, measured at Hδ and Hγ. The relative weakness of the spectrum at Hε and the ill-defined appearance of this line have combined to interfere with measurements at this point. But frequency curves have been constructed for each of the lines in Table IX and from such curves the data of this table have been obtained. In this table, R and V with subscripts refer to components in the absorption borders on the side of the emission line to the red and to the violet respectively. The quantities in the body of the table are distances in angstroms from the center of emission lines to associated border components.

The average values of the distances of the inner components, V_1 and R_1, from the center of the corresponding emission lines are proportional to the wave-length of these lines within limits of one or two tenths of an angstrom. This is in agreement with the writer's conclusions in connection with these features in the spectrum of β Lyræ. The average distances of the components in the borders of the Hγ in β Lyræ, from the center of the associated emission line were 2.68, 3.94, 5.37, and 7.01 angstroms, beginning with the inner components. These distances for β Lyræ average only one or two tenths of an angstrom less than the corresponding quantities for γ Cassiopeiæ and the general resemblance between these quantities for these two stars is clearly suggestive.

TABLE X. DISTANCES OF BORDER COMPONENTS FROM CENTERS OF EMISSION LINES.

	Hε	Hδ	Hγ	λ4584	Hβ
R_4	.....	6.30Å	.....	.....	.....
R_3	.....	5.10	5.51Å	.....	.....
R_2	.....	4.05	4.28	.....	.....
R_1	2.26Å	2.47	2.91	3.42Å	3.54Å
V_1	.....	2.59	2.80	3.30	3.54
V_2	.....	4.10	4.44	.....	.....
V_3	.....	5.21	5.58	.....	.....
V_4	.....	6.33?	6.94	.....	.....

Width of Emission Lines. Measures of the separation of the parts of the reversed emission lines in γ Cassiopeiæ and other similar stars have been published by Merrill.[2] These measures are of especial value because the variation of this separation from line to line and from spectrum to spectrum is obviously dependent upon physical conditions in these stars. For the same reason it is evidently desirable also to have available measures of the actual width of the emission lines in these spectra if such measures can be made with an accuracy sufficient to bring out existing variations.

During his study of the spectrograms of γ Cassiopeiæ, the writer has measured the width of the emission lines observed in all cases where the edges of such lines seemed well defined. Frequently, and especially in the cases of Hδ and Hγ, there was a sharp density gradient at the edge of these lines, where the emission component suddenly gave way to the wide absorption border. It was on this narrow gradient that the writer set the micrometer thread in determining the position of the edge of the emission line. The results of these measures of the width in angstroms of the emission lines of Hδ, Hγ, and Hβ are given for each spectrogram in the last three columns of Table VI. The mean results for these and other lines are found in Tables XI, XII and XIII.

Before making any deductions from the measures of the widths of the emission lines it seemed advisable to determine to what extent, if any, the measures of these widths were affected by over- and under-exposure. Accordingly the plates were surveyed with this in view and the spectrum in the neighborhood of each measured emission line was designated according to density as very weak, weak, normal, strong and very strong, indicated in Table VI by the letters, W, w, n, s, S respectively. For each line all the measures in each density class were combined into means with results as tabulated in Table XI. From the data of this table it appears that only in the case of the Hβ line is there a clear dependence of the measured width of the line upon the density of the spectrum, and, as would be expected, the measured width increases with spectral density. In the visual region the Hα line exhibits this effect still more strongly, having a bright core with fainter edges which increase greatly the apparent width of

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the line with lengthened exposure. But, allowing for the uncertainties affecting the results from these two lines, there appears to be much material in the present measures for comparative studies of the widths of the emission lines in the spectrum of γ Cassiopeiæ.

TABLE XI. AVERAGE WIDTHS OF EMISSION LINES GROUPED ACCORDING TO DENSITY.

DENSITY	WIDTH			
	Hδ	Hγ	λ 4584	Hβ
Very Weak	3.81Å	.....	.....	4.74Å
Weak	4.04	4.35Å	5.52Å	5.09
Normal	3.87	4.46	5.29	5.09
Strong	3.86	4.39	5.29	5.42
Very Strong	.....	4.39	.....	.....

Comparing the measured widths in the last three columns of Table VI, for any line on different plates there seems to be no evidence of a real variation in this quantity. Discrepancies, very probably due to uncertainties of measurement, between measures of the width of the same line on plates made within a few minutes of each other are of the same order as the differences between results from plates of different nights, indicating that these differences may be attributed to accidental errors of measurement.

Referring to the mean values of Table XIII, it is obvious that the width of the emission lines increases with increasing wave-length. Merrill[2] has examined the separation of the two parts of the emission lines in ϕ Persei and has found that this "separation apparently varies linearly with the wave-length or a little more rapidly, though this is not borne out by the Hα line." To facilitate the determination of the relation between emission line widths and wave-length in the spectrum of γ Cassiopeiæ the plot of Plate II has been drawn with the former as ordinates and the latter as abscissæ. Apparently a linear relation is strongly suggested here. The equation of this line is $d\lambda = 0.001885\ (\lambda - 2020)$. Making reasonable allowance for increase in width of lines due to slit width and diffraction, the constant, 2020, becomes smaller and may approach zero.

Relative Position of Emission and Central Absorption. In the cases of a number of the Class B spectra with reversed emission lines the narrow absorption appears to be centrally superposed upon the associated bright component. But in several cases the division of the emission line is clearly not symmetrical. In every case it seems desirable that the relative positions of the centers of the emission component and of the reversal should be determined quantitatively in so far as the measurability of the lines will warrant. Such data may throw light upon circulation, pressure and temperature conditions, and particularly upon pressure and temperature gradients, in the atmosphere of stars having spectra of this kind.

On sixty-one of the seventy plates in Table VI, measures were made of the relative positions of the centers of the emission lines and of the

TABLE XII. MEAN WIDTHS OF EMISSION LINES BY SEASONS.

YEARS AND MONTHS	WIDTHS OF EMISSION LINES									
	Hδ	NO. OF MEAS.	λ 4233	NO. OF MEAS.	Hγ	NO. OF MEAS.	λ 4584	MEAS. NO. OF	Hβ	NO. OF MEAS.
1911, July–October	3.96Å	17	4.48Å	3	4.30Å	25	5.23Å	7	5.10Å	26
1912, Sept.–1913, Feb.	4.01	13	4.18	3	4.43	22	5.34	11	5.04	23
1913, Aug.–1914, Feb.	3.80	12	3.73	4	4.46	19	5.32	5	5.04	21
1911–1914	3.94	42	4.09	10	4.40	66	5.30	23	5.06	70

Means of emission line widths by seasons are found in Table XII and the final means for all lines measured are given in Table XIII. Of variation from season to season there seems to be no evidence in Table XII.

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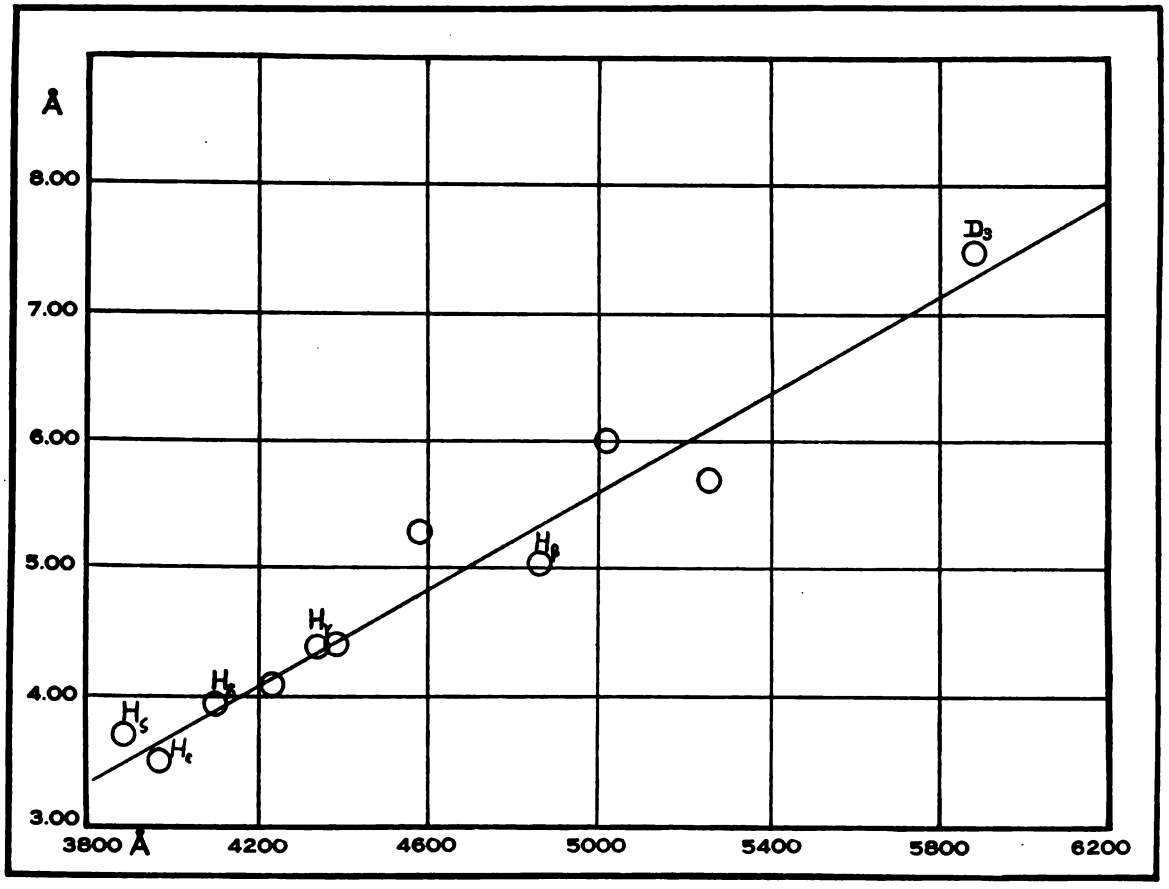

PLATE II. VARIATION OF WIDTH OF EMISSION LINES WITH WAVE-LENGTH.
EQUATION OF LINE, $\Delta\lambda = 0.001885\ (\lambda - 2020)$

TABLE XIII. FINAL MEANS OF MEASURES OF WIDTHS OF EMISSION LINES.

WAVE-LENGTHS	WIDTHS	MEAS.	REMARKS
3889	3.7-Å	1	Hζ
3970	3.5-	4	Hε
4102	3.94	42	Hδ
4233	4.09	10	Fe
4341	4.40	66	Hγ
4384	4.42	5	Fe
4584	5.30	23	Fe
4862	5.06	70	Hβ
5018	6.0±	4	Fe
5260	5.7±	5	Group of three lines. λ's 5169, 5278, 5317.
5876	7.5±	1	D_3
6563	9.	2	Width of core. Hα

associated narrow reversals. The results expressed in kilometers per second for each plate are given with accompanying weights in columns 7 and 8 of Table VI in the sense, emission — absorption (E — A). Only the hydrogen lines, Hε, Hδ, Hγ, and Hβ were used in these determinations and no measure was included unless both the emission and the absorption components were observed, thus eliminating errors due to assumption of wave-lengths. The values of E — A were combined into means for each plate since, as appears below, the mean values indicate that this quantity does not vary from line to line over limits that would be appreciable here. In measurements of the emission components of these lines settings were made on the edges and also upon the centers but in the reductions only the

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latter were used. Indeed these bisections of wide emission lines yielded more consistent results than did the measures on the narrow reversals.

The velocities in column 7 of Table VI, representing the relative displacements of emission and absorption components (E — A), in the hydrogen lines of γ Cassiopeiæ, apparently do not vary more than would be expected of such quantities when determined from relatively few spectral features, not well defined. That the variations found in these quantities are accidental is supported by the fact that the differences between the quantities, E — A, for plates made at intervals of a few minutes on the same night are no less on the average than those for plates of different nights, the average variation of E — A between successive determinations on the same night being ± 9.4 km. and for different nights, ± 8.7 km.

The lack of probability of the existence of a periodic variation in the quantities, E — A, for the hydrogen lines of γ Cassiopeiæ is well brought out also by a study of the distribution curve of the observations of column 7 of Table VI, as compared with the probability curve according to the method proposed by Schlesinger.[26] The data for such a distribution curve are found in Table XIV where, in column two is given the number of velocities for the velocity interval in the first column.

The graphical representation of the material in Table XIV is made in Plate III. In this plot the abscissæ are velocities; the ordinates are numbers of plates. The probability curve shown in this figure corresponds to a probable error for a single determination of E — A of ± 5.0 km. and has a maximum ordinate at + 0.4 km. These values of the probable error of a single determination and of the maximum ordinate of the probability curve are in satisfactory accordance with values of these quantities found in other ways.

TABLE XIV. DISTRIBUTION BY VELOCITIES OF E — A.

VELOCITY INTERVAL		NUMBER OF OBSERVATIONS
km.	km.	
— 27 to	— 23	0
22	18	1
17	13	3
12	8	6
7	— 3	9
— 2	+ 2	19
+ 3	7	15
8	12	3
13	17	3
18	22	1
+ 23	+ 27	0

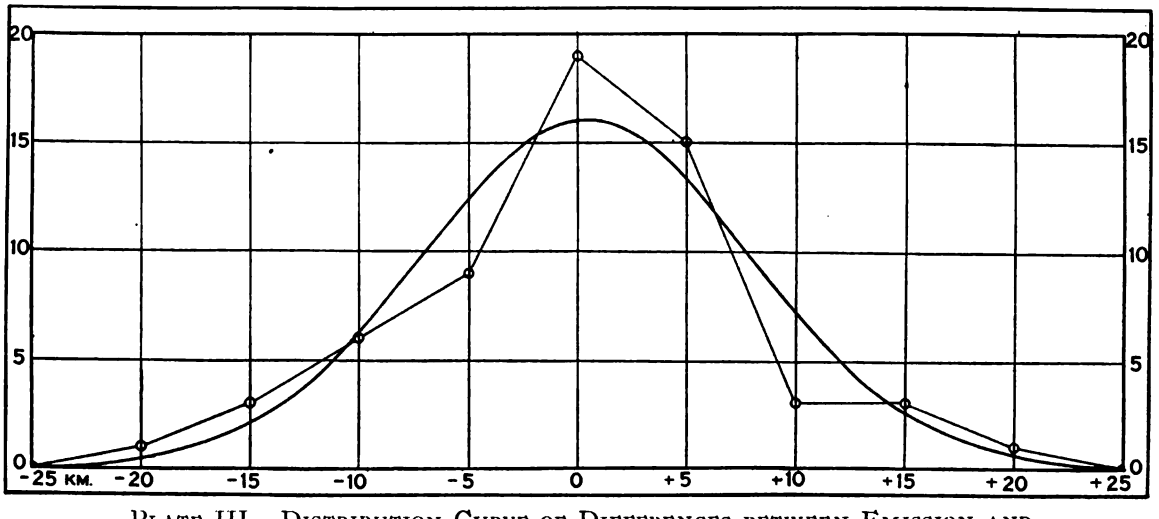

PLATE III. DISTRIBUTION CURVE OF DIFFERENCES BETWEEN EMISSION AND NARROW ABSORPTION LINE VELOCITIES COMPARED WITH ERROR CURVE

[26] *Astrophysical Journal*, Vol. 41, page 162.

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Further, the plotted points in the figure follow the probability curve as closely as might be expected in a case of this kind. There is no clear indication that the quantity studied here is variable. If a variation exists it is probably small, or it may consist in a slow change in a period counted in years.

To test the possibility of a small slow change from year to year, or of a variation from line to line in the quantity, E — A, for γ Cassiopeiæ, means of this difference have been formed by years for each hydrogen line, in Table XV. These means with their probable errors are found in this table; and in the last two lines, means and probable errors, both in kilometers and angstroms, for the whole set of plates. It is interesting to note that these means for Hε and Hγ are persistently negative and for Hδ positive, but such means are so nearly of the same order as their probable errors that there seems to be no reason to conclude that they indicate actual divergence from the mean for all the lines Considering then the means for all lines by years the results in this table indicate within limits of a few hundredths of an angstrom that the quantity, E — A, for the hydrogen lines considered did not vary from year to year. The final means for all the measures show that it is probable that the quantity, E — A, did not differ from zero, for the hydrogen lines measured, by an amount exceeding a few hundredths of an angstrom, or, in other words, that the absorption reversals did not deviate, on the average, from the centers of the emission lines of hydrogen by quantities greater than this.

Radial Velocities. Seventy radial velocities of γ Cassiopeiæ, derived from spectrograms made on thirty-five different nights, are found in column three of Table VI. These velocities have been made to depend upon the emission and central absorption components of Hδ and Hγ and upon the emission component of Hβ, since these features stand well above all others in this spectrum in point of measurability. This combination of emission and absorption components into means, which in general would not be wise, was justified by the indication, brought out above, that the absorption reversals occupy a fixed central position in the emission lines within limits that are negligibly narrow. As only the three lines mentioned were used here, the velocities in column three of Table VI and all the wave-lengths derived in this paper depend upon the following wave-lengths: 4101.92 Å for Hδ, 4340.63 Å for Hγ, and 4861.53 Å for Hβ; and through the use of both their emission and absorption components the wave-lengths for Hδ and Hγ have had a double influence on the results. The residuals for the velocities in column three of Table VI from a grand mean of —7.3 km. are found in column five and the corresponding number of lines measured with plate weights on the system used throughout this paper are found in the next two columns.

In view of the announcement by Hartmann and of the inclusion of γ Cassiopeiæ in lists of spectroscopic binaries, it is of interest to study the radial velocities of Table VI for indications bearing upon the question of variability. And apparently we may approach this investigation in several different ways. In the first place we may compare the differences between velocities from plates made at intervals of a few minutes on the same night with differences for successive

TABLE XV. RELATIVE DISPLACEMENTS; EMISSION MINUS ABSORPTION REVERSALS.

INTERVALS	DISPLACEMENTS; E — A				
	Hε	Hδ	Hγ	Hβ	MEANS
1911, July—Oct.	—6.– ± 10 km.	+ 1.1 ± 1.6 km.	— 1.2 ± 1.2 km.	+ 15.– ± 5.– km.	+ 0.32 ± 1.0 km.
1912, Sept.—1913, Feb.	— 2.– ± 10	+ 2.9 ± 2.1	— 1.8 ± 1.6	+ 8.3 ± 4.–	+ 0.77 ± 1.2
1913, Aug.—1914, Feb.	—7.– ± 7	+ 3.4 ± 2.0	— 2.7 ± 1.6	— 3.4 ± 4.–	— 0.44 ± 1.1
1911—1914	—5.5 ± 5.–	+ 2.3 ± 1.0	— 1.8 ± 0.8	+ 4.5 ± 3.–	+ 0.24 ± 0.63
1911—1914	— 0.072 Å	+ 0.032 Å	— 0.026 Å	+ 0.073 Å	+ 0.004 ± 0.010 Å

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plates of different nights. Apparently, if the star's velocity is variable in a period of a few days or weeks, the latter differences should exceed the former on the average. In Table XVI these differences are compared for each season and for the entire series of plates. On the average the

TABLE XVI. VELOCITY DIFFERENCE FOR SUCCESSIVE PLATES.

YEARS	DIFFERENCES FOR SAME NIGHT	DIFFERENCES FOR DIFFERENT NIGHTS
1911	± 5.1 km.	± 7.2 km.
1912-1913	4.9	6.8
1913-1914	7.6	5.9
1911-1914	5.7	6.7

differences for different nights are slightly in excess of those for the same night but this excess is very probably due largely to accident, as the results for 1913-1914 seem to indicate, or possibly in part, to the constancy of instrumental errors for plates made in rapid succession. However, if we consider this difference real, it suggests a velocity variation of about five kilometers for γ Cassiopeiæ, if the period is measured in days or weeks. Apparently many observations would be required to establish the reality of such a variation.

Again we may examine the evidence of velocity variation for γ Cassiopeiæ on the basis of probable errors. Thus, we may combine the probable error of the average plate, determined from the internal line residuals, with the element of error known to arise normally from instrumental sources, systematic measurement errors, etc., thereby deriving a probable error for a single plate which should agree closely with values of this same quantity obtained from the residuals of column four of Table VI, unless the velocities of this table are variable. Following this procedure we find the probable error of the average plate of Table VI, determined from the internal line residuals, to be ± 3.0 km. Assuming that the uncertainties due to instrumental sources, systematic measurement errors, etc., average ± 1.0 km. we obtain a value of ± 3.1 km. for the probable error of a single average plate. From the residuals of Table VI we obtain a value of ± 3.3 km. for this quantity. If we assume that this difference between 3.1 and 3.3 is due to variable velocity of γ Cassiopeiæ and if the velocities of Table VI be considered to be well distributed over a velocity curve not far from circular, the double amplitude of such a curve would be about four kilometers.

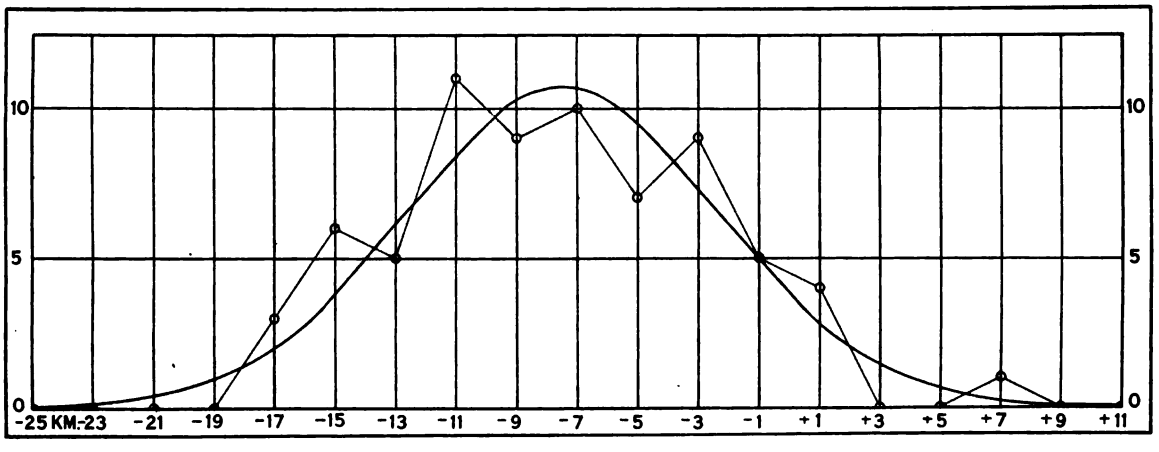

PLATE IV. COMPARISON OF DISTRIBUTION CURVE OF VELOCITIES OF γ CASSIOPEIAE WITH THE PROBABILITY CURVE

Again, to detect evidence of variation in the velocities of Table VI, we may compare their frequency or distribution curves with the probability curve in the manner employed above for the quantities, E — A. In the upper curve of Plate IV this is done for the individual velocities with ordinates corresponding to numbers of plates. The probability curve corresponds to a

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TABLE XVII. RADIAL VELOCITIES OF GAMMA CASSIOPEIAE AVERAGED BY SEASONS.

EPOCHS	RADIAL VELOCITIES					
	Hδ EMISS.	Hδ ABS.	Hγ EMISS.	Hγ ABS.	Hβ EMISS.	MEAN
	km.	km.	km.	km.	km.	km.
1911, Sept.	−10.6 ± 1.3	−12.8 ± 1.8	−5.5 ± 1.0	−5.4 ± 1.4	−7.5 ± 1.5	−8.0 ± 0.57
1912, Dec.	− 7.0 ± 2.8	− 8.0 ± 1.6	−9.2 ± 1.5	−8.5 ± 1.6	−9.4 ± 1.5	−8.5 ± 0.74
1913, Nov.	− 2.6 ± 2.0	− 5.6 ± 2.0	−4.0 ± 1.2	−1.7 ± 1.7	−8.7 ± 1.5	−4.9 ± 0.84
1911 to 1914	− 7.2 ± 1.2	− 8.9 ± 1.0	−6.3 ± 0.6	−5.1 ± 0.9	−8.5 ± 0.8	−7.21 ± 0.40

probable error for a single average plate of ± 3.50 km., with maximum ordinate at −7.3 km. In the curve of Plate V the plates are grouped by nights into normal places. The probability curve here corresponds to a probable error of ± 2.50 km. and the maximum ordinate is at −7.5 km. There seems to be nothing in this comparison to indicate that the velocities of Table VI are variable in a short period.

There remains the possibility that there is a slow variation in the velocity of γ Cassiopeiæ in a period to be counted by years. To test this, the velocities of Table VI have been combined into means by seasons in Table XVII for each measured feature separately. E and A denote emission and absorption respectively and the probable errors in the last column are based on the final mean of −7.2 km. and thus are larger than they would be if based on the means by years. The differences, E − A, which may be read from this table differ by unimportant amounts from similar differences derived above. These differences arise because all measures, whether both components are present or not, are included here. The velocities in this table do not establish any changes from line to line or from year to year but they do leave open the possibility of a slow variation with a period and amplitude similar in character to that apparently suspected by Hartmann. Apparently the observations now available are not sufficient to give γ Cassiopeiæ a place in catalogs of established spectroscopic binaries.

In addition to the radial velocities discussed above a number of determinations were made

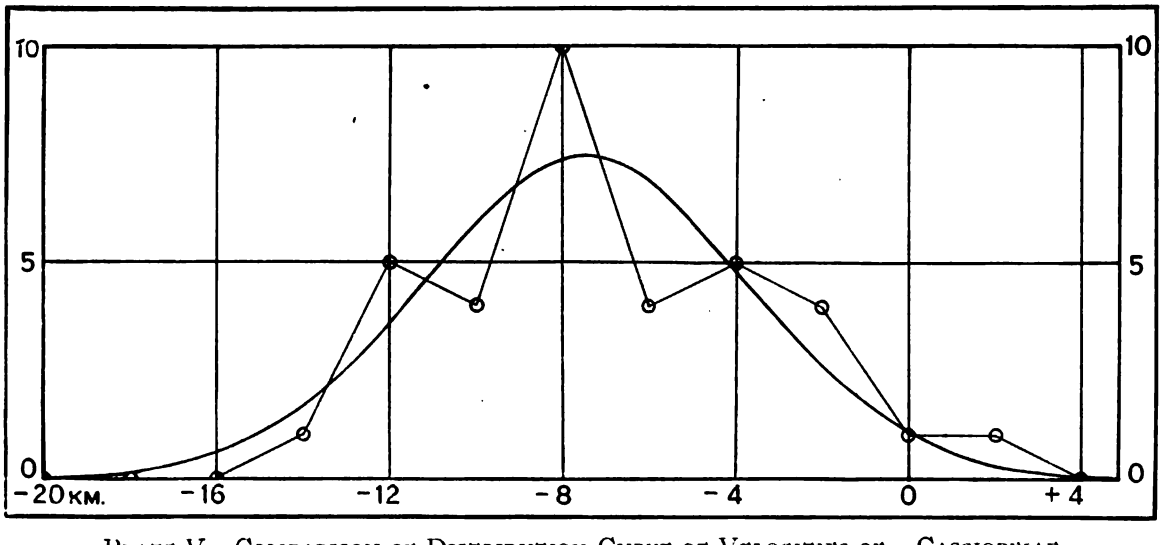

PLATE V. COMPARISON OF DISTRIBUTION CURVE OF VELOCITIES OF γ CASSIOPEIAE BY NIGHTS WITH THE PROBABILITY CURVE

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from lines of very poor quality. Such results are based on the wave-lengths given in the last column of Table IV. They are assembled for the whole series of plates in the following table.

TABLE XVIII. RADIAL VELOCITIES DERIVED FROM INFERIOR LINES.

LINE	ELEMENT	COMPONENT	VELOCITY	PROB. ERROR	NO. OF MEASURES
K	Ca.	Absorption	−3.5 km.	±2.0 km.	38
He	H.	Emission	+3.2	±4.5	5
He	H.	Absorption	−1.1	±2.8	29
λ 4045	He.	Absorption	+4.0	±2.0	51
Hβ	H.	Absorption	−3.9	±3.6	8

Combining all radial velocities of γ Cassiopeiæ measured at Ann Arbor into a final mean, with weights based upon probable errors, the result for the epoch of these observations (1912.8 ±) proves to be −6.4 ± 0.6 km. The inclusion of results from inferior lines may improve the final value through the use of more varied data but it actually increases the computed probable error of the mean. The final mean of the Ann Arbor velocities of γ Cassiopeiæ is compared below with the results of other observers.

	KM. PER SEC.
Vogel and Scheiner, 1888-9,	−3.5
Hartman, 1900-06,	−8.5
Campbell, 1896,	−2.6
Moore, 1903,	−2.8
Merrill, 1910-11,	−6.4
Merrill, 1912,	−2.6
Curtiss, 1912.8,	−6.4

SUMMARY

The foregoing paper is the second of a contemplated series of studies of bright line stellar spectra of Class B. The present report is devoted more particularly to a consideration of the spectrum and radial velocity of the star, γ Cassiopeiæ. The investigations of the present paper include: a study from several points of view of the subgroup of stellar spectra of Classes B to F, containing emission lines; a review of the published observations of the spectrum of γ Cassiopeiæ; a discussion of the evidence relating to the variability of features in this spectrum; a tabulation and review of published radial velocities and wave-lengths of this star; a study of the wave-lengths and intensity of the measurable lines between Hζ and Hα on seventy-five spectrograms of γ Cassiopeiæ made at the Detroit Observatory in the years, 1911 to 1914; quantitative studies of structures of the emission lines in these spectrograms; and determinations and critical discussion of radial velocities from seventy spectrograms of γ Cassiopeiæ made at this Observatory.

The principal deductions from a study of the sub-group of stellar spectra of Classes B to F, containing emission lines are:

1. These spectra, so far as known, tend strongly to group within the B division of the Draper Classification.

2. The apparent brightness of stars with these spectra is much greater on the average for Classes Bo to B5 than for Classes B8 to A2. Magnitude appears to become fainter on the average in stars of so-called greater effective age in this sub-group.

3. The strength referred to the neighboring continuous spectrum and number of the emission lines of hydrogren in these spectra are not clearly dependent upon spectral class, or, in other words, upon so-called effective age, so far as available data go.

4. From Merrill's study of thirty-eight of these spectra in Class B, it appears that the metallic emission lines tend to occur only in the early sub-divisions of this class or, in other words, under conditions thought to characterize stars in the earlier stages of development in this subgroup. And when these lines occur they are stronger on the average when the hydrogen emission is stronger.

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The principal results of the published investigations of the spectrum of γ Cassiopeiæ by former observers are:

1. The spectrum of γ Cassiopeiæ is of Class B, with the hydrogen lines of the Huggins series from $H\zeta$ to $H\alpha$, each consisting of a wide emission line apparently superposed upon a very wide and diffuse absorption band. Except in the case of $H\alpha$ the wide emission line is centrally reversed. In addition to the presence of the hydrogen emission this spectrum differs from the norm of Class B in the occurrence of faint D_3 emission of helium and of many metallic emission lines, largely due to iron, and, probably without exception, found in the bright line spectrum of the solar chromosphere. The dark Orion lines in this spectrum are weak and ill-defined.

2. Visual observations have indicated that the emission lines, $H\beta$, D_3 and $H\alpha$ were strongly variable in intensity in the period, 1872-1888; the photographic record, which was obtained since that time, reveals no certain variation in these lines or in other emission lines.

3. The hydrogen emission lines decrease rapidly in intensity in succession toward the violet. At the same time the broad underlying dark band and the narrow dark reversal become more conspicuous, so far as observed, as the bright bands become fainter.

4. Tests of the $H\beta$ line have not revealed any certain polarization.

5. A variation in the radial velocity of γ Cassiopeiæ, announced by Hartmann, is not confirmed by observations made at Lick Observatory up to March, 1910.

The writer's deductions from his review of the published observations of the spectrum of γ Cassiopeiæ are:

1. Indications of uncertainty affecting visual observations and the constancy of the photographic record make doubtful the physical reality of the reported capricious changes during the era of visual observations (1872-1888) in the brightness of the emission lines of $H\beta$, D_3, and $H\alpha$. Evidence of variation in the metallic emission lines is not conclusive.

2. The success attending visual observations of D_3 emission in the interval from 1872 to 1888 indicates that this feature of the spectrum of γ Cassiopeiæ probably has been stronger, at least periodically, than at present.

3. The twenty-five radial velocities derived at Potsdam and the Lick Observatory support the conclusion that more observations would be necessary to establish the reality of Hartmann's announced variation.

The principal results of the study of the spectrograms of γ Cassiopeiæ made at Ann Arbor are as follows:

1. Determinations have been made of the wave-lengths and intensities of 112 features from $H\zeta$ to $H\alpha$ in this spectrum, with many suggested identifications.

2. It appears that the b group of magnesium is not certainly present in this spectrum.

3. Comparison of wave-lengths measured in this spectrum by Miss Maury, Sidgreaves, Baxandall, Merrill, and the writer, indicates similarity in characteristics at different epochs.

4. In the Ann Arbor spectrograms (1911-1914) there has been found no evidence of real variation from plate to plate or from year to year in the intensity of the emission lines of $H\delta$, $H\gamma$, $H\beta$, D_3 $H\alpha$ and of the metallic emission lines, nor in the central absorption of $H\epsilon$, $H\delta$, $H\gamma$, $H\beta$ and the metallic lines, nor in the dark λ 4026 and the K line. Estimates of the intensity relatively to the neighboring continuous spectrum of the two parts of the emission components of λ 4584 and the hydrogen lines, $H\zeta$, $H\epsilon$, $H\delta$, $H\gamma$ and $H\beta$, furnish no clear indication of variation in the relative brightness of these features of the spectrum of γ Cassiopeiæ. The indication is that constancy was the rule in this spectrum during the period of these observations.

5. The metallic emission lines share the well known structure of the hydrogen lines.

6. The wide absorption borders of the emission lines are probably resolvable into narrow components. Narrow components of these absorption borders measured in this spectrum resemble those observed by the writer in the spectrum of β Lyræ, in point of distance from the center of the corresponding emission line.

7. Measures of the widths of the emission lines on the Ann Arbor plates of γ Cassiopeiæ indicate no real changes in these quantities from plate to plate nor from year to year.

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8. A linear relation,

$$d\lambda = 0.001885\ (\lambda - 2020),$$

between measured width and wave-length of all emission lines, is strongly suggested in the spectrum of γ Cassiopeiæ. On correction of measured line widths for slit width and diffraction, the constant, 2020, becomes smaller and may approach zero.

9. Determinations of the relative position of emission and central absorption of Hϵ, Hδ, Hγ, and Hβ for sixty-one Ann Arbor spectrograms do not indicate any real variation in this quantity from line to line, from plate to plate, nor from year to year. If periodic variations exist they are probably small or so slow that observations covering many years would be required to reveal them.

10. The absorption reversals occupy the centers of the emission lines of hydrogen within limits of a few hundredths of an angstrom on the average and no certain deviation from this rule is found for individual lines.

11. Radial velocities determined from seventy spectrograms of thirty-five different nights in the years, 1911 to 1914, establish no variations from line to line, from plate to plate, nor from year to year.

12. A study of probable errors and a comparison of the distribution of the Ann Arbor observations with the probability curve indicate independently that there is no periodic variation in the radial velocity of γ Cassiopeiæ greater than a few kilometers in range unless the cycle covers many years.

13. The final radial velocity of γ Cassiopeiæ from all the Ann Arbor spectrograms is -6.4 ± 0.6 km., for the epoch, 1912.8. Combining all other known measures except those of Vogel and Scheiner, assigning equal weight to each determination, the radial velocity of this star is found to be -5.8 km. The final velocity based on all known measures is -6.2 km.

CONCLUSION.

With further studies in view of stellar spectra similar to that of γ Cassiopeiæ, an exhaustive discussion of the material now at hand might well be deferred. It remains to consider briefly, in connection with the results available, the theories which have been proposed to account for the peculiarities of γ Cassiopeiæ's spectrum and for the characteristics of the sub-group to which this star belongs.

In 1867, Huggins[5] inferred that the nearly uniform light of γ Cassiopeiæ suggested that the luminous hydrogen in this star forms a normal part of the photosphere. About twenty-three years later, with much more information on which to base a conclusion, Scheiner[27] proposed two explanations for the presence of bright lines in the spectra of stars like γ Cassiopeiæ: first, that they are produced by an atmosphere of hydrogen and helium at a higher temperature than the nucleus; second, that they are due to radiation from an atmosphere so extensive that the emissive spectrum of those portions of the atmosphere which project out beyond the star's real disk when superposed upon the absorption spectrum of the central parts overpowers it. The second explanation was offered by Scheiner as the more reasonable one and in this form is probably generally accepted today to account for the bright lines in stellar spectra of Classes B and A. In extension of this explanation Scheiner suggested that the width of the lines in these spectra was dependent upon the density of the atmosphere emitting them, and their structure upon its density gradients.

In 1894,[28] Scheiner had not accepted Campbell's observations of a bright Hα coexistent with a dark Hβ line. Apparently also he was not aware that the hydrogen lines beyond Hζ, in the spectrum of γ Cassiopeiæ, are dark or neutral. Frost[29] and Kayser[30] pointed out that such a condition was not necessarily inconsistent with Kirchhoff's Law and Scheiner,[31] in 1908, accounted for it very simply on the basis of his own theory of an extensive atmosphere. It was necessary only to add the assumption that the hydrogen emission lines decrease in intensity toward the violet much as they do in the vacuum tube. Superposed on the corresponding absorption lines

[27] *Scheiner's Astronomical Spectroscopy.* Frost's Translation, page 250.
[28] *Ibid.*, page viii.
[29] *Astrophysical Journal*, Vol. 2, page 182.
[30] *Handbuch der Spectroscopic*, Vol. II, page 356. *Astrophysical Journal*, Vol. 14, page 313.
[31] *Populäre Astrophysik*, page 592.

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produced by the atmosphere immediately in front of the nucleus, the visibility of the bright lines, beginning with the brightest, Hα, in comparison with the continuous spectrum, depends upon the extent and effective luminosity of the atmospheric projection, the strength of the neighboring continuous spectrum and the character of the background, for each line.

The hypothesis of an extensive atmosphere seems not inconsistent with the facts brought out above with reference to the spectrum of γ Cassiopeiæ. The constancy of the spectrum now observed and the variation suspected formerly in γ Cassiopeiæ and observed clearly in other stars such as Pleione and f¹ Cygni, seem not to negative this hypothesis in any way. The similar nature of the hydrogen and metallic emission lines suggests certain interesting conclusions with reference to the distribution of these substances. It might indicate that hydrogen and iron at least were well mixed in such an extensive atmosphere, and the fitting in of the widths of the metallic emission lines with those of the hydrogen lines would suggest an identical cause of widening. The symmetrical structure of the emission lines and the central position of the narrow absorption, considered as a reversal, should throw light upon the causes of broadening of spectral lines in an extensive atmosphere.

In connection with the very wide emission lines in γ Cassiopeiæ, conditions known to produce small effects are clearly of secondary importance. Such may be the Doppler effect of the component of velocity of the vibrating particle in the line of sight, changes of pressure, and possibly disturbances due to the presence of free electrons.

A source of apparent line broadening which may produce important effects in the supposedly extensive atmosphere of stars like γ Cassiopeiæ, is great thickness of the incandescent stratum of vapor. Scheiner[32] states that the spectroscopic equivalence of the density and thickness of such an incandescent gaseous stratum renders more difficult the formation of an opinion as to the constitution of a celestial object from the appearance of its spectrum and Zöllner expresses himself in similar vein. Kayser[33] and others point out that the width of a line is increased by the thickness of the radiating layer only when the intensity curve of such a line falls off gradually to the edge. It is a question then to how great an extent this factor has affected the lines in this spectrum. According to Scheiner the effect of increased thickness of the radiating stratum is to reduce the difference in brightness of two contiguous parts of the spectrum. The intensity curves of the emission lines in the spectrum of γ Cassiopeiæ seem not inconsistent with the hypothesis of an extensive atmosphere having the function of a radiating stratum.

Possibly the factor of greatest potency in increasing the width of single lines in stellar spectra is thought to be vapor density. In a star with a spectrum of Class B it is well known that the mean density is probably very low and that the greatest density of the radiating gaseous stratum may be less than that of the earth's atmosphere at sea level. However the combination of pressure and temperature, both probably high, may be competent to produce much of the widening of lines actually observed. But widening of lines through pressure is inseparably bound together with change of wave-length, and atmospheric pressure can hardly exist without pressure and density gradients. The narrow reversals of the emission lines of γ Cassiopeiæ indicate that such gradients exist, associated also with reasonable temperature gradients. It would be thought then that such reversals, produced under lower pressure conditions, would not occupy the center of these very wide emission lines even within limits determinable with a single-prism spectrograph. However balancing causes may be at work. Indeed the actual maximum of the emission lines from the lower stratum may be strongly displaced by pressure but such maxima differing in position for different layers might not form any well defined feature when combined in the integrated line and might be masked in any given star by several factors tending to smooth out the intensity curve of an emission line.

Among the factors tending to alter or smooth out the intensity curves of the emission lines in the spectrum of γ Cassiopeiæ great thickness of

[32] *Scheiner's Astronomical Spectroscopy.* Frost's Translation, page 130.

[33] *Handbuch der Spectroscopie*, pages 246, 296.

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the emissive strata has been mentioned. A second possible factor is rotation. We do not know to how great an extent the atmosphere at various levels shares in the rotation of early stars nor how rapid such rotation may be, though in some cases it has been thought to be rapid enough to result in fission and the assumption of strongly ellipsoidal form. If the angular rotation of the atmosphere is as rapid as that of the nucleus, the center of the line corresponding to high levels will be broadened most thus tending to equalize the density of the line over its width and also, of course, the total width of the line may be increased. The relatively small width of the central absorption in the emission lines of γ Cassiopeiæ's spectrum may indicate that rotation does not play a very important role in the case of this star.

Atmospheric circulation, which, as the presence of iron vapor probably at high levels seems to indicate, may be rapid and far reaching in the case of γ Cassiopeiæ, could affect very appreciably the intensity curves of the emission lines in this spectrum. Probably the effects of the radial component of motion in such currents would be a widening of the lines and, since the ascending currents would be more highly emissive and more effective on the side of the star toward the observer, a displacement toward the violet. In the outer strata, producing the narrow absorption lines, such currents would probably not be so effective. Thus we might expect an apparent displacement of the emission line to the violet with reference to the reversal. This might balance or even overcome in some cases the opposite relative displacement due to pressure. In this way the central position of the narrow reversals in the emission lines of γ Cassiopeiæ may be accounted for even though the chief cause of widening is vapor density due to pressure, combined with high temperature.

The probability that the Zeeman or Stark effects are important among the factors affecting the intensity curves of emission lines in γ Cassiopeiæ seems small in view of the tests for polarization and separation made by Merrill.[2] And other suggested explanations such as the relative motion of two stars, may be dismissed without discussion. At present, Ekman's[40] resonance theory to account for the widening of spectral lines is developed hardly far enough for application here.

The approximately linear relation between emission line widths and wave-length, found in γ Cassiopeiæ, seems consistent with hypotheses of widening due to rotation and convection currents. Not enough is known about this relation for lines widened by vapor density or pressure to draw definite conclusions but it seems at least unexpected that the iron and hydrogen emission line widths in γ Cassiopeiæ, if they are due to vapor density and pressure combined with high temperature, should together bear any simple relation to wave-length. However, at least the fact of increase of width with wave-length for lines broadened by vapor density accords with theory and experiment.

It may be suggested that the possible role of electrical discharge must not be forgotten in connection with the light of these stars. In hot stars of low density, convection currents and other relative internal motions may produce great electrical disturbances with discharge which, especially in low pressure regions, may pass very freely. At least part of the light emitted by the atmosphere of γ Cassiopeiæ may be accounted for in this way. It is well known that the lines of a gas through which an electric spark is passing are broadened even under relatively low pressures. Possibly the emission lines in the spectrum of γ Cassiopeiæ have been broadened in this way. If so, a lack of appreciable displacement in them might not be unexpected.

Referring to the sub-group of stars to which γ Cassiopeiæ belongs Scheiner in *Die Spectralanalyse der Gestirne*[34] makes the statement: "There can be nothing more natural than to assume that the stars of Class Ic (exemplified by γ Cassiopeiæ and β Lyræ),[35] which have far more powerful atmospheres than those of Class Ia, are in a preliminary state from which they

[40] *Annalen der Physik*, Vol. 24, page 580.

[34] *Scheiner's Astronomical Spectroscopy.* Frost's Translation, page 250. See reference 27.

[35] Parentheses inserted. These stars are cited as types in Frost's translation of *Scheiner's Astronomical Spectroscopy.*

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will pass over gradually into that of Class Ia (exemplified by Vega and Sirius).[35] They would therefore be considered as representing the earliest or 'youngest' phase of stellar evolution." Scheiner reiterates this view as a possibility in his *Populäre Astrophysik*,[36] and at least one other authority[37] has expressed agreement in the opinion expressed in the first sentence quoted.

On the other hand Miss Clerke[38] considered that the passage of dark line helium stars through a bright-line phase can be inferred to occur by exception in stellar history as a consequence perhaps of peculiarities of internal constitution, perhaps of unusual external influences.

Quite recently Doctor P. W. Merrill[39] has observed two bright line stars of Class A and Class A2 and has found that the characteristics of these two stars in the spectral region studied appear to be the same as those of bright line stars of Class B. Also nine spectra of Class Oe5 were photographed to test their relation to bright line spectra of Class B, since it is to be expected that bright Hα, at least, would be found in a considerable proportion of Class Oe5 spectra if they precede immediately in evolutionary order those of Class B and if the bright line spectra of the latter class are those of stars particularly early in development. But no bright lines were found. On the basis of the evidence at hand it was not possible to assign stars with bright line spectra of Class B to a place by themselves in evolutionary progression. Quite in accord with Miss Clerke's suppositions, Merrill suggests that it may be that special physical conditions, not uniquely connected with a particular epoch in effective age, give rise to bright hydrogen lines in stellar spectra of Classes B and A. Photographs at the Detroit Observatory of spectra of Class Oe5 confirm Merrill's result.

In support of the opinion that bright hydrogen lines are not uniquely connected with a particular epoch of stellar age we may add the statistical fact, brought out above, that the strength and number of emission lines of hydrogen in the spectra of the sub-group here discussed are not clearly dependent so far as known on so-called effective age. Possibly we may go farther and assert that the evidence indicates that stars with bright line spectra of Classes B to F form an evolutionary series running parallel with normal stars, differing only in some internal peculiarity—probably the possession of an abnormally extensive atmosphere.

Thus it seems difficult to agree with Scheiner in his opinion that the hydrogen bright line stars like γ Cassiopeiæ are the "youngest" stars. Too many spectra of this group are already developed into Class A. But the point may be raised that such spectra in Class A have never contained and will never show the dark Orion lines; that they are newly formed stars of which the atmospheres are so proportioned or constituted that the Orion lines are not evident. Further, the suggestion might be made that stars with spectra of Classes O and Oe5 and stars with bright line spectra in Classes B to F form two groups of "youngest" stars; the one evolved from the planetary nebula and the other from the irregular nebula. But the burden of opinion at present probably would favor the view that all stars develop in successive stages indicated by the spectral sequence, O, Oe5, B, A, and so forth, and that the occurrence of bright lines superposed on some of the absorption lines in many spectra of Classes B, A and F does not indicate for the stars emitting them a position in the evolutionary succession different from that of stars having similar spectra without *visible* emission lines.

A striking circumstance in connection with this group of stars is the strong tendency brought out above for average magnitude to become fainter with advancing spectral class. The indication would be that members of this group in Class O would average bright and that further discoveries of members of this group in Classes F and G may be expected when fainter stars are studied. Is it not probable that extensive atmospheres about these stars, or the lower layers of such extensive atmospheres, develop relatively strong general absorption, as compared with other stars of the same effective age, when Classes A and F are reached, thus making relatively faint this portion of the sub-group of bright line stars?

[35] *Populäre Astrophysik*, page 595.
[36] See reference 29.
[37] *Problems in Astrophysics*, page 279.
[38] *Lick Observatory Bulletins*, Vol. VII, page 23.

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Another point is raised by the occurrence of strong hydrogen emission lines in Class A spectra. If stellar atmospheres are condensing and cooling as spectra develop from Class B to Class A, should we not find, for stars which give strong emission line spectra in Class A, forerunners with emission more intense than any so far observed? Apparently not, for it is the *relative* intensity of emission lines and neighboring continuous spectrum that determines the former's visibility, and on the average this relative intensity may remain the same while a spectrum develops from Class B to Class A, in a given case, even though the absolute brightness of these lines is much greater in Class B than in Class A. In some cases the emission lines are known to be fading relatively to the continuous spectrum. In other cases they may be found to be growing relatively stronger.

At many points in the above discussion attention is called again to the need of observations of spectra of Classes B, A, and F with bright lines, and of Classes O and Oe5. The program of observations at this Observatory has been extended to include spectra of Classes O and Oe5, and studies of bright line spectra of Classes A and F are contemplated.

Ann Arbor, Mich., December 3, 1915.

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STUDIES OF CLASS B STELLAR SPECTRA CONTAINING EMISSION LINES

CHANGES IN THE SPECTRUM OF f^1 CYGNI

By RALPH H. CURTISS

In *Harvard Circular* No. 98, announcement was made of peculiarities in the Class B spectrum of f^1 Cygni (1900.0, $\alpha = 20$ h. 56.4 m., $\delta = +47^\circ\ 9'$; mag. = 4.9). In an objective prism spectrogram of November 15, 1904, the hydrogen lines from Hζ to Hβ inclusive were found to consist of fine bright lines superposed on strong dark bands. In accordance with the well known rule of brightness of hydrogen emission in spectra of Class B, the intensity of these lines was found to diminish with diminishing wave-length. Other fine bright lines were visible.

Nine days later bright Hδ, Hγ and Hβ were photographed, the last two named lines being on the edge of shorter wave-length of the accompanying dark lines. The change observed between these two spectrograms seemed to indicate a binary character for this star, one component having bright hydrogen lines.

Since the emission of Hζ was visible on November 15, 1904, with an observed increase in the brightness of the other hydrogen lines with increasing wave-length it seems probable that the Hβ line and, in accordance with our knowledge of other stars, the Hα line must have been strongly bright. From the Harvard description it is difficult to determine whether the bright lines were reversed as in many of these stars or whether there was a single sharp emission component in each hydrogen band.

On a single-prism spectrogram of this star, of May 29, 1911, Doctor P. W. Merrill found Hγ probably bright in absorption and Hβ possibly bright in absorption, but the Seed 30 emulsion used was not effective in bringing out fine detail and small constrasts. On a spectrogram of June 6, 1912, with a single-prism instrument adjusted for visual light, the same investigator found Hβ and Hα indistinctly bright. He considered this spectrum to be peculiar and possibly variable. (*Lick Observatory Bulletins,* Volume VII, page 172.)

Twelve spectrograms of this star have been made with the single-prism spectrograph of this Observatory on Seed 23 plates. These are listed with the corresponding date of observation in the accompanying table.

In no case was emission observed in connection with the Hζ and Hϵ lines. These were always wide dark lines much like the lines of helium which are also present in this spectrum. Probably this is also true of the Hδ line though in some cases this line appeared asymmetrical, in one case in 1911 the presence of narrow emission on the side of greater wave-length was suspected, and in another case in that year the limits of an emission line may have been measured. But such emission line if it existed was not brighter than the neighboring continuous spectrum and was revealed only by the absorption of the underlying dark band.

The Hγ and Hβ lines on the seven plates of 1911, with the exceptions noted below, showed the structure which is characteristic of the hydrogen lines in many of the peculiar spectra of Class B. An emission line, divided by a dark reversal into two components, was superposed on an absorption band so wide that dark borders flanked the emission on either side. The reversal in this case was so wide that the components of the emission line were relatively narrow. Further the reversal was observed to be displaced toward the violet relatively to the center of the emission line in all but one case where measures could be made. Thus the emission component of shorter wave-length was narrower than the other. Also observations indicated that this sharper component of shorter wave-length tended to be the stronger.

The emission of Hβ averaged slightly brighter than the neighboring continuous spectrum in 1911, while the emission of Hγ was very nearly of the same intensity as the continuous spectrum in the Hγ region. Indeed without the evidence

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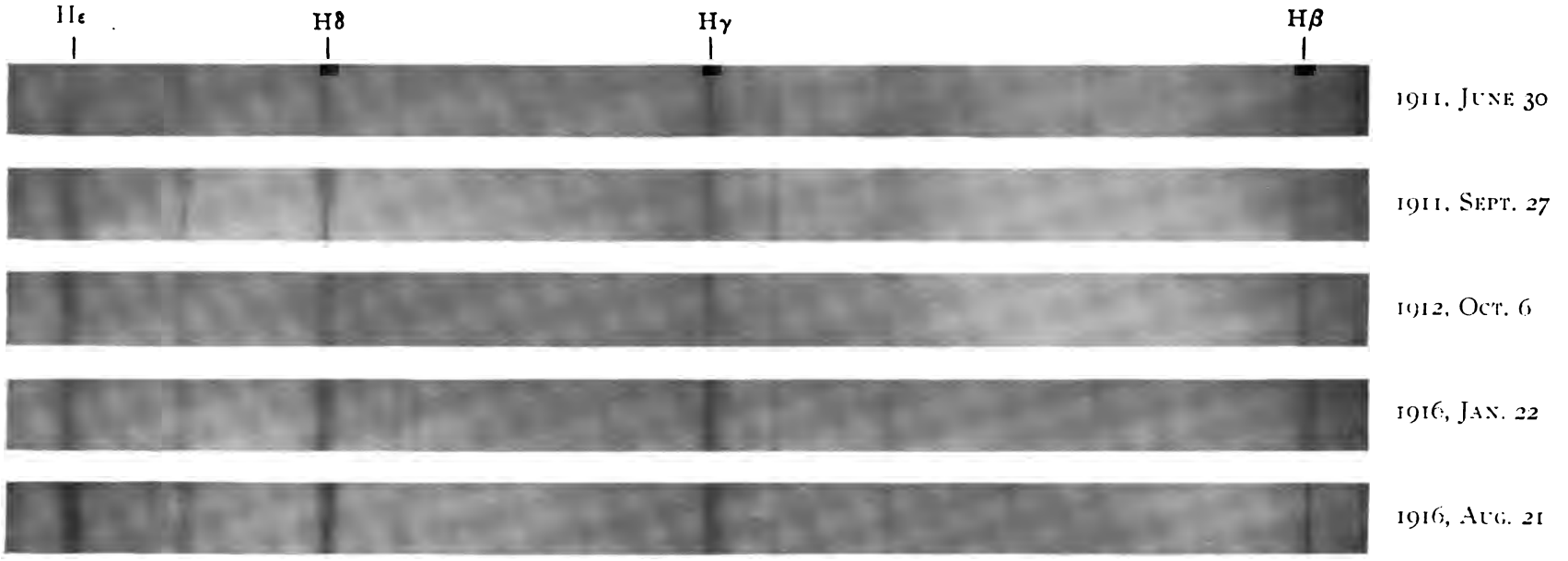

Plate D. Spectra of f¹ Cygni Showing Progressive Changes in Hε, Hδ, Hγ and Hβ, from 1911 to 1916

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THE ANN ARBOR OBSERVATIONS OF f¹ CYGNI.

PLATE NO.	DATE G. M. T.	VELOCITIES			INTENSITY OF CENTRAL ABSORPTION			WIDTH OF EMISSION	
		REVERSAL	WT.	E—A	Hδ	Hγ	Hβ	Hγ	Hβ
	d	km.		km.				Å	Å
65	1911 June 30.807	+33	2½	+33	10	10	9	7.20	9.80
74	July 1.797	+33	2½	....	11	10	9	....	....
87	5.803	+37	2½	+45	11	12	9	....	8.71
	Second measure	+38	3		11	10	9	....	....
114	July 11.871	+36	2½	+29	14	10	9	7.50	9.24
150	26.772	+33	2½	+17	14	12	12	7.69	10.36
162	30.772	+27	2½	— 5	12	11	10	....	8.71
227	Sept. 27.694	— 7	2½	+26	..	..	..	7.54	9.80
	Second measure	+ 1	3	+28	11	10	11	7.54	8.97
1197	1912 Oct. 6.673	— 9	3	....	15	12	12	....	....
3407	1915 Dec. 9.414	— 6	1	....	24	15	15	....	9.71
3416	14.588	—23	1½	....	25	16	16	....	....
3420	1916 Jan. 6.583	—24	½	....	..	..	16	....	....
3427	22.568	— 9	2½	....	25	18	18	....	....
	MEANS			+25				7.48	9.33

of the absorption borders on either side and the better definition of the edges of the reversal, the existence of emission at Hγ would hardly have been suspected. Adopting Miss Maury's expressive term, the line was neutral.

The absorption border of shorter wave-length was stronger and was resolved in some cases into narrow dark lines. On six of the seven spectrograms of 1911 the absorption border was visible on the edge of shorter wave-length of Hγ and on four of these spectrograms it was visible on the edge of greater wave-length. The absorption border of shorter wave-length at Hβ was visible on all plates of 1911 and that of greater wave-length on all except plate 74. The invisibility of the absorption borders of Hγ and Hβ in five cases out of twenty-four, in 1911, seems ascribable to lack of suitable photographic contrast rather than to variation during this season. There was no plate showing less than two of these absorption borders of Hγ and Hβ.

On the plate of 1912, and on those of 1915 to 1916, the structure of the Hγ and Hβ lines appeared to be changed. The absorption borders were weaker or absent; the reversals, wider and less well defined. On plate 3420, which was sensitized according to Wallace's formula, underexposed continuous spectrum was faintly visible in the Hα region but no trace of emission was identified. Apparently at present there remain in the spectrum of f¹ Cygni only very uncertain and possibly vanishing traces of the structure, characteristic of emission lines in Class B spectra, which was probably pronounced in 1904 and was clearly shown in 1911. The Ann Arbor observations indicate the existence of a progressive change but the possibility of a short period variation, which was suggested at Harvard, is not excluded.

The velocities determined from the central absorption of the hydrogen lines Hδ, Hγ and Hβ are found in column three of the accompanying table. They appear to vary slowly, though over a wide range, toward smaller values. Whether this change is due to varying radial velocity or to alterations in the structure of the lines due to internal changes in the star can not be decided. The Hδ line, which showed no certain emission, and the helium lines so far as they can be measured, indicated the same velocity variation, but in these cases also internal changes in the star may be responsible for the measured

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variations. If the variation is one of velocity the orbital period may be long, though a short period is not excluded.

In column four the excess of the velocity for the emission lines over that for the reversal is tabulated, where measured. The mean relative displacement amounts to 0.36 Å at $H\gamma$.

In columns five, six and seven the average increase from 1911 to 1916 in the intensities of the central absorption of $H\delta$, $H\gamma$ and $H\beta$ is brought out.

Columns eight and nine give the width of the emission lines of $H\gamma$ and $H\beta$, corrected for width of slit by the formula, $\Delta\lambda = 0.040\, d\lambda/dR$. If we assume a linear relation between line width and wave-length in this case, we arrive at the equation,

$$\Delta\lambda = 0.00356\ (\lambda - 2240).$$

CONCLUSION.

From the descriptive and numerical data given above it seems apparent that there have taken place in the spectrum of f^1 Cygni changes involving the fading of the hydrogen emission lines, the effacement of the structure of these lines, an increase in the intensity of the narrower dark lines of hydrogen and a variation in their wave-length. There is evidence that this change is a progressive one.

Ann Arbor, Mich., Jan. 20, 1916.

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STUDIES OF CLASS B STELLAR SPECTRA CONTAINING EMISSION LINES

CHANGES IN THE SPECTRUM OF H. R. 985

By RALPH H. CURTISS

The spectrum of the star, H. R. 985 or B. D. + 65°340, (1900.0, $\alpha = 3h\ 11.2m$; $\delta = +65°\ 17'$; B. D. mag., 4.5; Harvard photographic mag. 4.86) in Camelopardalis was suspected of peculiarities by Espin on September 22, 1895. The total light was pronounced white; the type of spectrum was not determined but the F line of hydrogen was recorded as possibly bright. In *Harvard Annals,* Volume 56, Epsin is credited with the discovery of the peculiarity of this spectrum.

Previous to this observation by Espin, bright Hα had been observed in this spectrum by Campbell, and the photographic spectrum had been studied on sixteen objective prism plates at Harvard by Miss Maury whose detailed description, written several years at least before 1897, is found on page 105 of Volume 28 of the *Harvard College Observatory Annals.* With a note now inserted in parenthesis this description reads as follows:

"The system of dark lines in the spectrum of this star belongs to Group IV, Division b, but all the lines are exceedingly wide. (This last remark is not true of the H and K lines nor of the hydrogen reversals.) A distinct bright band lies almost centrally on the wide dark band Hβ. A fainter and narrower bright band or line lies upon Hγ, its position being commonly of slightly greater wave-length than the center. The lines Hϵ and Hδ appear in most photographs unequally divided, with the strong component occupying the true central position for each hydrogen line, and the weak and hazy component having the greater wave-length. In some plates, however, both these lines and Hγ appear triple. Other lines of the spectrum sometimes appear unequally divided, with the stronger component of shorter wave-length."

In Volume 56 of the *Harvard College Observatory Annals* this spectrum is classified as B2p, and the lines Hγ, Hβ and Hα are noted as bright.

In the *Lick Observatory Bulletins,* Volume VII, page 167, Doctor P. W. Merrill reports the salient features found on four spectrograms of this star made between July 28 and August 13, 1912, with single-prism spectrographs. Hα was a strong bright line; Hβ and Hγ were "double bright on absorption"; Hδ was probably "faint bright on absorption". Strong helium absorption was observed and traces of a few metallic emission lines were seen.

The twenty spectrograms made at the Detroit Observatory are grouped in two epochs—the first group of nine plates extending from October 26 to December 16, 1912, the second from January 22 to April 17, 1916.

On the plates of each of these groups the characteristics of the lines appeared to be essentially constant. A central reversal divided each hydrogen emission line into two parts and the edges of a broad underlying absorption band were visible on either side of the emission line. This structure was brought out very clearly in the case of Hβ, less clearly in Hγ, rather indistinctly in Hδ and quite indistinctly in Hϵ and Hζ. As in other stars of this sub-group the emission components decreased and the central absorption increased in intensity from Hβ to Hζ. Broad helium dark lines, too diffuse for useful velocity determinations, but sometimes giving evidence of the structure characteristic of the hydrogen lines, were observed at λ's 4009, 4026, 4121, 4144, 4388, 4472, and 4922. Metallic emission lines showing faintly the structure characteristic of the hydrogen lines were measured at λ's 4233, 4584, 4925 and 5018. In addition a narrow dark K line, probably fixed in position, was observed on many plates of both groups.

But in the interval between the two series of plates a striking change had taken place in the intensities of the two parts on either side of the central reversal of the emission bands of hydro-

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gen. Briefly, in the first series of plates the part of the emission component having greater wave-length, so far as observed in the hydrogen lines, was stronger than that of smaller wave-length in nearly every case, whereas in the second series the reverse was the case. In Table I, the extent of this change for the different hydrogen lines is brought out. The table gives the estimated intensities of the parts of each hydrogen emission line referred to the neighboring continuous spectrum for each plate, the serial number of which is given in column 1, the date in Table II. R at the head of a column denotes the part of the emission component of greater wave-length; V the corresponding part of smaller wave-length.

Variations in the estimates of line intensities from plate to plate in the 1912 series, as well as in the 1916 series, are present and appear to be due to the well known uncertainties of photographic contrast, and partly perhaps in the case of Hϵ to the neighboring H line. It is not thought that they establish any variation within either series. Very probably the means for 1912 and for 1916 in this table represent quite well the relative intensities of the two parts of the hydrogen emission lines, as well as their intensities referred to the continuous spectrum at these epochs. Thus the part of greater wave-length of the Hβ emission component lost one-third of its strength between 1912 and 1916 while at the same time the part of smaller wave-length gained a similar amount in brightness; and the other hydrogen lines observed in the photographic region exhibited similar changes which, however, were less striking, largely perhaps because these lines were relatively fainter than Hβ.

The edges of the hydrogen emission lines were best defined when associated with strong emission components. And this was true also of the absorption borders of the emission lines. In

TABLE I. BRIGHTNESS OF PARTS OF EMISSION COMPONENTS REFERRED TO THE CONTINUOUS SPECTRUM.

PLATES	Hζ		Hϵ		Hδ		Hγ		Hβ	
	V	R	V	R	V	R	V	R	V	R
1360	0.6	0.8	0.6	0.8	0.6	0.9	...	...	1.3	1.6
1404	0.6	0.8	...	...	0.6	0.9	1.0	1.1	1.3	1.6
1449	0.4	0.8	0.7	0.7	0.6	0.8	0.9	1.1	1.2	1.5
1477	...	...	0.4	0.6	0.4	0.5	0.9	1.2	1.0	1.5
1504	...	...	...	...	0.4	0.9	0.9	1.1	1.3	1.5
1509	...	...	...	...	0.7	0.9	1.0	1.1	1.0	1.4
1532	...	...	...	...	...	...	...	...	1.0	1.4
1539	...	...	...	...	0.4	0.5	0.9	1.0	1.2	1.5
1554	...	...	0.4	0.6	0.6	0.8	0.9	1.1	1.2	1.4
MEAN 1912	0.5	0.8	0.5	0.7	0.5	0.8	0.9	1.1	1.2	1.5
3429	...	...	0.8	0.6	0.8	0.6	1.3	0.9	1.6	1.0
3430	...	...	0.7	0.4	0.7	0.4	1.2	0.8	1.5	0.9
3436	...	...	0.6	0.3	0.7	0.4	1.0	0.8	1.5	1.1
3437	...	...	0.7	0.5	0.8	0.5	1.1	0.8	1.5	1.1
3450	...	...	0.5	0.6	0.8	0.5	1.1	0.8	1.6	1.0
3454	...	...	...	...	0.5	0.5	1.2	0.9	1.6	1.0
3461	...	...	...	...	0.9	0.5	1.1	0.8	1.5	1.1
3463	0.6	0.4	0.7	0.4	0.7	0.6	1.2	0.8	1.4	1.1
3470	0.6	0.4	0.5	0.8	0.7	0.7	1.1	0.8	1.5	1.0
3471	...	...	0.6	0.8	0.8	0.5	1.0	0.8	1.5	1.1
3494	...	...	0.8	0.8	0.9	0.8	1.0	0.8	1.5	1.0
MEANS 1916	0.6	0.4	0.7	0.6	0.8	0.5	1.1	0.8	1.5	1.0

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plates of the second group the edge of greater wave-length with its absorption border, even in the case of Hβ, was frequently difficult to observe.

Hα was observed as a strong emission line on one plate (Number 3430) of the second group; the only plate of the set sensitized to red light. On the scale used in column 24 of Table II, the intensity of this line would be about twenty. It was accompanied by strong absorption borders and no reversal was visible. Its width corrected for the width of the slit was about 5.5Å, but in view of the structure of the Hβ line it is possible that this measurement of width applies to only part of the normal Hα emission band. No other lines were noted clearly in the visual region.

In Table II, are given essentially complete data from twenty spectrograms relative to radial velocities, line intensities and line widths in the photographic region of the spectrum of this star. In this table in the column of dates the numbers before the decimal point refer to the day of the year. The abbreviation, Abs., refers to the central dark reversal of the hydrogen lines; I, to the intensity of lines on the scale used by the writer in his study of γ Cassiopeiae; p, to the weight of the observed velocities; E — A, to the differences between results for the broad emission components and the dark reversals of the same line. With these explanations the contents of the table will be readily understood.

For a case of this kind in which changes in the structure of the lines are observed and in which therefore more than the usual caution must be exercised in the interpretation of observed displacements as Doppler effects, it is unsafe to derive mean velocities without investigating first the behavior of individual lines. Accordingly in Table II the data for each measurable line are listed. Considering the velocities for any line in either the 1912 or the 1916 series the variations from plate to plate are not greater than we would expect for lines of this character if the velocity were not changing. In the first series it seems possible from the means for each line that the measured velocities tend toward algebraically smaller values with increasing wave-length, whereas in the second series the reverse seems to be suggested. However this indication is not strong enough to detract seriously from the value of plate means based on all the hydrogen lines. Furthermore since the velocities in columns 19 and 28 of Table II do not indicate any departure of the absorption reversals from the centers of the emission lines it appears allowable to combine velocities from emission lines with those from absorption lines in the plate means. Such plate means are found in column 29 of Table II. Possibly there is a slight progressive tendency toward algebraically greater values in the plate velocities through the series for 1912 but the divergence is small at most from the average of —3.1 km. for the series. No variation is suggested in the plate velocities of the 1916 series but the final average of + 41.9 km. differs forty-five kilometers from the 1912 average, a variation extending apparently to all the hydrogen lines so far as the measures give evidence and suggesting unmistakably a Doppler shift of the whole spectrum between 1912 and 1916.

As noted above it is necessary to proceed cautiously with the interpretation of shifts in this spectrum as Doppler effects. Displacements both apparent and real may be associated with the changes in structure of the hydrogen lines. Thus apparent displacement of the dark reversals might be expected to result from the observed changes in the relative intensity of the two parts of the emission lines which border the reversals. The somewhat large displacement of the Hβ absorption component as compared with the displacement of the other lines, between the 1912 and the 1916 series, may be due to this effect. But if this effect is important here we should expect the emission lines to show displacement in the opposite direction between the two series whereas there seems to be little or no evidence of this in the case of Hβ and none in Hγ. In the case of Hβ it is conceivable that the greater variation in the position of the central reversal between the 1912 and 1916 series, as compared with that of the emission component, is attributable to this cause. Probably in the plate means the extent of this effect does not exceed a few kilometers.

The K line of calcium on the plates of both series is narrow, having an intensity between

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TABLE II. VELOCITIES, LINE INTENSITIES, AND LINE WIDTHS. H. R. 985.

PLATE NO.	DATE G. M. T.	Hζ			He			Hδ			Hγ									Hβ									PLATE MEANS			K-LINE	
		ABS.			ABS.			ABS.			ABS.			EMISSION				E–A		ABS.			EMISSION				E—A						
		VEL.	I	P	VEL.	I	P	VEL.	I	P	VEL.	I	P	VEL.	P	WIDTH	P	VEL.		VEL.	I	P	VEL.	I	P	WIDTH	P	VEL.	VEL.	P	VEL.	P
	1912	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19		20	21	22	23	24	25	26	27	28	29	30	31	32
	d	km.			km.			km.			km.			km.		Å		km.		km.			km.			Å		km.	km.		km.	
1360	300.757	+23	4	1	— 1	4	2	— 2	3	2	— 1	3	2	....	..	....	..	....		—37	2	4	—19	18	4	7.13	4	+18	—14	15	+23	1
1404	308.745	....	.	.	....	..	..	+23	4	2	—11	3	2	— 1	2	5.34	2	+10		--28	2	2	—17	16	2	6.66	2	+11	— 7	10	+ 4	1
1449	315.774	....	.	.	—11	4	2	+ 2	3	2	+ 4	4	2	+16	1	4.83	1	+12		— 5	2	4	— 1	16	4	6.81	2	+ 4	— 1	15	—11	3
1477	324.715	....	.	.	....	..	..	— 9	4	2	+ 6	3	4	— 6	1	5.78	1	—12		+ 1	2	4	— 8	15	2	6.75	2	— 9	— 1	13	+12	3
1504	327.777	....	.	.	....	..	..	+11	3	1	+ 5	3	2	— 7	1	5.40	1	—14		— 9	2	2	— 2	16	3	6.42	2	+ 7	— 1	9	+ 3	2
1509	332.677	....	.	.	....	..	..	—15	3	1	+13	2	1	+27	1	4.83	1	+14		—26	1	1	....	15	..	....	..	....	± 0	4	....	..
1532	335.657	....	.	.	....	..	..	— 3	4	1	....	..	..	....	..	....	..	....		....	..	..	+ 3	14	1	6.75	1	....	± 0	2	—27	1
1539	339.701	....	.	.	+11	4	2	— 7	3	1	— 4	2	2	....	..	....	..	....		—19	2	3	—17	15	2	6.39	1	+ 2	— 8	10	....	..
1554	349.635	....	.	.	....	..	..	+ 6	4	2	+22	3	4	+17	2	5.04	1	— 5		— 2	2	2	— 2	15	2	6.36	2	+ 0	+10	12	— 8	1
MEANS	1912	+23	4	1	± 0	4	6	+ 2	3.4	14	+ 6	2.9	19	+ 8	8	5.22	7	± 0		—15	1.9	22	— 9	16	20	6.58	16	+ 6	—3.1	..	— 6	12
	1916																															
3429	22.672	+31	8	2	+36	4	4	+45	4	4	+34	3	4	....	..	....	..	....		+61	2	3	+61	15	3	6.90	2	± 0	+44	20	— 5	2
3430	22.730	+19	4	1	+28	3	1	+29	3	2	+45	2	4	....	..	....	..	....		+51	1	4	+47	14	2	6.81	2	— 4	+42	14	— 9	1
3436	60.604	....	.	.	....	..	..	+21	4	1	+59	3	1	+58	1	....	..	— 1		....	..	..	+49	14	2	7.10	2	....	+47	5	—19	4
3437	60.633	+26	5	2	+64	4	2	+30	4	2	+48	2	2	+66	1	5.53	1	+18		+20	2	2	+26	14	2	6.48	2	+ 6	+38	13	—20	3
3450	75.605	....	.	.	+41	6	1	+19	4	2	+36	4	2	....	..	....	..	....		+43	2	2	+43	14	1	6.28	1	± 0	+35	8	—14	1
3454	76.637	....	.	.	+51	9	1	+58	5	2	+34	4	4	+38	1	5.42	1	+ 4		+56	2	2	+54	15	2	7.10	2	— 2	+47	12	— 8	2
3461	83.618	....	.	.	....	..	..	+35	5	2	+54	3	3	+46	1	5.32	1	— 8		+57	2	4	+39	14	2	6.45	1	—18	+49	12	—14	4
3463	90.635	+22	5	0	+38	5	1	+37	4	2	+36	3	3	....	..	....	..	....		....	..	..	+33	15	3	6.48	2	....	+35	9	— 1	2
3470	97.582	+53	5	1	+51	4	1	+49	3	2	+29	3	2	+33	1	6.25	1	+ 4		+43	2	2	+45	15	2	6.90	2	+ 2	+43	11	— 1	2
3471	97.616	+37	8	0	+56	2	2	+31	3	2	+36	3	3	....	..	....	..	....		+35	2	2	+41	15	2	6.28	2	+ 6	+39	11	— 8	2
3494	108.598	+43	5	1	+54	4	1	+32	5	2	+36	3	2	+29	1	5.05	1	— 7		+42	2	2	....	14	..	....	..	....	+38	9	—12	2
MEANS	1916	+33	6	7	+46	4.6	14	+37	4.0	23	+40	3.0	30	+45	6	5.51	5	+ 2		+48	1.9	23	+44	14	21	6.71	18	— 1	+41.9	..	—11	25

NOTE.—The spectrograms of this table were made on Seed 23 plates except numbers 1509 and 1532 for which Seed 30 plates were used. The normal exposure for this spectrum on a Seed 23 plate was about thirty minutes. Plates 1504, 1509, 1539, 3436, 3437, 3450 and 3494 were made by Mr. Mellor; plate 1477 by Messrs. Hess and Crump. The other twelve spectrograms were made by the writer.

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two and three, but as a rule is not well defined. The velocities derived from this line in the 1912 series do not indicate variation nor divergence from the values derived from the hydrogen lines. The velocities from the K line in the 1916 series do not indicate internal variation but they differ markedly from the corresponding results for hydrogen. Probably the K line is fixed in position in the spectrum of this star and is displaced from its zero position by an amount corresponding to — 9 km. per second of velocity.

In addition to the results in Table II velocities were derived from measures of the broad helium lines. But such velocities were not reliable enough to determine whether these spectral features shared the displacements of the hydrogen lines.

Two measures of the emission component of Hδ gave results in accord with velocities derived from the corresponding reversals.

The line intensities in Table II do not establish any changes in the strength of the absorption reversals nor in the total intensity of the Hβ emission line. The latter observation is in accord with the quantities in Table I.

Measures of the positions of the emission components of Hγ and Hβ for the determination of velocity were made both by bisecting the entire line and by setting on its edges. The latter measures furnished the determinations of the width of the emission components of Hγ and Hβ in columns seventeen and twenty-six of Table II. Since the emission component of the Hγ line averaged no stronger than the neighboring continuous spectrum, and since one-half of the Hβ emission was always relatively weak, the measures presented some difficulty. They cannot be said to establish any changes in the widths of these lines.

The mean widths of the Hγ and Hβ emission components are 5.34Å and 6.65Å respectively. The width of the nearly monochromatic comparison lines in this region is 20 microns. If this be converted into angstroms and subtracted from the measured widths of emission components the results above become 4.58Å and 5.47Å respectively. If we assume a linear relation between wave-length and line width, as suggested by studies of φ Persei and γ Cassiopeiae, the equation of such a line becomes

$$\Delta \lambda = 0.00170\ (\lambda - 1640).$$

SUMMARY

The bright line Class B spectrum of H. R. 985, though showing in the hydrogen lines the structure typical of such spectral features in objects of this sub-group, is of unusual interest because of marked changes observed between the years 1912 and 1916, in the relative intensities of the two parts into which the emission components of hydrogen were divided by the narrow central absorption. At the same time no certain changes in the *total* intensities of the lines in this spectrum were thought to be registered, and *within* the series of plates in 1912, October to December, and in 1916, January to April, no changes in the relative intensities of a given part of any emission line were found to be established.

The hydrogen lines showed as a whole a relative displacement of forty-five kilometers between 1912 and 1916 on the assumption, probably well justified, that such displacement was common to all of them; but within the series in 1912 and within that of 1916 no change in the measured velocities was established, though the possibility of some change during the 1912 series is suggested. The mean velocity for the 1912 series was — 3.1 km. and for the 1916 series, + 41.7 km.

The K line was narrow and apparently fixed in position throughout the whole series of plates. The mean velocity derived from it was — 9 km.

Though wide lines of helium and several metallic emission lines were present in this spectrum no features in the photographic region other than the hydrogen lines were available for velocity determinations.

The widths of the emission components of Hγ and Hβ were not found to be certainly variable and were found to be in the mean 4.58Å and 5.47Å respectively.

CONCLUSION

Probably the most plausible conclusion suggested by the above observations of the spectrum of the star, H. R. 985, is that at least two bodies

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are involved, forming a system with a long orbital period—possibly about 2.3 years. The radial velocity variation may be attributed to orbital motion but on this basis the explanation of the probably simultaneous changes in the emission lines is not obvious, though it must appear reasonable that the two variations are related.

Apparently the variations which the Ann Arbor observations have brought out in the spectrum of this star account for Miss Maury's puzzling description quoted above of the Harvard objective prism plates of H. R. 985.

Evidently this star must be observed over a relatively long time interval, even though the variable phenomena prove to be periodic, before the story of its spectral variations can be written.

April 24, 1916.

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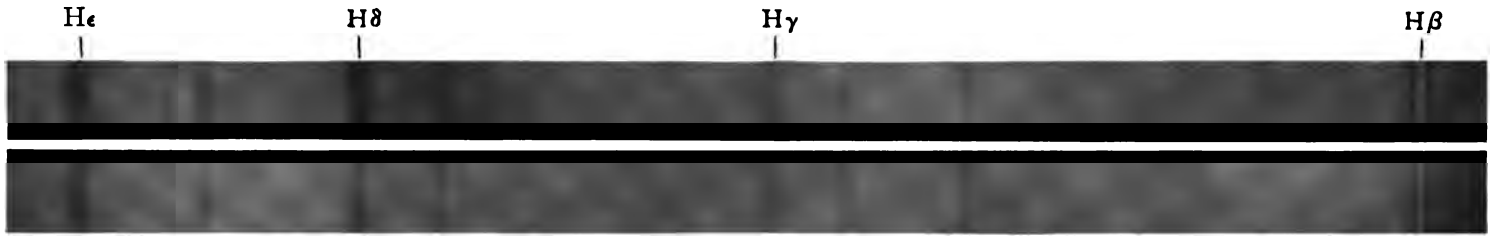

PLATE E. SPECTRA OF THE STAR, H. R. 985, SHOWING CHANGES IN Hε, Hδ, Hγ AND Hβ BETWEEN 1912 AND 1916

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SPECTROSCOPIC OBSERVATIONS OF STARS OF CLASS Md.

By PAUL W. MERRILL

INTRODUCTION

A large majority of all stars, telescopic as well as naked-eye, shine continually with a steady light, but an occasional one, known as a variable star, is subject to fluctuations in brightness. As exceptions to the usual order, such objects, naturally, have been of great interest to all watchers of the skies, and much attention has been devoted to them by professional and amateur astronomers, resulting in the accumulation of a large amount of information concerning them.

Upon close scrutiny it is found that nearly all variable stars are periodic, that is, they repeat their cycles of change after more or less regular intervals. Moreover, they naturally divide into two rather distinct groups according to the length of this interval, one group having periods of a few days, and the other completing their cycles of mutation in about a year. To distinguish between the two groups conveniently, they are called short-period and long-period variables.

Eighty-five per cent of all the long-period variables are red stars whose spectra are characterized by absorption bands of titanium oxide and also contain bright hydrogen lines. Spectra of this kind are comprised in Secchi's third type, or are of Class Md in the Harvard system. Every star known to belong to this spectral class is without exception a variable of long or irregular period. The fact that such a spectrum always indicates this type of variability is a remarkable one which has never received a satisfactory explanation. The present paper deals with stars having spectra of this kind.

HISTORICAL

Light Variations.—The first historical discovery of stellar variability relates to a star of Class Md. It was made in 1596, before the invention of the telescope, by Fabricius who noted the apparent creation of a star in the constellation Cetus. It later proved to be a long-period variable with a change in brightness carrying it in less than six months from complete invisibility without optical aid to a rather conspicuous position among the naked-eye stars. In 1603, Bayer, in the course of assigning designations to the naked-eye stars and ignorant of the variability discovered by Fabricius, gave this star the Greek letter omicron, so that it is known today as *o* Ceti. On account of its wonderful changes in light it was later given the special name, Mira, by Hevelius.

As its characteristics are typical of stars of this kind, it naturally serves as the example of long-period variation. The period approximates a year in length; the variation in light is great, usually from four to seven magnitudes, i. e., an increase varying from forty to six hundred and thirty times its minimum brightness, not being perfectly regular either in time or limiting magnitudes. The rise to maximum is more rapid than the decline. The star is decidedly red in color.

The second historical discovery of a variable star, in 1669, was of the short-period star Algol, but the next three discoveries were long-period stars of Class Md, given in this table:

STAR	YEAR OF DISCOVERY	PERIOD
o Ceti	1596	332 days
R Hydræ	1670	425
χ Cygni	1696	406
R Leonis	1782	313

The four stars listed are all of this kind which were discovered prior to 1800. During the nineteenth century 223 were found, 131 of them during the decade 1890 to 1900, a fact which illustrates the efficiency of the new methods of photography and spectroscopy. From 1900 to 1911 inclusive, 136 additional members of this group were discovered, making a total of 363 variable stars definitely known to have spectra of Class Md. Lists for the years since 1911 are not available. Only 15 of these stars are ever brighter

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than the sixth magnitude, or within the range of naked-eye observation. The writer has secured successful spectral photographs with the Ann Arbor 37½-inch reflector and attached spectrograph with reasonable exposure times down to magnitudes as low as 8.5. Within this limit there are 225 stars, most of which could not be profitably studied with less powerful apparatus. In any case, their observation is rather difficult on account of their faintness and variability. In many instances they are bright enough to be photographed for only a few weeks each year, at which time they may be in too close proximity to the sun for observation, so that the gathering of complete data for them would be a long program for a large observatory, and would necessarily require a station in the southern hemisphere since forty-six per cent of the 225 stars lie in the southern half of the sky.

While periodic in a general way, the successive cycles of long-period stars are by no means identical, so that it is impossible to predict exactly when the next maximum of a given star will occur, or exactly how bright it will be upon that occasion. This enhances the difficulty of observing such a star spectrographically and necessitates a special watch of each star near the predicted time of its maximum. The observing list, chosen from stars known to have reached a certain brightness in the past, is apt to contain stars whose maxima in any particular year are disappointingly faint.

Spectroscopic Observations.—Omicron Ceti, the brightest as well as the best known member of its group, has been by far the most extensively studied with the spectroscope. The continuous and absorption spectrum belongs to type III, as described by Secchi more than fifty years ago, having strong dark flutings which terminate abruptly on the violet side and shade off gradually in the other direction. The bright hydrogen lines were first detected, by means of photography, at Harvard, in 1886. The star has been photographed there at the time of nearly every maximum since that date, but complete descriptions of the photographs have never been published. Comparison with other similar stars early led to the following generalizations for variable stars of Class Md: at maximum the bright hydrogen lines are very strong relatively to the continuous spectrum; usually Hγ and Hδ are strong, and Hε is especially weak; there are great differences in the relative intensities of the bright lines in different stars and even variations in the same star. Brief descriptions of many spectra of Class Md have been published by the Harvard College Observatory, referring largely to the bright lines. Ten subdivisions, Md 1-10, were used by Mrs. Fleming, apparently depending both upon the relative strength of the bright hydrogen lines, particularly Hγ and Hδ, and upon the appearance of the continuous and absorption background.

Numerous other observers have photographed the spectrum of *o* Ceti at various maxima, as follows:

Vogel	1896	Potsdam
Sidgreaves	1897–98	Stonyhurst
Campbell	1898	Lick Observatory
Stebbins	1902	Lick Observatory
Plaskett	1906–07	Ottawa
Slipher	1906–07	Lowell Observatory
Wright	1909	Lick Observatory

Vogel[1] found the bright lines of hydrogen broad and strong, and somewhat displaced to the red, i. e., indicating a motion of recession. The series was seen from Hγ to Hι with the exception of Hε. No other bright lines were visible.

The following year the spectrum had much the same appearance on Sidgreaves' plates.[2] It remained substantially constant during two and a half months of observation, but there were changes in the relative intensities of different portions of the spectrum, showing that the blue light decreased during the interval as compared with the yellow green. Of the bright hydrogen lines, Hγ and Hδ were extraordinarily brilliant, Hε was absent, and Hβ uncertainly seen. He remarked upon this as follows: "But it is not easy to reconcile the comparatively weak absorption at this part of the band in Mira with its supposed absorbing action on the very energetic radiation of Hβ. * * * It seems more probable that *o* Ceti shows a condition of hydrogen radi-

[1] *Sitzungsberichte der Koeniglichen Preussichen Akademie der Wissenschaften zu Berlin*, 1896, p. 395.

[2] *Monthly Notices, R. A. S., 58*, 344, 1898.

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ance not yet met with in the laboratory, in which Hα and Hβ have fallen out of the spectrum."

At the next maximum in 1898, portions of the spectrum were photographed with three-prism dispersion by Campbell[3]. The radial velocity from the dark lines was measured and appeared constant at +62km. per second. Bright Hγ and Hδ were not monochromatic but broadened, at times appearing triple. Additional bright lines ascribed to iron were seen, which grew relatively stronger as the continuous spectrum faded. The bright lines, approximately consistent among themselves, gave a radial velocity some 15 km. less than that yielded by the absorption lines.

The spectrum of *o* Ceti was thoroughly studied during the decreasing phase in 1902, by Stebbins[4], at the Lick Observatory. His measures showed a difference of about 15 km. per second between the radial velocities derived from the bright lines and from the dark lines, in confirmation of the earlier observations by Campbell. Many details of the spectrum are recorded as studied with both high and low dispersion. An instructive diagram showing the relative intensities of numerous bright lines as the star waned is included. The changes are so great and of such a nature that their reality cannot be doubted.

The spectrum of *o* Ceti was also photographed by Plaskett[5] during December, 1906, and January, 1907, using three-prism dispersion. His radial velocities are in excellent agreement with those of Campbell and Stebbins. He confirmed the variations in the relative intensities of the emission lines. V. M. Slipher also secured spectrograms at this maximum[6], which are of particular interest and importance in that they contain the red end of the spectrum.

Observations which showed that the components of the triple bright Hγ line in the spectrum of *o* Ceti are not strongly polarized were made by Wright[7] in 1909. The multiple character is then not due to a magnetic field of constant direction.

[3] *Astrophysical Journal, 9,* 31, 1899.
[4] *Lick Observatory Bulletin, 2,* 78, 1903.
[5] *Journal, R. A. S., Canada, 1,* 45, 1907.
[6] *Astrophysical Journal, 25,* 66 and 235, 1907.
[7] *Lick Observatory Bulletin, 6,* 60, 1910.

Observations of χ Cygni are described by Eberhard in *Astrophysical Journal 18,* 198, 1903. Hydrogen and other bright lines were present and altered their relative intensities as the star waned, Hδ decreasing and 4308Å Fe increasing in strength. Velocities from bright and dark lines were discrepant as in *o* Ceti. In conclusion he refers to the highly interesting fact that χ Cygni and *o* Ceti "exhibit a precisely identical behavior, both as to the spectrum and as to the variations of the spectrum; whence it is highly probable that this sort of spectrum is typical for the long-period variables of that class." This idea is borne out in several respects by observations to be described in the present paper.

In *Monthly Notices R. A. S., 72,* 546, 1912, Espin gave an account of visual observations by himself and Maunder of R Cygni, a star with a peculiar spectrum but resembling this general type. Attached to the article is a description of a few spectrograms of the star made by Wright at Mt. Hamilton.

Professor R. H. Curtiss, while at Lick Observatory, obtained numerous spectrograms of W Cygni[8], a star possessing a spectrum which at times comes within the limits of Class Md. Measurements of these plates by the writer are recorded in the present paper.

One of the most important investigations bearing upon the nature of these stars was made in the laboratory by Fowler[9]. His experiments identified the conspicuous and characteristic flutings seen in spectra of Class M with those produced in the outer portions of the titanium arc and presumably due to titanium oxide. A photograph of the spectrum of *o* Ceti by Slipher was compared by Fowler with a negative copy of one of his exposures upon the arc; the two are remarkably and convincingly similar.

Summary of Spectroscopic Observations.—Nearly all long-period variable stars show essentially similar spectra known as Class Md, consisting of a continuous spectrum with numerous absorption lines and the absorption flutings of titanium oxide, with the addition of the bright

[8] *Lick Observatory Bulletin, 3,* 41, 1904.
[9] *Proc. Roy. Soc., 73,* 219, 1904.
Mon. Not. R. A. S., 69, 508, 1909.

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lines of hydrogen, which do not, in general, follow their laboratory intensities.

Two stars, *o* Ceti and χ Cygni, have been studied at the maximum phase, the former extensively. The bright lines, including a few due to other elements than hydrogen, show variable relative intensities, and are displaced to the violet about 0.25 Å with respect to the dark lines.

Radial velocities have been published for four stars:

o Ceti,
χ Cygni,
L_2 Puppis,[10]
R Cygni,

There is no evidence of variation in radial motion in any of the four.

No star has been followed spectroscopically through its entire cycle. The practical difficulties are the faintness of the stars at minimum, and the fact that the periods of many are almost a year so that a certain portion of the cycle is often lost for several successive years by the proximity of the sun.

ANN ARBOR OBSERVATIONS

The present observations were undertaken with the hope that they might be of value in two directions: first, to extend the investigations of *o* Ceti and χ Cygni, in a partial measure to numerous other stars for comparison, as tending to aid in the interpretation of this interesting type of variability; and, second, a determination of a considerable number of radial velocities as a contribution to our knowledge of the relation of these stars to those of other spectral classes. My observational material bears principally on the bright lines, and largely on those due to hydrogen. The exposures were often too short to record other bright lines or the continuous spectrum satisfactorily.

The Ann Arbor spectrograms were all obtained with the one-prism spectrograph in connection with the 37½-inch Cassegrain reflector. The first plates were secured with the instrument as described by Professor Curtiss in these *Publications*, Vol. 1, p. 37. On May 20, 1914, a prism of light flint glass was substituted for the original one of O:102 glass.[11] This prism was readjusted on Oct. 22, 1914, changing the dispersion somewhat.

Bright Lines.—There follows a record of the data yielded by the bright lines of forty-three stars, forty of them having been observed at Ann Arbor. The exposure time required here for a star of a given magnitude varied greatly with the character of the spectrum as well as with the observing conditions. For stars not fainter than 8.5 visual magnitude, good images of the stronger bright lines could usually be obtained in two hours. The great relative intensity of the emission lines is illustrated by the fact that they are frequently well exposed on plates where the continuous spectrum is very weak or invisible.

The principal comparison lines (titanium spark) together with the bright stellar lines are given below with their tabular micrometer settings for a screw of 0.5 mm. pitch. The origins of the stellar lines are indicated. In a few instances the wave-lengths first used were changed slightly in the following tables; in these cases the value is given to the tenth of an angstrom only.

Magnitude data from the American Association of Variable Star Observers has been an aid in making out the program since many of the stars could be observed only within a few weeks of maximum, and that epoch cannot be satisfactorily predicted. I am indebted to the president, Mr. William Tyler Olcott, and especially to Mr. Chas. B. Lindsley, who sent a considerable number of current magnitude determinations of the stars on my list.

The data appearing in the first line for each star are taken from the *Annals of the Harvard College Observatory*. The number appearing before the name of the star indicates the star's position; the first and second figures give the hour of right ascension, the third and fourth the minute, the fifth and sixth the degree of declination; italics denote southern declination. The time of the maximum was kindly supplied, in most cases, by Mr. Leon Campbell, through the

[10] *Lick Observatory Bulletin*, 7, 127, 1913.

[11] *Detroit Observatory Publications*, 1, 77, 1915.

[12] Clerke, *Problems in Astrophysics*, p. 362.

[13] Campbell, *Stellar Motions*, p. 303.

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courtesy of Professor Pickering. The column headed "Phase" gives the fraction of the period, before (—), or after (+), the time of maximum. The magnitude of the variable was estimated at the time of each spectroscopic observation by comparing it with adjacent stars in the four inch finder, using, in most cases, Hagen's magnitudes for the comparison stars. The estimated magnitude appears immediately after the date; if considered unreliable, the value is enclosed in parentheses. The intensities of the bright lines are roughly absolute: 1 indicating the weakest image of the rather wide slit, which could be measured with accuracy; "v" means visible, but weaker than 1. The velocities are in kilometers per second; a colon indicates uncertainty.

TABLE I.

WAVE-LENGTH	PRIOR TO 1914, MAY 22		MAY 22—OCT. 22		AFTER 1914, OCT. 22	
	MICR.	$\frac{DV}{DR}$	MICR.	$\frac{DV}{DR}$	MICR.	$\frac{DV}{DR}$
4981.93	36.097		37.616		39.428	
4885.27	39.192		40.531		42.360	
4861.53 Hβ	39.988	1825	41.280	1938	43.113	1926
4856.1	40.170		41.451		43.284	
4841.04	40.686		.....		43.773	
4512.93	53.584		.....		.....	
4399.90	58.938		59.042		61.015	
4367.8	60.561		60.557		62.540	
4340.63 Hγ	61.975	1311	61.875	1406	63.874	1394
4338.1	62.107		62.000		64.000	
4163.87	72.132		71.346		73.425	
4112.8	75.421		74.405		76.505	
4101.98 Hδ	76.150	1091	75.078	1174	77.186	1157
4082.63	77.454		76.295		78.413	
4078.65	77.727		76.548		78.668	
3924.69	89.272		.....		89.456	
3905.66 Si	90.847	917	88.695	993	90.925	984
3904.93	90.908		88.751		90.982	
3889.20 Hζ	92.239	903	89.978	979	92.222	969
3882.73	92.794		90.490		92.739	

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001755. T CASSIOPEIAE.

R.A. $0^h\ 17^m.8$; Decl. $+55°\ 14'$.

Class, Md8; Magnitude, 6.9 to 12.3. Period, 445 days. Time of maximum, double, 1914 Nov. 13, magnitude 8.3; 1915 March 13, magnitude 8.4.

G. M. T.	MAG.	PHASE	VELOCITY AND INTENSITY Hδ
1914 Nov. 27.61	8.5	0.0±	—27: underexposed.

001838. R ANDROMEDAE.

R.A. $0^h\ 18^m.8$; Decl. $+38°\ 1'$.

Class, Md2. Magnitude, 6.0 to 14.9. Period, 411 days. Time of maximum, May 25, 1915.

G. M. T.	MAG.	PHASE.	VELOCITY AND INTENSITY Hβ	Hγ	Hδ	MEAN
1915 May 30.84	7.3	0.0	—29:(1½)	—18::(v)	—38:(v)	—30
July 5.81	8.5	+0.1	—39 (1½)	(v)	—45:(v)	—41
					Mean,	—36

Both plates underexposed, and measures not very accurate.

012502. R PISCIUM.

R.A. $1^h\ 25^m.5$; Decl. $+2°\ 22'$.

Class, Md7. Magnitude, 7.6 to 13.5. Period, 344 days. Time of maximum, 1914 Nov. 14.

G. M. T.	MAG.	PHASE.	VELOCITY AND INTENSITY Hβ	Hγ	Hδ	Hζ	MEAN
1914 Nov. 6.69	8.3	0.0	—47::(½)	—58 (5)	—59 (4)	...	—58
Nov. 22.69	8.2	0.0	—66: (1½)	—58 (5½)	—59 (5)	(v)	—59
						Mean,	—59

021024. R ARIETIS.

R.A. $2^h\ 10^m.4$; Decl. $+24°\ 35'$.

Class, Md4. Magnitude, 7.5 to 13.7. Period, 187 days. Time of maximum, September 7, 1914.

G. M. T.	MAG.	PHASE.	VELOCITY AND INTENSITY Hβ	Hγ	Hδ	MEAN.
1914 Sept. 16.77	8.6	0.0	+106 (4)	+ 99 (4—)	+ 99 (2)	+100
Sept. 24.79	8.5	+0.1	+101:(2—)	+ 98 (2)	+106:(1)	+100
Sept. 26.78	8.5	+0.1	+102 (3—)	+109 (3)	+ 94 (2)	+102
					Mean,	+101

021143. W ANDROMEDAE.

R.A. $2^h\ 11^m.2$; Decl. $+43°\ 50'$.

Class, Md. Magnitude, 6.5 to 14.0. Period, 395 days. Time of maximum, February 23, 1915.

G. M. T.	MAG.	PHASE.	VELOCITY AND INTENSITY Hγ	Hδ	Hζ	MEAN
1915 Feb. 28.63	7.7	0.0	—40 (2)	—47 (5)	(v)	—44

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021403. *o* CETI.

R.A. $2^h\ 14^m.3$; Decl. $-3°\ 26'$.

Class, Md9. Magnitude, 1.7 to 9.6. Period, 332 days. Time of maximum, February 1, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY						
					Hγ	Hδ	Hζ	Hη	Hθ	H_4	MEAN
1914	Dec.	25.48	5.6	—0.1	+58(1+)	+51(2)					+54
		25.57	5.6	—0.1	+56(3)	+46(5)					+52
1915	Jan.	29.55	3.7	0.0	+55(7)	+47(12)	+64(3)	+69(1+)	+34(1—)	+64(1)	+53
	Nov.	5.68	6.6	+0.8	(Barely v)	(1)					....
		12.66	6.2	+0.8	+57(1)	+42(3)					+49
										Mean,	+52

A series of three-prism measures in 1897-1898 by Campbell and Wright (*Astrophysical Journal 9*, 32, 1899) of the bright Hγ line yielded a mean radial velocity of + 55 km. The velocity from bright Hδ was + 49 km. Measures in November, 1898, on four bright lines gave a mean of only + 44 km. In 1902, Stebbins (*Lick Observatory Bulletin, 2*, 93, 1902) found the mean of three bright lines with three prisms to be + 44 km., and the mean of four with one prism to be + 48 km.

The mean velocity given by Plaskett from measures of the bright Hγ line on fourteen three-prism plates, taken in December 1906 and January 1907, is + 46.1 km. (*Journal R. A. S., Canada, 1*, 53, 1907).

REMARKS ON ANN ARBOR SPECTROGRAMS.

1915 Jan. 29. The bright lines are somewhat stronger than on December 25.

1915 Nov. 5, 12. The bright lines are weak not showing the contrast with the continuous spectrum exhibited by the other plates. On Nov. 5, Hγ is so weak as not to be readily visible. It is easily seen on Nov. 12, but is not conspicuous as it is not much stronger than the adjacent continuous spectrum. This is strikingly at variance with its appearance on my earlier plates, and as observed by others. For instance, referring to three-prism observations in 1897 and 1898, Campbell says "If an exposure of an hour was required for recording the dark line spectrum, an exposure of two minutes under the same conditions would record the Hγ band," and in January 1907, Plaskett estimated "that the bright Hβ had an intensity about 15 times that of the continuous spectrum in that region, Hγ about 25 times and Hδ at least 50 times." The star was considerably past maximum at this time.

050953. R AURIGAE.

R.A. $5^h\ 9^m.2$; Decl. $+53°\ 28'$.

Class, Md. Magnitude, 6.5 to 13.8. Period, 459 days. Time of maximum, April 12, 1915.

G. M. T.			MAG.	PHASE.	VELOCITY AND INTENSITY		
					Hγ	Hδ	Hζ
1915	March	7.70	8.6	—0.1		—14 (2)	(v)
		12.72	8.5	—0.1	(barely v)	+ 1 (2)	
						Mean, — 9:	

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061702. V MONOCEROTIS.

R.A. $6^h\ 17^m.7$; Decl. $-2°\ 9'$.

Class, Md7. Magnitude, 7.2 to less than 12.9. Period, 332 days. Time of Maximum, February 17, 1915.

	G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY				
					Hγ	Hδ	Hζ	Hη	MEAN.
1915	Feb.	19.64	7.6	0.0	+14 (3)	+12 (5)	+21 (1)	(v)	+15
		19.68	7.6	0.0	+26 (1)	+12 (2)			+19
								Mean,	+16

065355. R LYNCIS.

R.A. $6^h\ 53^m.0$; Decl. $+55°\ 28'$.

Class, Md1. Magnitude, 7.0 to 13.8. Period, 379 days. Time of maximum, March 4, 1915.

	G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY			
					Hβ	Hγ	Hδ	MEAN
1915	Feb.	28.70	8.2	0.0	+16 (8)	+ 9 (3)	−6 (1)	+ 9
		28.74	8.2	0.0	+20:(7)	+16 (3½)	−3 (1)	+10
	Mar.	12.78	8.1	0.0	+12 (5½)	+19 (3)	(½)	+16
		28.71	8.3	+0.1	+ 9 (3)	+11 (2)		+10
							Mean,	+11

071044. L_2 PUPPIS.

R.A. $7^h\ 10^m.5$; Decl. $-44°\ 29'$.

Class, Md6. Magnitude. 3.4 to 6.2. Period, 140 days. Observations by D. O. Mills Expedition, Santiago, kindly communicated by Director Campbell, as follows:

Two-prism plates 1911. Bright lines are single and narrow.

Br. Hγ and Hδ	+49
Br. Hγ and Hδ	+49
Br. Hγ	+57

One-prism plates in 1914.

Br. lines	+50
Br. lines	+51
Mean of all	+51

084803. S HYDRAE.

R.A. $8^h\ 48^m.4$; Decl. $+3°\ 27'$.

Class, Md4. Magnitude, 7.5 to 13.0. Period, 256 days. Time of maximum, April 3, 1915.

	G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY				
					Hβ	Hγ	Hδ	Hζ	MEAN
1915	March	26.61	8.3	0.0	+74 (1½)	+79 (3)	+74 (3—)	+77 (½)	+77

085008. T HYDRAE.

R.A. $8^h\ 50^m.8$; Decl. $-8°\ 46'$.

Class, Md4. Magnitude, 7.0 to 13.1. Period, 289 days. Time of maximum, February 22, 1915.

	G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN
1915	Feb.	19.73	8.0	0.0	−10 (2)	− 9 (2+)	−10
		26.67	8.0	0.0	−13 (2)	−14 (3)	−14
						Mean,	−12

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093934. R LEONIS MINORIS.

R.A. $9^h\ 39^m.6$; Decl. $+34°\ 58'$.

Class, Md8. Magnitude, 7.0 to 13.0. Period, 370 days. Time of maximum, January 14, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY							
					Hγ	Hδ	3905	Hζ	Hη	Hθ	H_ι	MEAN
1915	Jan.	29.68	8.0	0.0	+ 5(2)	— 6(5)		(v)				— 1
		29.76	8.0	0.0	—13(2)	—13(6)		(1)				—13
	Feb.	19.83	8.2	+0.1	— 8(4+)	—10(9)	—3(1)	+5(3)	+4(2)	—9(1)	+4(⅔)	— 4
											Mean,	— 6

1915 Feb. 19.83, bright Fe 4202.20 (Rowland) gives a velocity of + 1km.

094211 R LEONIS.

R.A. $9^h\ 42^m.2$; Decl. $+11°\ 54'$.

Class, Md10. Magnitude, 4.6 to 10.5. Period, 313 days. Time of maximum, February 22, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY					
					Hγ	4202	Hδ	3905	Hζ	MEAN
1913	Dec.	14.98	8.6	—0.2			(v)			
		21.88	8.4	—0.1			0(2)			0
1914	Feb.	1.80	6.7	0.0	0(3)		+4(9)			+2
		27.74	6.7	+0.1	—2(3)	(1)	0(7)		+5:(1)	0
		28.76	6.8	+0.1	—4(5)	—1(2)	0(11)	+2(1)	+7:(2)	0
	Mar.	14.74	6.8	+0.1	—3(5)	+3(2)	—1(10)	(v)	+3 (1)	0
		19.63	6.85	+0.2	—3(1)		—5(2+)			—4
	Apr.	11.60	8.0	+0.2	—1(4)	—1(2+)	+4(7)		(v)	0
		30.61	8.1	+0.3	(v)		(v)			
									Mean,	0.0

Observation by Lick Observatory, kindly communicated by Director Campbell.
Three-prism plate, bright Hγ, — 10 km.

103769. R URSAE MAJORIS.

R.A. $10^h\ 37^m.6$; Decl. $+69°\ 18'$.

Class, Md8. Magnitude, 7.0 to 13.5. Period, 302 days. Times of maximum, May 14. 1914, March 1, 1915.

G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY						
				Hβ	Hγ	Hδ	3905	Hζ	Hη	MEAN
1914	May 30.64	8.1	+0.1		+27(6)	+20(9)		+33 (2—)	(v)	+25
1915	Feb. 19.83	(7.0)	0.0		+25(7)	+23(8)	(v)	+35:(2)		+26
	26.74	(6.8)	0.0	+10::(v)	+17(5)	+21(7)		+28 (1+)	(v)	+19
									Mean,	+23

A narrow maximum, possibly a bright line, was measured on 1915, February, 26.74, at 4138.75, Å (reduced to zero velocity).

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121418. R CORVI.

R.A. $12^h\ 14^m.4$; Decl. $-18°\ 42'$.

Class, Md6. Magnitude, 7.5 to 12.6. Period, 318 days. Time of maximum, May 19, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN
1915	May	9.68	7.6	0.0	−36 (2)	−40 (2+)	−39
	June	4.62	7.8	+0.1	−29 (1)	−28 (1+)	−29
						Mean,	−34

123160. T URSAE MAJORIS.

R.A. $12^h\ 31^m.8$; Decl. $+60°\ 2'$.

Class, Md6. Magnitude, 6.4 to 13.1. Period, 257 days. Time of maximum, April 27, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN.
1914	Apr.	11.75	8.3	−0.1	−106 (2)	−112 (3)	−109
	May	2.68	8.1	0.0	−101 (2)	−110 (3)	−105
		16.74	8.4	+0.1	−107 (1)	−108 (1½)	−107
						Mean,	−107

123307. R VIRGINIS.

R.A. $12^h\ 33^m.4$; Decl. $+7°\ 32'$.

Class, Md5. Magnitude, 6.4 to 12.1. Period, 146 days. Time of maximum, April 14, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN
1915	Mar.	28.79	7.9	−0.1	−23 (3)	−42 (4)	−32
	Apr.	16.66	7.9	0.0	−27 (1)	−26 (⅔)	−27
	May	7.60	8.1	+0.1	−46 (2−)	−40 (1)	−44
						Mean,	−35

123961. S URSAE MAJORIS.

R.A. $12^h\ 39^m.6$; Decl. $+61°\ 38'$.

Class, Md4. Magnitude, 7.3 to 12.5. Period, 226 days. Time of maximum, June 19, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hβ	Hγ	MEAN
1915	Apr.	25.66	8.5	−0.2	−2::		
	May	7.69	8.5	−0.2	−6::		
	June	11.68	8.1	0.0	+6 (2)	0 (2−)	+2
	June	25.66	7.8	0.0	+2 (3)	−4 (4)	−2
						Mean,	−1

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132422. R HYDRAE.

R.A. $13^h\ 24^m.2$; Decl. $-22°\ 46'$.

Class, Md. Magnitude, 4.0 to 9.8. Period, 425 days. Time of maximum, April 1, 1914, May 7, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY					
					Hβ	Hγ	Hδ	Hζ	Hη	MEAN
1914	May	28.65	(6.6)	+0.1		—22(3)	—24(5)	(v)		—23
1915	May	30.61	4.9	+0.1	—28(2)	—32(5)	—27(7)	—15(2)	(1)	—27
		30.64	4.9	+0.1	—28(1)	—28(5)	—30(7)	(1)	(v)	—29
									Mean,	—26

A narrow maximum, possibly a bright line, measured on 1915, May, 30.61 at 4202.38 Å (reduced to zero velocity.) If this is Fe 4202.20 (Rowland), it gives a velocity of — 15 km. reduced to sun.

Observations by Lick Observatory, kindly communicated by Director Campbell: two three-prism plates, bright Hγ, — 26 km.

134440. R CANUM VENATICORUM.

R.A. $13^h\ 44^m.6$; Decl. $+40°\ 2'$.

Class, Md9. Magnitude, 6.1 to 12.7. Period, 333 days. Time of maximum, April 2, 1914, March 5, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY				
					Hγ	Hδ	Hζ	Hη	MEAN
1914	May	30.74	(8.2)	+0.2	—24(2)	—20(3)			—22
	June	11.66	8.1	+0.2	—13(1)	—22(1½)			—19
1915	Mar.	26.69	7.3	+0.1	—29(2)	—24(5)	(1)	(v)	—26
		26.74	7.3	+0.1	—32:(2)	—25(5)	—13(1+)	(v)	—25
								Mean,	—24

142539. V BOOTIS.

R.A. $14^h\ 25^m.7$; Decl. $+39°\ 18'$.

Class, Md7. Magnitude, 6.9 to 11.0. Period, 256 days. Time of maximum, May 24, 1914, Feb. 29, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY				
					Hγ	Hδ	Hζ	Hη	MEAN
1914	May	28.74	7.75	0.0	—39 (1)	—31(2—)			—34
1915	Feb.	28.90	7.6	0.0	—48 (2)	—44(3½)	—46(1—)	(v)	—46
	Mar.	12.88	7.7	+0.1	—39:(2)	—41(3)			—40
								Mean,	—41

143227. R BOOTIS.

R.A. $14^h\ 32^m.8$; Decl. $+27°\ 10'$.

Class, Md4. Magnitude, 6.6 to 12.9. Period, 223 days. Time of maximum, April 5, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY			
					Hγ	Hδ	Hζ	MEAN
1915	Feb.	26.85	8.2	—0.2	—52(5)	—57(5—)		—54
		28.81	8.3	—0.2	—49(5)	—55(4)	(v)	—52
	Mar.	7.81	7.9	—0.1	—53(5)	—62(4)		—58
		26.81	7.3	0.0	—59(6)	—57(5)	(1)	—58
	Apr.	16.73	7.6	+0.1	—60(3)	—65(2)		—62
							Mean,	—57

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151731. S CORONAE.

R.A. $15^h\ 17^m.3$; Decl. $+31°\ 44'$.

Class, Md9. Magnitude, 6.7 to 12.7. Period, 361 days. Time of maximum, February 26, 1914, February 13, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY								
					Hβ	Hγ	Hδ	3905	Hζ	Hη	Hθ	H_ι	MEAN
1914	Apr.	9.90	8.6	+0.1		—13(2)	—24(5)						—21
1915	Feb.	26.93	7.3	0.0	—22(2)	—26(10)	—26(14)	—24(1)	—13(3+)	—13(2)	(1)	—12(1+)	—22
	Mar.	7.80	7.4	+0.1	—32:(1½)	—22(6)	—25(9)	—18:(1—)	—15(4)	—15(2+)	—27(1+)	— 8(1+)	—21
		7.94	7.4	+0.1	(1—)	—25(8)	—26(12)	(v)	—17:(2)	(1)	(v)	(v)	—24
												Mean,	—22

A narrow maximum, possibly a bright line, was measured as follows:—

1915	Feb. 26.93	4138.95Å	Wave length reduced to zero velocity.
	Mar. 7.80	4138.91	

This line was also measured by the writer in the spectrum of 103769 R Ursae Majoris, 4138.75 Å, and by Plaskett in the spectrum of o Ceti, 4138.78 Å (*Journal R. A. S., Canada, 1,* 55, 1907).

154615. R SERPENTIS.

R.A. $15^h\ 46^m.1$; Decl. $+15°\ 26'$.

Class, Md8. Magnitude, 5.6 to 13. Period, 357 days. Time of maximum, April 1, 1914, April 12, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY				
					Hβ	Hγ	Hδ	Hζ	MEAN
1914	Apr.	11.88	(6.5)	0.0	+ 5(7)	+13(14)	+9(10)	(v)	+10
		30.72	(6.3)	+0.1	+13:(1)	— 3(3)	+9(3)		+ 5
	May	2.79	(6.3)	+0.1	(1½)	(5)	(4)	Comps. poor	
		14.70	7.0	+0.1	(1)	+ 7(4)	+5(4)		+ 6
1915	Mar.	26.89	(7.6)	0.0	(?)	+19(5)	+5(8)	+22:(1)	+ 9
								Mean,	+ 8

162119. U HERCULIS.

R.A. $16^h\ 21^m.4$; Decl. $+19°\ 7'$.

Class, Md(8). Magnitude, 6.4 to 12.0. Period, 403 days. Time of maximum, May 5, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN
1915	Apr.	16.81	7.7	0.0	—35(1)	—43(4)	—40
		25.77	7.2	0.0	—43(1—)	—40(3)	—41
	May	7.79	7.6	0.0	—34(2)	—47(5)	—40
						Mean,	—40.4

The class of this star appears in *Annals H. C. O.,* Vol. 56, p. 203 as Md. However, from an examination of Mrs. Fleming's record book, I believe that she meant to classify it as Md8. This is in agreement with both the Harvard and Ann Arbor spectra.

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163137. W HERCULIS.

R.A. $16^h\ 31^m.7$; Decl. $+37°\ 32'$.

Class, Md5. Magnitude, 7.8 to 13.5. Period, 280 days. Time of maximum, July 11, 1915.

G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY		
				Hγ	Hδ	MEAN
1915 June	11.80	8.6	—0.1	—50(1½)	—62(2)	—57
	13.70	8.5	—0.1	—61(2)	—59(3—)	—60
					Mean,	—59

164715. S HERCULIS.

R.A. $16^h\ 47^m.4$; Decl. $+15°\ 7'$.

Class, Md6. Magnitude, 7.3 to 12.6. Period, 308 days. Time of maximum, July 28, 1914, May 26, 1915.

G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY				
				Hβ	Hγ	Hδ	Hζ	MEAN
1914 Aug.	8.66	7.8	0.0		—15(1)	—23(1+)		—21
	15.69	7.7	+0.1	(1—)	—22(3)	—22(3)		—22
1915 May	30.69	(7.4)	0.0		—21(1)	—22(2)	(v)	—21
							Mean,	—21.3

170215. R OPHIUCHI.

R.A. $17^h\ 2^m.0$; Decl. $-15°\ 58'$.

Class, Md8. Magnitude, 7.1 to 13.6. Period, 302 days. Time of maximum, June 24, 1915.

G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY		
				Hγ	Hδ	MEAN
1915 June	20.73	7.6	0.0	—64 (1)	—59 (1½)	—61
	27.70	7.6	0.0	—54 (3)	—59 (3)	—57
					Mean,	—59

171401. Z OPHIUCHI.

R.A. $17^h\ 14^m.5$; Decl. $+1°\ 37'$.

Class, Md(2). Magnitude, 7.5 to 12.5. Period, 348 days. Time of maximum, May 27, 1914, May 2, 1915.

G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY			
				Hβ	Hγ	Hδ	MEAN
1914 June	11.76	8.0	0.0	(1—)	— 86:(1)	—80:(1½)	—83
	18.74	7.8	+0.1	—103:(1)	— 84 (1½)	—92:(v)	—89
July	2.66	8.1	+0.1	— 97:(2)	—102 (3)	—95 (1)	—99
1915 May	30.76	7.8	—0.1	— 98 (3)	— 92 (2)	—99:(1)	—96
						Mean,	—93

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180531. T HERCULIS.

R.A. $18^h\ 5^m.3$; Decl. $+31°\ 0'$.

Class, Md3. Magnitude, 7.2 to 13.6. Period, 165 days. Time of maximum, Sept. 27, 1914; March 3, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY			
					Hβ	Hγ	Hδ	MEAN
1914	Sept.	16.60	8.4	—0.1		—135 (2)	—131 (2+)	—133
		24.60	(8.5)	0.0		—117:(2—)	—123:(2)	—120
	Oct.	2.63	8.3	0.0		—132 (2)	—125 (2)	—129
1915	Mar.	7.90	8.0	0.0	—138:(2)	—132 (4—)	—133 (3)	—132
							Mean,	—130

181136. W LYRAE.

R.A. $18^h\ 11^m.5$; Decl. $+36°\ 38'$.

Class, Md. Magnitude, 7.6 to 12.5. Period, 197 days. Time of maximum, 1915, Jan. 14.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY			
					Hβ	Hγ	Hδ	MEAN
1914	July	9.67	8.2	—0.0	(v)	—184 (2+)	—198 (2—)	—191
		9.78	8.2	—0.0	—183 (1+)	—179 (3)	—184 (2)	—182
		18.64	7.7	+0.1	—172:(1)	—184 (2)	—191 (2—)	—184
							Mean,	—186

It is an interesting coincidence, showing the great velocity of approach, that the bright Hγ line lies very nearly between the two components of the titanium comparison line 4338.10 Å. The bright hydrogen lines appear monochromatic, i. e., no wider than comparison lines of equal strength.

183308. X OPHIUCHI.

R.A. $18^h\ 33^m.6$; Decl. $+8°\ 44'$.

Class, Md8. Magnitude, 6.5 to 9.0. Period, 335 days. Time of maximum, Nov. 3, 1914, Oct. 9, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY			
					Hγ	Hδ	Hζ	MEAN
1914	Nov.	27.48	7.0	+0.1	—95:(1)	—89:(1—)		—91
1915	June	4.73	8.5	—0.4	Faint continuous; no bright lines.			
	July	5.67	8.1	—0.3		—85:		—85
	Aug.	6.67	7.6	—0.2	—76::(½)	—84 (2+)	—60::(v)	—83
	Aug.	9.63	7.55	—0.2	—93::(½)	—86 (2)	(v)	—86
							Mean,	—86

1915 July 5.67, bright Hδ does not appear monochromatic, being sharper on the violet edge.

193311. RT AQUILAE.

R.A. $19^h\ 33^m.3$; Decl. $+11°\ 30'$.

Class, Md9. Magnitude, 8.0 to <13. Period, 326 days. Time of maximum, Aug. 4, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN
1914	July	18.76	8.2	—0.1	—51 (2)	—61 (3)	—55
		25.66	8.3	0.0	—48 (2)	—58 (3½)	—54
		25.77	8.3	0.0	—50 (3)	—59 (4)	—54
						Mean,	—54.2

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193449. R CYGNI.

R.A. $19^h\ 34^m.1$; Decl. $+49°\ 58'$.

Class, Peculiar. Magnitude, 6.6 to 13.9. Period, 426 days. Observations by Wright at Lick Observatory. *Mon. Not. R. A. S., 72,* 548, 1912, as follows:

			PHASE	VELOCITY AND INTENSITY				
				Hα	Hβ	Hγ	Hδ	MEAN
1911	Dec.	5	+		—43	—30	—10:	—30
		11	+		—43 (10)	—30 (4)	—33 (2)	—34
1912	Jan.	13	+	—78::(10+?)	—56 (10)			
				Mean,	—47	—30	—26	—34

Several other bright lines were suspected.

194048. RT CYGNI.

R.A. $19^h\ 40^m.8$; Decl. $+48°\ 32'$.

Class, Md5. Magnitude, 6.7 to 12.0. Period, 190 days. Time of maximum, Oct. 2, 1914, April 12, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY				
					Hβ	Hγ	Hδ	Hζ	MEAN
1914	Sept.	24.67	7.6	0.0	—140: (2+)	—123 (4)	—129 (3)	—116::(½)	—127
		24.73	7.6	0.0	—135::(1)	—125 (3—)	—127 (2)		—127
		26.70	(7.6)	0.0	—130 (2)	—129 (4)	—119 (3)		—125
	Oct.	11.58	7.4	+0.1	—113::(2—)	—129 (4)	—129 (3)		—128
								Mean,	—126.8

Five plates were obtained by Frost and Parkhurst during the first half of December, 1905. Bright Hγ and Hδ gave a radial velocity of — 100 km. *Publications Astronomical and Astrophysical Society of America, 1,* 244, 1910.

194632. χ CYGNI.

R.A. $19^h\ 46^m.7$; Decl. $+32°\ 40'$.

Class, Md6. Magnitude, 4.0 to 13.5. Period, 406 days. Time of maximum, Dec. 13, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY					
					Hβ	Hγ	Hδ	Hζ	Hη	MEAN
1914	Nov.	27.52	5.1	0.0	—17 (2+)	—14 (3)	—20 (5)	—15 (1)	(v)	—16
		27.55	5.1	0.0	—24 (2½)	—13 (3)	—21 (5+)	(v)		—18
									Mean,	—17

Eberhard (*Ap. J., 18,* 198, 1903) records velocities from the bright lines as follows:—

1901; 26 plates from Aug. 2 to Nov. 23. Hγ —19.7 km.
9 plates from Sept. 7 to Nov. 23. Fe 4308 —20.3
1902; 18 plates from Sept. 22 to Dec. 12. Hγ —21.0

Hγ was hazy toward the red as was also Hδ. From 1901, Aug. 2 until Sept. 19, Hδ was considerably stronger than Hγ; from Oct. 3 to 15, Hγ and Hδ differed little from each other. On Oct. 26 they were equal; and on Nov. 9 and 23 Hγ was brighter than Hδ. The Fe line λ 4308 increased in brightness the fainter the star became. Other bright lines were observed; Hγ, Hδ, Hζ, Hθ, H_ι, 3905.8 Si?, Fe 4402 and other Fe lines.

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205923. R VULPECULAE.

R.A. $20^h\ 59^m.9$; Decl. $+23°\ 26'$.

Class, Md6. Magnitude, 7.5 to 12.1. Period, 137 days. Time of maximum, June 16, 1914, Nov. 2, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY			
					Hβ	Hγ	Hδ	MEAN
1914	May	30.84	8.4	−0.1		−15 (1½)	− 5:(1)	−12
	June	25.78	8.3	+0.1	−9:(½)	−21 (1½)	−27 (1)	−22
	July	4.81	8.7	+0.1		0::(v)	−13:(v)	−10
							Mean,	−17

210868. T CEPHEI.

R.A. $21^h\ 8^m.2$; Decl. $+68°\ 5'$.

Class, Md9. Magnitude, 5.1 to 10.5. Period, 387 days. Time of maximum, Nov. 13, 1914.

G. M. T.		MAG.	PHASE	VELOCITY AND INTENSITY					
				Hγ	Hδ	Hζ	Hη	Hθ	MEAN
1914 Nov.	6.56	(6.7)	0.0	−36 (2−)	−33 (4)	−22 (1)	(v)		−32
	6.60	(6.7)	0.0	−28 (3)	−31 (5+)	−20 (2−1)	(v)	(v)	−29
							Mean,		−30

213244. W CYGNI.

R.A. $21^h\ 32^m.2$; Decl. $+44°\ 56'$.

Class, Mc. Magnitude, 5.0 to 6.7. Period, 132 days. The following are measures made by the writer of plates taken by Professor R. H. Curtiss at the Lick Observatory in 1903, with spectrograph I. See *Lick Bulletin 3*, 41, 1904. I am indebted to Professor Curtiss and to Director Campbell for the opportunity of using them.

			RADIAL VELOCITY		
			Hγ	Hδ	MEAN
1903	Aug.	3	−21	−18	−19
	Aug.	7	−27	−36	−31
	Aug.	26	−24	−36	−30
	Sept.	13		−23	−23
				Mean,	−26

NOTES ON BRIGHT HYDROGEN LINES FROM THESE AND OTHER PLATES.

1903 July 19 Bright lines weak, if present.
Aug. 3 Hγ3, Hδ4, both showing fair contrast with the background.
Aug. 9 Probably the same as Aug. 3.
Aug. 14 Hγ4, Hδ5, contrast a little stronger.
Aug. 17 Contrast slightly stronger.
Aug. 26 Contrast slightly less.
Sept. 4 Contrast of bright lines decidedly less than on Aug. 26. The bright lines are not sharply distinguished from the background.
Sept. 6 About the same as Sept. 4.
Sept. 13 Hγ is weak, if present, not being as strong as several spaces of the continuous spectrum near-by. Bright Hδ is just seen.
Sept. 23–Dec. 28. (Ten plates.) Bright lines not distinguishable; very weak, if present.

The times of maximum were "early in August and in the middle of December." Minimum occurred about October 12. It appears then that bright lines may *or may not* be seen as the star reaches maximum.

An underexposed plate taken at Ann Arbor on July 2, 1914, shows Hδ, and possibly also Hγ, faintly bright.

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231425. W PEGASI.

R.A. $23^h\ 14^m.8$; Decl. $+25°\ 44'$.

Class, Md8. Magnitude, 7.5 to 13.5. Period, 342 days. Time of maximum, July 6, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY		
					Hγ	Hδ	MEAN
1914	July	25.85	7.8	+0.1	—39 (1)	—40 (3)	—40
		30.78	7.8	+0.1	—30 (1)	—32 (3)	—31
		30.86	7.8	+0.1	—31 (1)	—33 (3½)	—33
						Mean,	—35

231508. S PEGASI.

R.A. $23^h\ 15^m.5$; Decl. $+8°\ 2'$.

Class, Md8. Magnitude, 7.8 to 12.9. Period, 318 days. Time of maximum, July 16, 1915.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY				
					Hβ	Hγ	Hδ	Hζ	MEAN
1915	June	25.81	8.3	—0.1	(v)	—4 (4)	—9 (5)	(v)	—6
		27.81	8.2	—0.1		—7 (5)	—9 (4)	(v)	—8
								Mean,	—7

235350. R CASSIOPEIAE.

R.A. $23^h\ 53^m.3$; Decl. $+50°\ 50'$.

Class, Md8. Magnitude, 5.3 to 12.8. Period, 432 days. Time of maximum, Oct. 4, 1914.

G. M. T.			MAG.	PHASE	VELOCITY AND INTENSITY						
					Hβ	Hγ	Hδ	3905	Hζ	Hη	MEAN
1914	Aug.	15.77	(7.5)	—0.1		+25:(1)	+12 (2+)				+16:
	Sept.	9.68	6.5	—0.1	+10 (v)	+ 9 (15)	+ 8 (22)	+10 (1)	+15 (3+)	(v)	+ 9.7
		9.72	6.5	—0.1	(?)	+10 (12)	+ 6 (18)	(1—)	+21 (3)	(v)	+ 9.6
		9.75	6.5	—0.1		+ 8 (6)	+ 6 (10)		(v)		+ 7.
		16.70	6.5	0.0		+12 (3)	+ 8 (5)				+10.
	Oct.	11.64	6.4	0.0		+ 8 (2)	+ 6 (3)				+ 7.
										Mean,	+ 9.4

BRIGHT-LINE RADIAL VELOCITY

Sept.	9.68	4007.35 Å	+ 26 km.
	9.72	4007.13	+ 10

The assumed normal wave-length of 4007.0 was deduced from Stebbins' measures of the line in the spectrum of o Ceti.

The radial velocities from the bright lines are collected in Table II. In no instance is the motion thought to be variable. The residual velocities are computed on the assumption that the sun is approaching the point $\alpha = 270°\ 0'$, $\delta = +28°\ 0'$, with a speed of 20.0 km. per second. The mean taken without regard to sign is 36.5 km. The algebraic mean is —21.4 km. It is noteworthy, although possibly only a coincidence, that this value has the same sign and about the same magnitude as the average divergence between bright and dark line velocities to be discussed later in this paper.

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TABLE II. RADIAL VELOCITIES FROM BRIGHT LINES.

	STAR.	OBSERVED VELOCITY.	RESIDUAL VELOCITY.	REMARKS.
001755	T Cassiopeiae	— 27 km.	— 20 km.	
001838	R Andromedæ	— 36	— 31	
012502	R Piscium	— 59	— 65	
021024	R Arietis	+101	+ 96	
021143	W Andromedæ	— 44	— 44	
021043	o Ceti	+ 52	+ 42	
050953	R Aurigæ	— 9	— 12	
061702	V Monocerotis	+ 16	— 2	
065355	R Lyncis	+ 11	+ 9	
071044	L_2 Puppis	+ 51	+ 32	D. O. Mills Expedition.
084803	S Hydræ	+ 77	+ 65	
085008	T Hydræ	— 12	— 26	
093934	R Leonis Minoris	— 6	— 9	
094211	R Leonis	0	— 8	
103769	R Ursæ Majoris	+ 23	+ 30	
121418	R Corvi	— 34	— 36	
123160	T Ursæ Majoris	—107	— 98	
123307	R Virginis	— 35	— 31	
123961	S Ursæ Majoris	— 1	+ 9	
132422	R Hydræ	— 26	— 24	
134440	R Canum Venaticorum	— 24	— 12	
142539	V Bootis	— 41	— 27	
143227	R Bootis	— 57	— 43	
151731	S Coronæ	— 22	— 6	
154615	R Serpentis	+ 8	+ 25	
162119	U Herculis	— 40	— 22	
163137	W Herculis	— 59	— 40	
164715	S Herculis	— 21	— 5	
170215	R Ophiuchi	— 59	— 45	
171401	Z Ophiuchi	— 93	— 75	
180531	T Herculis	—130	—110	
181136	W Lyræ	—186	—166	
183308	X Ophiuchi	— 86	— 67	
193311	RT Aquilæ	— 54	— 36	
193449	R Cygni	— 34	— 15	Wright
194048	RT Cygni	—127	—109	
194632	χ Cygni	— 17	+ 1	
205923	R Vulpeculæ	— 17	— 2	
210868	T Cephei	— 30	— 17	
213244*	W Cygni	— 26	— 12	
231425	W Pegasi	— 35	— 27	
231508	S Pegasi	— 7	— 2	
235350	R Cassiopeiæ	+ 9	+ 17	
	Algebraic Mean		— 21.4	
	Arithmetic Mean		36.5	

* Plates by R. H. Curtiss at Lick Observatory, measured by Merrill.

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If each of the residual velocities be corrected by + 17 km., there will be 21 positive, 1 zero, and 21 negative values. The arithmetic mean, 32.1 km., will then be a minimum. This would not be reduced by using the dark line velocities where known. We may then take the average value of the residual motion as derived from the present data as 32 km., making them apparently the swiftest of any class of stars so far investigated.

There are strong indications, however, that the observed velocities are not wholly at random but are affected by a motion systematic with regard to the extensive group of stars from which the solar motion has been determined. The correction to remove the velocity due to the sun's motion is $V\odot \cos d$, where $V\odot$ is the sun's speed, and d is the angle between the star and the solar apex. The residual velocities are collected in Table III with respect to this correction, where $V\odot$ is taken as 20.0 km. This is done in order that the effect of assuming other values of the sun's speed may be easily estimated. In view of the small number of stars, their large individual motions, and their lack of uniform distribution over the sky, it has not been considered advisable to make a solution for the sun's motion. It is obvious, however, from Table III, that if the apex be assumed as $\alpha = 270°\ 0', \delta = +28°\ 0'$, a velocity much higher than 20.0 km. would result, or in other words, these 43 stars seem to have a systematic motion in the general direction of the solar ant-apex.

TABLE III. RESIDUAL VELOCITIES FROM BRIGHT LINES BY ZONES.

20 cos d	AVERAGE RESIDUAL VELOCITY.	NO. OF STARS.
— 20 to — 10	+ 22	5
— 9 to — 5	+ 8	3
— 4 to 0	— 16	5
+ 1 to + 5	— 22	4
+ 6 to + 10	— 15	6
+ 11 to + 15	— 23	7
+ 16 to + 20	— 48	13

Table IV exhibits the relation between residual velocity and maximum magnitude as taken from *Annals H. C. O., 56,* pp. 197 ff. It indicates that the fainter stars are moving with considerably greater rapidity than the brighter ones. Since only a few of these stars are as bright, even at maximum, as the stars of other spectral classes which have entered into average radial velocities, it remains to be seen whether the apparently great radial velocities of stars of Class Md should be considered as an effect of spectral class or of magnitude.

TABLE IV. AVERAGE RESIDUAL VELOCITY FROM BRIGHT LINES, AND MAXIMUM MAGNITUDE.

MAGNITUDE AT MAXIMUM.	AVERAGE RESIDUAL VELOCITY.	NO. OF STARS.
0.0 to 5.0	20 km.	6
5.1 to 6.5	34	11
6.6 to 7.2	35	13
7.3 to 8.0	48	13

With very few exceptions, the bright lines observed in these stars by the writer have appeared monochromatic with the dispersion used, i. e., no wider than titanium comparison lines of the same intensity. Other observers have frequently (but not always) found them narrow with three prism dispersion. A central reversal has, I believe, never been noted. These facts together with the rapid changes in intensity, which they undergo, have given the writer the impression that the emission lines are, in general, high level phenomena.

When a hydrogen tube is made luminous in the laboratory by the passage of an electric discharge, the spectral lines decrease in strength in the order $H\alpha$, $H\beta$, $H\gamma$, $H\delta$, etc. This is also the order of the hydrogen emission lines of stars of Class B. But in stars of Class Md, as is well known, it is not always so. Table V indicates the nature and the extent of the deviations.

TABLE V. INTENSITIES OF BRIGHT LINES.

TITANIUM BANDS	$H\beta : H\gamma$	$H\delta : H\gamma$	$H\delta : H\beta$	NO. OF STARS.
Not seen	1.7	0.4	0.2	4
Not prominent	0.7	1.0	1.5	6
Strong	0.3	1.2	4.6	18
Very strong	0.2	2.3	13.4	8

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In many or all stars the relative intensities are variable, hence, an attempt has been made to estimate the average, or characteristic ratio for use in Table V. Published descriptions, the Ann Arbor spectrograms, and notes made by the writer at Cambridge on spectra in the collection of the Harvard College Observatory, have been used. When one of the three lines, Hβ, Hγ, Hδ, was not seen at all, it was assigned the greatest intensity which the photographic conditions seemed to warrant. This process could scarcely accentuate the peculiarities observed in Table V since, aside from errors in estimating, the effect would be to smooth out differences between the lines. The departure from the laboratory (normal?) relative intensities seems to proceed in close accord with the strength of the absorption bands of titanium oxide. One cannot be sure whether the titanium oxide is directly responsible, or whether the two effects exist in parallel from a common cause.

It is suggested that the numerical ratios of intensity, Hδ : Hγ, and Hδ : Hβ might be used as criteria for the subdivision of the spectral Class Md.

According to the writer's experience 3905 Å (Si?) is in general the most prominent emission line not in the hydrogen series, occurring in Md spectra. An emission line at 4202 Å, presumably due to iron, as well as unidentified emission lines at 4138.9 Å and 4007.1 Å, has been measured in a few stars. All four of these lines had been previously observed in the spectrum of *o* Ceti.

The miscellaneous bright lines in spectra of Class Md may prove of great importance not only in theories of long-period variation, but in widely different connections. Observers should not fail to note them whenever possible.

ABSORPTION LINES

Due to underexposure, the absorption spectrum is measurable on only a small proportion of the Ann Arbor spectrograms of these stars.

In measuring the plates no attempt was made to include all the lines, but settings were made only on those which seemed capable of yielding fairly reliable velocities. The normal wave-lengths, contained in Table VI, were gathered from various low-dispersion measures of solar and late type stars, and from Stebbins' determinations in *o* Ceti, only one or two values being taken from Rowland. In Table VI

o = Stebbins' measures of *o* Ceti,
R = Rowland,
L = Miscellaneous one-prism measures of stars of Classes G, K, M.

TABLE VI.
ASSUMED NORMAL WAVE-LENGTHS OF ABSORPTION LINES IN CLASS Md.

Å.		Å.	
3982.3	o	4272.0	o L
3998.84	o	4274.96	R Cr
4005.4	L Fe	4289.84	o L
4031.3	o	4303.1	o
4063.78	L Fe	4314.6	L
4071.7	o L	4319.1	o L
4091.42	o L	4326.00	o L Fe
4092.7	o L	4330.2	o
4096.18	o	4347.16	o
4109.44	o	4368.4	L
4112.07	o	4375.4	L
4123.75	o	4379.35	o
4134.4	o	4384.63	o L Fe
4140.0	o	4389.75	o
4149.8	o	4395.3	o L
4152.5	o	4404.92	L
4160.0	o	4408.3	o
4164.9	o	4415.25	L Fe
4187.25	o	4482.0	L
4190.9	o	4489.8	L
4215.95	L	4522.94	Ti
4226.90	R Ca	4535.8	o
4250.7	o L	4738.45	o
4254.50	R Cr		

021430. o CETI.

Measures of seven three-prism plates in 1897-98 by Campbell and Wright gave a mean velocity of + 62.3 km., for the absorption lines, (*Ap. J.*, 9, 31, 1899). One-prism plates taken in 1902 yielded a mean velocity of + 66 km. from six absorption lines as measured by Stebbins, (*Lick Observatory Bulletin*, 2, 93, 1902).

Measures of two three-prism plates in December, 1906, by Plaskett, gave a mean velocity of + 65.4 km. for the absorption lines, (*Jour. R. A. S. Canada*, 1, 48, 1907).

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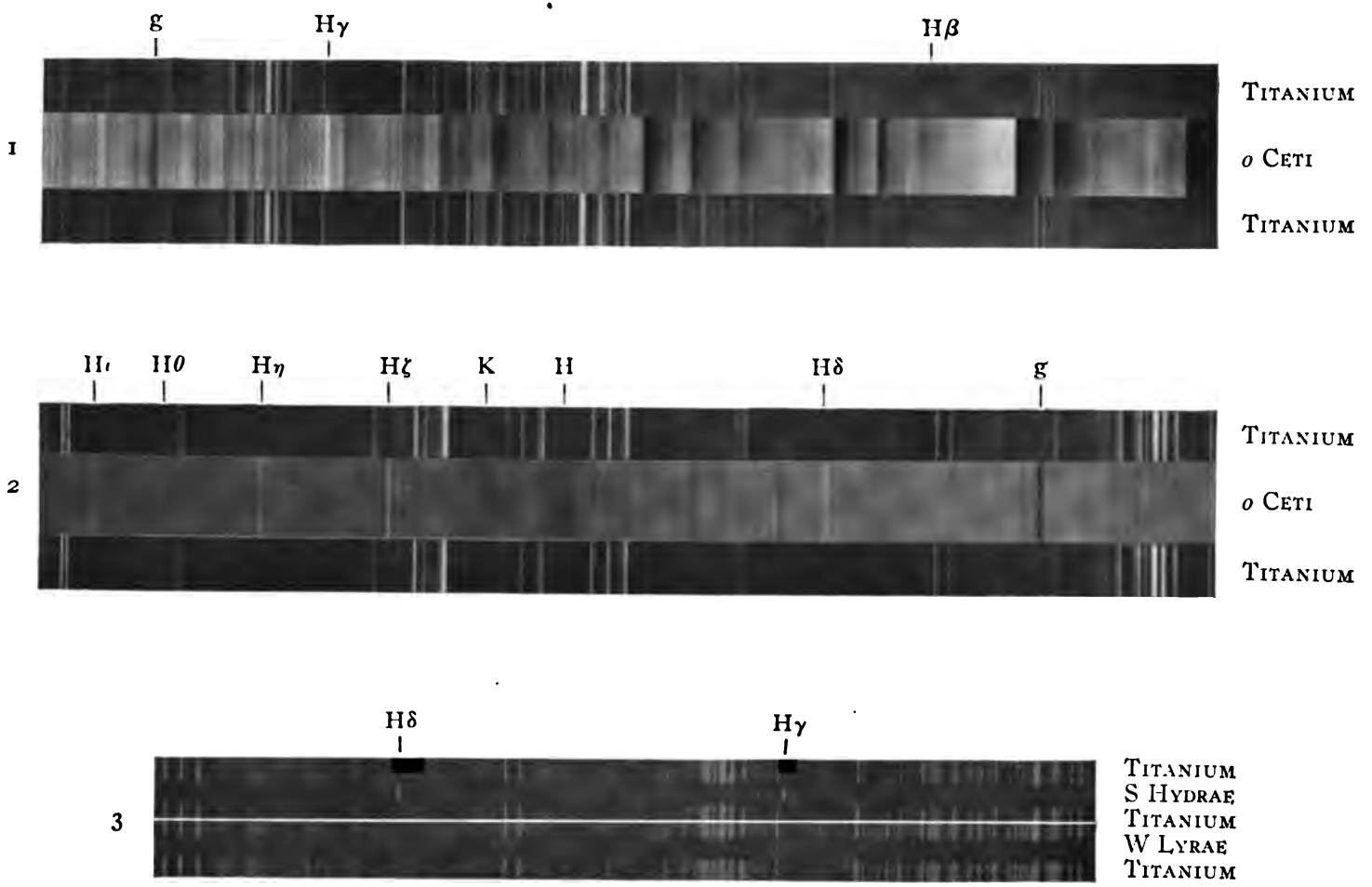

Plate F. 1 and 2. Overlapping Sections of the Spectrum of the Long Period Variable Star, Omicron Ceti, with Titanium Spark Comparison, 1915, January 29

3. Spectra of the Long Period Variable Stars, S Hydrae and W Lyrae, Showing Opposite Displacements of Hδ and Hγ Emission Lines

Dates: S Hydrae, 1915, March 26; W Lyrae, 1914, July 9

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ANN ARBOR OBSERVATIONS.

G. M. T. 1914 DEC. 25.57.		1915 JAN. 29.55.		1915 NOV. 12.66.	
PLATE WAVE-LENGTH.	RADIAL VELOCITY.	PLATE WAVE-LENGTH.	RADIAL VELOCITY.	PLATE WAVE-LENGTH.	RADIAL VELOCITY.
3887.74		4000.04	+ 90 km.	4064.62	
3891.87		4032.13	+ 62	4072.88	+ 87 km.
3983.49	+ 90 km.	4056.48		4078.85	
3999.82	+ 74	4065.14		4097.19	+ 74
4006.72	+ 99	4073.15	+117	4110.66	+ 89
4032.06	+ 57	4078.99		4112.93	+ 63
4065.13		4097.34	+ 85	4192.12	+ 88
		4125.36	+117	4216.70	+ 53
4079.62		4161.24	+ 90	4251.68	+ 69
4097.72	+113	4217.03	+ 77	4255.82	+ 93
4125.16	+102	4228.18	+ 91	4326.97	+ 67
4228.32	+101	4255.88	+ 97	4385.50	+ 59
4255.80	+ 92	4327.33	+ 93	4390.64	+ 61
4273.33	+ 93	4385.77	+ 77	4572.69	+ 58
4276.34	+ 97	4396.53	+ 84	4577.49	
4348.58	+ 98	4406.30	+ 94	4739.68	+ 78
4385.77	+ 77	4409.82	+104		
4396.74	+ 98	4740.03	+100		
4409.80	+102				
4739.66	+ 77				
Mean	+ 92.0		+ 92.7		+ 72.2
To sun	— 26.1		— 29.0		— 9.9
Obs. V.	+ 65.9		+ 63.7		+ 62.3

Mean velocity from absorption lines (Ann Arbor Observations) + 63.9 km.

065355. R LYNCIS.

PLATE WAVE-LENGTHS.

G. M. T. 1915 FEB. 28.70.	1915 FEB. 28.74 UNDEREXPOSED.
4524.03 Å	4607.26 Å
4586.70	4699.01
4621.89	4753.75
4634.61	
4723.93	
4806.62	
4816.76	

To sun — 22 km.

There is a broad absorption band containing narrower absorption lines, having its center about 4645-50 Å. The absorption spectrum differs greatly from that of the other stars of Class Md, not resembling them in this respect much more than it does stars of Class N.

The spectrum bears a considerable resemblance to that of R Cygni, as observed by Wright. See remarks under that star.

071044. L_2 PUPPIS.

Observations by D. O. Mills Expedition, kindly communicated by Director Campbell, as follows:

1911, TWO-PRISM PLATES, ABSORPTION LINES.

	+ 51.6 km.
	+ 53.2
	+ 53.0
Mean.....	+ 52.6

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1914, ONE-PRISM PLATES, ABSORPTION LINES.

	+ 53.5
	+ 55.5
Mean.....	+ 54.5
Mean of all..	+ 53.1

Remark by Campbell, "This star seems to be an exception in that absorption and radiation lines give approximately equal velocities."

093934. R LEONIS MINORIS.

G. M. T. 1915 FEB. 19. 83.

PLATE WAVE-LENGTH	RADIAL VELOCITY
4124.05	+ 22 km.
4290.41	+ 40
Mean	+ 31 ±
To sun	— 7
Obs. Velocity	+ 24 ±

094211. R LEONIS.

Measure by Lick Observatory, kindly communicated by Director Campbell: one three-prism plate, absorption lines give + 11 km.

ANN ARBOR OBSERVATIONS.

G. M. T. 1914 FEB. 28.78.		1914 MARCH 14.74.	
PLATE WAVE-LENGTH.	RADIAL VELOCITY.	PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4124.34	+ 43 km.	4124.27	+ 38 km.
4255.04	+ 38	4254.64	+ 10 p.
4290.27	+ 30	4290.18	+ 24
4303.61	+ 35		
4326.55	+ 38	4326.74	+ 51 p.
4377.22			
4380.07	+ 49		
4385.05	+ 29		
4389.57		4390.48	+ 33 p.
4662.11			
4738.60	+ 10 p.	4739.30	+ 54 p.
4943.40			
Mean	+ 35.6		+ 37.3
To sun	— 8.0		— 14.7
Obs. V.	+ 27.6		+ 22.6

Mean velocity from absorption lines (Ann Arbor Observations) + 25 km.

103769. R URSAE MAJORIS.

G. M. T. 1915 FEB. 19.83.		1915. FEB. 26.74.	
PLATE WAVE-LENGTH.	RADIAL VELOCITY.	PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4124.46	+ 51 km.	4227.43	+ 38 km.
4216.72	+ 38	4315.38	+ 54
4384.93	+ 20 p.	4326.79	+ 55
4396.10	+ 55	4384.72	+ 6 p.
4571.73		4405.59	+ 45
4607.24		4408.79	+ 33
4739.94		4536.31	+ 34
Mean	+ 46.5		+ 40.3
To sun	— 7.8		— 9.5
Obs. Vel.	+ 38		+ 31

Mean velocity from absorption lines, + 34 km.

132422. R HYDRAE.

Observations by Lick Observatory, kindly communicated by Director Campbell: two three-prism plates give for the absorption lines —3 km.

ANN ARBOR OBSERVATIONS.

G. M. T. 1915 MAY 30.61.

PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4326.62	+ 43 km.
4330.34	+ 10
4380.04	+ 47 p.
4384.59	— 3 p.
4395.53	+ 16
4489.83	+ 2 p.
4536.12	+ 21 p.
4739.06	+ 39
Mean	+ 24
To sun	— 18
Obs. V.	+ 6

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142539. V BOOTIS.

G. M. T. 1915 FEB. 28.90.

PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4164.09	— 58 km.
4226.47	— 31 p.
4254.00	— 35
4325.66	— 24
4383.67	— 66
4481.39	— 41
4534.85	— 63
4570.91 p.	
4737.87	— 37 p.
Mean	— 44.4
To sun	+ 11.5
Obs. V.	— 33

143227. R BOOTIS.

G. M. T. 1915 FEB. 28.81.

PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4122.58	— 85 km.
4288.77	— 75 p.
4318.34	— 53
4320.36	
4346.44	— 50
4387.63	— 68
4394.60	— 48
4404.09	— 56
4414.51	— 50
Mean	— 60.6
To sun	+ 17.8
Obs. V.	— 43

151731. S CORONAE.

G. M. T. 1915 FEB. 26.93.		1915 MARCH 7.30.	
PLATE WAVE-LENGTH.	RADIAL VELOCITY.	PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4090.27	— 11 km.	4139.72	— 20 km.
4092.64	— 4	4226.66	— 17
4123.72	— 2	4254.39	— 8
4139.59	— 30	4326.17	+ 12
4187.05	— 14	4384.21	— 29
4226.74	— 11	4389.67	— 5
4254.26	— 17	4394.85	— 31
4271.69	— 22	4404.84	— 5
4274.77	— 13	4408.06	— 16
4314.32	— 19	4571.36	
4325.87	— 9	4737.70	— 48
4384.00	— 43	4842.93	
4408.21	— 6		
4535.57	— 15		
4737.79	— 42		
Mean	— 17.2		— 16.7
To sun	+ 17.1		+ 15.3
Obs. V.	— 0.1		— 1.4

Mean velocity from absorption lines, — 0.7 km.

154615. R SERPENTIS.

G. M. T. 1914 APR. 11.88.

PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4216.33	+ 27 km.
4254.53	+ 2
4227.06	+ 11
4271.99 p.	
4314.91	+ 22
4326.40	+ 28
4368.85	+ 31
4384.63	0 p.
4390.12	+ 25 p.
4405.14	+ 14
4523.40	+ 30
4571.58 p.	
4737.97 p.	
Mean	+ 19.8
To sun	+ 12.0
Obs. V.	+ 32

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193449. R CYGNI.

Observations by Wright at Lick Observatory, *Mon. Not. R. A. S., 72,* 548, 1912.

The spectrum as observed by Wright in December, 1911, and January, 1912, is similar to that of R Lyncis as observed at Ann Arbor in February, 1915, in the following respects: general appearance; relative intensities of bright hydrogen lines; broad absorption band with center about 4645 Å; several prominent absorption lines as shown in the following table, which gives measured wave-lengths, not reduced for radial velocity.

	R CYGNI	R LYNCIS	DIFFERENCE.	
			Å	KM.
Em.	4101.4 Å	4101.8 Å	0.4	29
Em.	4340.2	4340.9	0.7	62
Abs.	4606.2	4606.9	0.7	46
Abs.	4620.9	4621.6	0.7	45
Abs.	4633.7	4634.3	0.6	39
Em.	4860.7	4861.7	1.0	62

The differences for various lines are fairly consistent, and are probably due to differences in radial motion, and errors of observation.

Mrs. Fleming classified the spectrum of R Cygni on different plates as Md1, Md2, Na, Pec.

194632. χ CYGNI.

Eberhard (*Ap. J. 18,* 198, 1903) records radial velocities from the dark lines as follows:

	G. M. T.	
1901 Aug.	9.38	+2.5 km.
	10.39	+2.3
1902 Sept.	24.31	−1.3
	26.33	−3.3
Mean		+0.1

213244. W CYGNI.

The following are measures made by the writer, of plates taken by Professor R. H. Curtiss at Lick Observatory in 1903, with Spectrograph I (*Lick Bulletin, 3,* 41, 1904). I am indebted to Professor R. H. Curtiss and to Director Campbell for the opportunity of using them. The measures were reduced by means of the table found in *Lick Bulletin, 3,* 29, 1904.

DATE: 1903 AUG. 3.		1903 AUG. 7.		1903 AUG. 26.		1903 SEPT. 13.	
TABULAR WAVE-LENGTH.	RADIAL VELOCITY.	TABULAR WAVE-LENGTH.	RADIAL VELOCITY.	TABULAR WAVE-LENGTH.	RADIAL VELOCITY.	TABULAR WAVE-LENGTH.	RADIAL VELOCITY.
4063.7	−13 p.	4384.5	−29	4118.8	−54	4024.8	−58
4132.5	−33	4404.9	−29	4187.6	−25	4046.0	−35
4216.0	−24			4191.7	−45	4092.8	−28
4227.0	−28			4384.5	−25	4118.8	−32
4275.1	−30			4395.0	−27	4128.0	−16
4384.5	−52 p.			4404.9	−2 p.	4134.5	−43
						4187.6	−12
						4395.0	−52
				4571.8	−19	4571.8	−44
Mean	−29.3		−29		−30.3		−35.0
To sun	+10.3		+9		+4.5		−0.7
Obs. V.	−19		−20		−26		−36

Mean velocity from absorption lines, −27 km. The range is not significant.

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235350. R CASSIOPEIAE.

G. M. T. 1914 SEPT. 9.68.		1914 SEPT. 9.72.	
PLATE WAVE-LENGTH.	RADIAL VELOCITY.	PLATE WAVE-LENGTH.	RADIAL VELOCITY.
4030.86		4030.87	
4064.10	+ 24 km.	4072.04	+ 25 km.
4078.03		4124.19	+ 32
4143.57 p.		4134.59	+ 14
4191.59	+ 49	4139.99	0
4194.74		4150.20	+ 29
4216.15	+ 14	4153.17	+ 48
4227.36	+ 33	4191.31	+ 29
4251.25	+ 39	4216.11	+ 11
4254.64	+ 10	4227.19	+ 21
4275.26	+ 21 p.	4254.79	+ 13
4314.84	+ 17	4314.70	+ 7 p.
4326.31	+ 22	4326.57	+ 40
4375.75	+ 24	4376.04	+ 44
4384.70	+ 5	4384.55	− 5
4395.27 p.		4405.16	+ 16
4405.33	+ 28	4408.79	+ 33
4408.62	+ 22	4571.60	
4571.70		4738.30 p.	
4738.68	+ 13		
Mean	+ 22.9		+ 22.3
To sun	+ 12.7		+ 12.6
Obs. V	+ 35.6		+ 34.9

Mean velocity from absorption lines, + 35 km.

The values in the last column of Table VII are not in all cases the exact differences of the numbers in the preceding columns because more weight was given to those Ann Arbor plates upon which both sets of lines were measured, and the values of other observers were taken into account where available.

The relative displacement of the bright and dark lines appears characteristic of the stars of Class Md. The two stars which do not show it, L_2 Puppis and W Cygni, are differentiated from the others in that they have short periods and small magnitude ranges. The presence of the bright lines and their peculiarities seem to depend upon the activity of the star but not to represent the direct cause of it.

TABLE VII. RADIAL VELOCITIES FROM BOTH BRIGHT AND DARK LINES.

STAR.		DARK LINES.	BRIGHT LINES.	ADOPTED DARK MINUS BRIGHT.
021403	o Ceti	+ 64 km.	+ 52 km.	+ 14 km.
071044	L_2 Puppis	+ 53	+ 51	+ 2
093934	R Leonis Minoris	+ 24 ±	− 6	+ 29 ±
094211	R Leonis	+ 25	0	+ 24
103769	R Ursæ Majoris	+ 34	+ 23	+ 12
132422	R Hydræ	+ 6	− 26	+ 26
142539	V Bootis	− 33	− 41	+ 11
143227	R Bootis	− 43	− 57	+ 13
151731	S Coronæ	− 1	− 22	+ 21
154615	R Serpentis	+ 32	+ 8	+ 23
194632	χ Cygni	0	− 17	+ 20
213244	W Cygni	− 27	− 26	+ 1
235350	R Cassiopeiæ	+ 35	+ 9	+ 26

GENERAL REMARKS ON LONG-PERIOD VARIATION—CLASS Md.

The spectra and light-curves of these stars are so essentially similar to one another that we may confidently presuppose the same general explanation in all cases; and since no star shows variation in radial motion, or the characteristics of eclipsing systems, we have no evidence that the light changes are due to the influence of a companion star. It is true, however, that only a very few stars have had their radial motions measured at times other than near maximum, but it does not seem probable that many, if indeed any, will prove to be spectroscopic binaries.

In view of the lack of spectroscopic observations which trace the spectral variations through the minimum phase, the writer offers the following "Tentative outline of spectral variations" merely as a basis for amplification and, if need be, correction. It is not a description of the history of any particular star (which would be extremely valuable) but depends upon miscellaneous observations from various sources.

TENTATIVE OUTLINE OF SPECTRAL VARIATIONS—CLASS Md.

1. Minimum. Absorption spectrum similar to that of Class M.

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2. Rise to Maximum. After the star has risen from one to three magnitudes (perhaps one-third or one-half the whole ascent) the bright hydrogen lines appear, Hδ appearing first, followed by Hγ, Hζ and others, and possibly Hβ and Hα.

3. Maximum. The bright hydrogen lines are very strong and are displaced to the violet relatively to the absorption lines by the equivalent of about 20 km. The series often extends to Hι (3771 Å) but seems to end there abruptly. Hε is conspicuous by its absence or decided weakness.

4. Decline to Minimum. The spectrum remains nearly the same as the star drops one or two magnitudes, the bright hydrogen lines fading slowly, and other bright lines, notably 3905 Å Si?. etc., become relatively stronger. This continues until minimum except that the secondary bright lines become faint at or just before, minimum.

HYPOTHESES TO EXPLAIN VARIABILITY.

Comparison of the curves of long-period variables with the graph of solar spottedness has led to the idea that there may be an analogy between the two phenomena.[12] The dimming of the stellar surface by spots would probably not be considered sufficient to account for the great variations observed, and in fact the analogy points in the opposite direction, namely, that stellar light-maximum corresponds to spot maximum on the sun. Observations of the solar constant seem to show that the sun radiates more heat at spot maximum than at other times, which appears consistent with its greater internal activity. Thus in the star there may be recurrent periods of activity which greatly increase its brightness and alter details in its spectrum. However, the corresponding effects observed upon the sun are so exceedingly slight in comparison that the use of them to account for stellar variability of this type is a long and a precarious extrapolation.

The so-called "geyser theory" does not differ very essentially from the above, except in presupposing the formation of a viscous, and perhaps even solid, crust at minimum which is finally broken through by the gradually increasing pressure of the gases imprisoned beneath it.[13] The resulting uprush of gas is suggested to account for the relative velocity of approach yielded by the emission lines. It is hard to see, however, how it could be sustained at a constant value over as long an interval as the observations have shown it. And it is difficult to understand why large quantities of hydrogen should become imprisoned under a heavy crust.

When we remember the great dimensions of these stars and the large amounts of energy concerned the enormous changes of brightness which they undergo in the course of a few months are perplexing occurrences. The spectral changes seem small in comparison with the variations in total light. In view of these considerations, as well as others, it might be well to keep in mind the possibility that a star of this class may not actually change its brightness so greatly, but that at time of minimum a screen is interposed between it and the earth, presumably in the immediate vicinity of the star. It is suggested that this might be composed of condensing gases, possibly calcium vapor, in the upper atmosphere of the star. Calcium exists at high levels in the solar, and in many stellar atmospheres, and especially in these stars if we admit that the hydrogen line Hε is blotted from the spectrum by the absorption of calcium H. The cloud formed by condensation would conserve the heat radiated from the photosphere to space so that the temperature of the materials immediately above the photosphere would increase until the overlying veil is again vaporized and the star shines out brightly. It is easy to conceive that these phenomena would be periodic and would cause variations in the spectrum, particularly in chromospheric emission. Possibly electrical effects having their origin in the evaporation of the cloud are effective in stimulating the hydrogen and other gaseous emission. If so the source would be at a high level, which seems to accord with observation especially if we assume that the relative displacement of bright and dark lines is due to pressure.

The tentative idea submitted in the above paragraph might be conveniently styled the veil theory. The writer proposes that it be considered as an alternative working hypothesis.

Ann Arbor, 1916, January 7.

[12] Clerke, *Problems in Astrophysics*, p. 362.

[13] Campbell, *Stellar Motions*, p. 303.

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A SPECTRUM OF THE P CYGNI TYPE

By PAUL W. MERRILL

INTRODUCTION.

One of the first generalizations drawn from extensive observations of stellar spectra was the recognition of a single sequence in which there was a place for nearly every star examined. As a matter of convenience certain definitely characterized types were chosen as a framework for classification first by Secchi, later by Vogel, and Lockyer, and lastly and on the most complete basis by investigators at the Harvard College Observatory. In a manner somewhat, but not wholly, arbitrary the remaining objects are said to be intermediate, or in case they do not fit into the scheme at all, anomalous. The Harvard Classification of Stellar Spectra, using the letters B, A, F, etc., to designate the divisions, is almost universally adopted in this country, and is largely used in Europe. Corresponding to a wider range of observation and increased knowledge the system has been slowly extended to include more and more objects, for some of which no provision had been made at first, so that now there remain outstanding only a few isolated specimens or small groups of intractable objects. The past few years have witnessed considerable success in establishing connections and relationships between these and the better understood types of the regular classification. That this field is considered an important one is shown by the large amount of labor spent in cultivating it, and its fertility is evidenced in results obtained.

Occupying a prominent position among these outstanding objects is the typical spectrum of a nova or new star. The spectrum of such a star usually changes rapidly, and very curiously the later epochs seem to include two other remarkable types of spectra, namely those known as nebular, and Wolf-Rayet, or Class O. There seems to be a fairly direct sequence connecting Class O spectra with Class B, which is usually accounted the first division in the main progression. And recently Wright has shown in several instances that a planetary nebula has a Wolf-Rayet star as a nucleus, and infers that in general planetary nebulae are condensing to form Wolf-Rayet stars. In this connection it is interesting to note that in the later history of the changes of a nova spectrum a period characterized as nebular is followed by one which Adams has shown to possess strong resemblances to Wolf- Rayet spectra.

After a sudden rise from obscurity a nova usually fades again in a few weeks or months to a low magnitude, but there is one notable exception to this rule. In 1600 a star appeared near the intersection of the arms of the great cross of the constellation Cygnus. As usual there were marked fluctuations in brightness but in this object they were measured by years rather than by days and weeks. They continued for about three-quarters of a century leaving the star at the fifth magnitude where it has remained ever since, well visible to the unaided eye. Moreover the spectrum retains certain characteristics associated with the earlier stages of typical novae. It was classified at Harvard as "B1 with hydrogen lines bright".

While this star, known as P Cygni,[1] may eventually prove valuable in the study of nebulae and Wolf-Rayet stars, it has an immediate application in the interpretation of the complicated spectra of novae near maximum. In 1913 there occurred certain small but rather striking changes[2] in the hydrogen lines of P Cygni which related that star still more closely to typical novae through a peculiar feature of the spectrum exhibited by both. The study of the star's spectrum may be of value in its application to the more general problem of novae both in general and in detail. For instance we must admit that the conditions causing the peculiar distribution of the light in many of the lines can under some conditions exist for a long period of years without much change; and we may be furnished the opportunity in P Cygni of viewing the complete

[1] *Harvard Annals, 28,* 101, 1895; *Astrophysical Journal, 35,* 286, 1912; *Popular Astronomy, 22,* 133, 1914.

[2] *Lick Observatory Bulletin, 8,* 24, 1913.

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cycle of nova history leisurely and much more thoroughly than when it is hurried through in a few weeks, or when the star fades to a magnitude at which it is difficult to observe. And, on the other side, Wright's probable identification of nitrogen in the spectrum of Nova Lacertae[3] is rendered more certain by the fact that nitrogen lines are undoubtedly prominent in the spectrum of P Cygni.

For the reasons outlined above the announcement by Miss Cannon in *Harvard Annals, 76,* No. 3, p. 31, of ten additional "Spectra of the P Cygni Type" is of much interest. Since the Harvard plates of these faint stars are of small dispersion and without comparison spectra it was impossible to decide from them just how far the resemblance to P Cygni extended. A detailed comparison of these spectra with P Cygni and with each other may prove of considerable value in the interpretation of these curious phenomena, and may help remove them from their apparent isolation by bringing to light some connections with more familiar types.

A single one of these ten stars is available for examination at northern observatories. Five one-prism spectrograms of this object, DM. + 11° 4673, have been obtained at Ann Arbor, and serve as the basis for the discussion which follows.

DM. + 11° 4673.

R.A. 1900 $21^h 46.^m2$; Decl. 1900 + 12° 9'; Mag. 7.7.

JOURNAL OF OBSERVATIONS.

PLATE NO.	DATE, G. M. T.	DURATION OF EXPOSURE.	OBSERVER.
3228A	1915 May 21.82	$2^h 20^m$	Merrill.
3259B	June 13.78	2 10	Merrill.
3332A	July 23.78	3 30	Merrill.
3340B	Aug. 6.81	4 4	Merrill.
3368A	Nov. 12.54	3 0	Dawson. Merrill.

THE HYDROGEN LINES.

The only conspicuous lines in the spectrum are those due to hydrogen. As in P Cygni they consist of strong bright portions superposed on the continuous spectrum, apparently in their normal positions except for Doppler effects, with absorption borders on their more refrangible edges. In DM. + 11°4673 the absorption borders are much less intense, and in some cases might be overlooked unless the exposure were exactly correct to show them most distinctly. Bright Hβ, Hγ, Hδ were first observed by Mrs. Fleming,[4] more[5] than 22 years ago, while bright Hα was seen shortly after by Campbell. As is usual in stars of early types the bright hydrogen series decreases in strength toward the shorter wave-lengths, but in this star the accompanying dark lines show no decided increase. Both bright and dark lines are narrow. Settings on the bright portions give consistent velocity results while the displacements of the dark lines are not to be interpreted as due to radial motion.

In Table I the correction for the solar motion is + 11.5 km., making the residual velocity + 27.5 km. which is rather high for a star of Class B.

In Table II are given the measured displacements of the dark hydrogen lines with respect to their bright components.

The displacement decreases with wave-length. The slight discrepancy shown by Hε is probably to be explained by the influence of the absorption line H, of calcium, since K is visible as a dark line. The values are decidedly less than the corresponding ones for P Cygni. Both bright and dark portions are narrower and weaker than in P Cygni. Whatever the physical conditions may be which cause the peculiar appearance of these lines, they are evidently less intense in the fainter star.

In one or two instances a narrow, weak, dark line is seen on the redward edge of a bright hydrogen line.

BRIGHT LINES NOT DUE TO HYDROGEN

In addition to the hydrogen lines there are many fainter emission lines largely of metallic origin. There are also numerous absorption lines, a condition giving rise to a difficulty in interpretation well known to those who have studied a complicated spectrum of this kind. The difficulty

[3] *Lick Observatory Bulletin, 6,* 95, 1911.

[4] *Astronomy and Astrophysics, 13,* 502, 1894.

[5] *Astrophysical Journal, 2,* 180, 1895.

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TABLE I. VELOCITIES FROM BRIGHT HYDROGEN LINES.

KILOMETERS PER SECOND.

PLATE.	Hβ	Hγ	Hδ	Hε	Hζ	Hη	Hθ	MEAN.
3228A	+18	+ 7	+19					+15
3259B	+ 6	+13	+16	+17				+13.2
3332A	+23	+16	+18	+23	+20	+19		+19.9
3340B	+18	+ 7	+17	+19	+20	(+36)	(+10)	+16.5
3368A	+14	+15	+ 8	+22	+19	+10	(+ 8)	+14.6
Mean	+15.9	+11.8	+15.7	+20.5	+19.6	+19	(+ 9)	+16.0

TABLE II. BRIGHT MINUS DARK.

PLATE.	Hγ	Hδ	Hε	Hζ	Hη
3228A	1.45 Å	1.13 Å			
3259B	1.51	1.27	1.04 Å	1.2 Å	
3332A	1.63	1.22	1.34	1.05	
3340B	1.55	1.30	1.25	1.15	
3368A	1.83	1.25	1.42		1.01 Å
Mean	1.59 Å	1.23 Å	1.26 Å	1.12 Å	1.0 Å
	110 km.	90 km.	95 km.	86 km.	78 km.
P Cygni					
Frost[6]					
1-prism	2.73 Å	2.38 Å			
Merrill[7]					
3-prism	2.48 Å				

is enhanced in this spectrum by the general weakness of the lines except those of hydrogen, and by the fact that a single line is apt to have both bright and dark components, so that the true character of some of the poorer lines remains in doubt.

It is a striking fact that as regards the fainter emission lines DM.+11° 4673 and P Cygni seem to have nothing at all in common. The P Cygni lines, almost wholly non-metallic, are not found in the fainter star. Several of the bright lines of DM. +11°4673 are accompanied by very weak absorption components, particularly 4244 Å, 4287 Å, 4352 Å, 4583 Å, and 4629 Å. Quite in contrast with P Cygni, the absorption border is seen about as frequently on the red as on the violet edge. The following table gives in the first column the measured wave-lengths of emission lines of the star under consideration. They have been reduced for the velocity measured on each plate from the bright lines of hydrogen. The displacements given in the third column are therefore relative. To obtain actual velocities with respect to the sun the hydrogen velocity +16.0 km. must be applied.

This star seems to occupy an intermediate position between P Cygni (and novae?) and those stars of Class B showing bright hydrogen lines such as γ Cassiopeiae and φ Persei. The lines of hydrogen show the characteristic structure of the P Cygni lines, though in a less marked degree, but this phenomena does not clearly extend to the other lines, although that is the case in P

[6] *Astrophysical Journal, 35, 286, 1912.*

[7] *Lick Observatory Bulletin, 8, 24, 1913.*

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TABLE III. BRIGHT LINES.

DM. + 11°4673.	SOLAR CHROMOSPHERE.	DISPLACEMENT.	NOTES.
			All lines observed as bright unless otherwise noted.
3855.9 Å	3856.40 Å Fe, Si.	−41 km	β Lyræ 3853.76, 56.16, 62.73.
3905.3	3905.67 Si, Cr.	−33	Bright line near here in spectra of Class Md, Si?
4232.8	4233.40 Fe, Cr.	−45	β Lyræ 4233.33; γ Cassiop. 4233.60.
4244.0	Several near.		
4287.0	Several near.		
4351.7	4352.02 Cr, Mg.	−33	β Lyræ 4352.08; γ Cassiop. 4353.3; φ Persei 4353.0.
4358.6	Prob. blend of 2 or 3.		
4384.9	4383.70 Fe.	(+82)	Near He 4388.10 which is dark in DM. + 11°4673; γ Cassiop. 4384.25.
4549.2	4549.80 Ti, Fe, Co.	−35	β Lyræ 4549.64; φ Persei 4549.2.
4583.2	4584.04 Fe, V.	−51	β Lyræ 4584.02; γ Cassiop. 4583.76; φ Persei double, mean about 4583.8.
4628.7	4629.63 Ti, Fe, Co.	−58	β Lyræ 4629.79; γ Cassiop. 4629.3; φ Persei double, mean 4629.5.
Present	4924.14 Fe.		γ Cassiop. 4925.7; φ Persei 4924.

DOUBTFUL BRIGHT LINES.

Based on a single measure, or upon discordant measures.

3821.0	3820.57 Fe.		Close double, perhaps affected by dark He 3819.78.
3887.1	3886.46 Fe, La.		Real line? Near Hζ.
4121.6	4121.48 Co.		β Lyræ double, mean 4121.02; He 4120.97.
4177.6	4177.70 Y, Fe.		Double; γ Cassiop. double, mean 4177.4; φ Persei 4177.1.
4347.0	Several near.		
4416.5	Several near.		
4505.3			
4511.5			β Lyræ 4512.16 dark.
4555.2	Several near.		β Lyræ 4555.83 dark; φ Persei 4555.3.
4569.8			
Present	5018.61 Fe.		γ Cassiop. 5018.3; φ Persei 5018.4.

REFERENCES:

Solar Chromosphere, Mitchell, *Astrophysical Journal, 38,* 407, 1913.
β Lyræ, Curtiss, *Pub. Allegheny Observatory, 2,* 73, 1911.
γ Cassiopeiæ, Curtiss. This volume, p. 1.
φ Persei, Jordan, *Pub. Allegheny Observatory, 3,* 31, 1913.
Merrill, *Lick Obs. Bulletin, 7,* 166, 1913.
Compare also η Carinæ, Moore and Sanford, *Lick Obs. Bulletin, 8,* 55, 1914.

Cygni. As is evident from the last column of Table III the faint emission lines are those present in the spectra of φ Persei and stars of its class. If novae are caused by the plunge of a rapidly moving star into a nebula, is it possible that circumstances of a similar but less violent character are responsible for the bright lines of φ Persei and other stars of Class B? In any event the relations existing between these objects would seem to warrant close investigation.

Most of the bright lines of DM. + 11°4673 show a negative displacement of about 40 km.

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with respect to the bright lines of hydrogen. They seem therefore to yield a velocity more nearly comparable with that from the dark lines, principally of helium. The observations of all lines except hydrogen are rather inaccurate, but the effect seems fairly definitely indicated. If real, the phenomenon is at variance with P Cygni as in that star bright lines of various elements give nearly consistent results. A similar effect of about the same magnitude has been observed in the spectrum of η Carinae.[8]

ABSORPTION LINES.

There are numerous dark lines which are not seen to have bright components. They show little contrast with the continuous spectrum and cannot be measured with great precision on the Seed 27 emulsion employed on account of the faintness of the star. Most of those which can be identified are due to helium.

TABLE IV. ABSORPTION LINES.

NORMAL WAVE-LENGTH.	VELOCITY.	NO. OF PLATES.	PLATE WAVE-LENGTH REDUCED TO SUN.	NOTES.
3933.82 K, Ca.	0 km. ±	3	3995.2	β Lyræ 3995.17; P Cygni N; γ Cassiop. 3995.8.
4026.34 He	—15	3	4010.8 ±	He 4009.42.
4120.97 He	—14	1	4088.6	γ Cassiop. 4089.7; Si 4089.02; P Cygni.
4143.92 He	— 2	3	4116.4	γ Cassiop. 4116.5; Si 4116.35; P Cygni.
4388.10 He	—11	5	4129.4	β Lyræ 4128.20; Si 4128.19.
4471.68 He	—19	2	4180.3	β Lyræ 4179.07.
4552.76 Si	—13	5	4182.6	
			4184.9	γ Cassiop. 4185.
			4517.5	
			4567.1	Near Si lines.
			4573.4	Near Si lines.
			4606.8	
			4781.6	
			(4922)	He 4922.10 present but not measured.

[8] *Lick Observatory Bulletin, 8,* 134, 1915.

The somewhat insecure relationship to P Cygni is reinforced by the behavior of the absorption lines which were measured for velocity. They yield a velocity algebraically less than that from the bright hydrogen lines; this is also true of the absorption lines of P Cygni.

SUMMARY.

1. The hydrogen lines of DM. + 11°4673, a star announced by Miss Cannon as "of the P Cygni type", are bright, with weak dark components on their more refrangible edges. The bright hydrogen lines agree in giving a radial velocity of +16.0 km.

2. Numerous emission lines of metallic origin are present in the spectrum. In several cases they are accompanied by very weak absorption borders which are seen as frequently on the red as on the violet edge. These bright lines yield velocities differing considerably from those of hydrogen, being about 40 km. less.

3. A number of weak absorption lines have been measured. Those which are identified are mainly due to helium and silicon. Five helium lines and 4552 Å of silicon give a velocity of —13 km. The velocity from K of calcium is about zero but the determination is of small weight.

4. The intensity curves of the hydrogen lines are qualitatively similar to those of P Cygni. Several dark lines (He, Si, Ca) are displaced to the violet with respect to the bright hydrogen lines. The resemblance to P Cygni seems to stop with these characteristics.

5. DM. + 11°4673 may be considered in some respects as intermediate between P Cygni and the well known bright-line stars of Class B.

6. It seems probable that future investigation will disclose facts of interest in regard to the relationships between the following objects:

(a) Novae.
(b) P Cygni, η Carinae.[9]
(c) Stars of the P Cygni Type—Miss Cannon's List.
(d) Bright-line Stars of Class B, e.g., γ Cassiopeiae, ϕ Persei, β Lyrae.

April 7, 1916.

[9] *Harvard Annals, 76,* 36, 1915.

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A STUDY OF THE SPECTRUM OF ζ_1 URSAE MAJORIS[1]

By LAURENCE HADLEY

INTRODUCTION AND STATEMENT OF THE PROBLEM

As early as the middle of the seventeenth century ζ Ursae Majoris ($\alpha = 13^h\ 20^m$, $\delta = +55°\ 23'$) was observed as a double star. This was the first discovery of its kind and was made by Riccioli in 1650. It was again observed as a visual double star in 1700 by Kirch. However, the position angle and distance of this pair were not measured until 1750, when they were recorded by Bradley as 143°.1 and 13".88, respectively. Many measures have been made and published since that time. The recorded values of the position angle seem to show a very slow relative motion which probably does not exceed 0°.025 per year. The distance between the two components, ζ_1 and ζ_2, as given by the best observers for more than a half century, does not vary greatly on either side of 14.4 seconds of arc. So slow are the changes in these quantities that thousands of years will have passed before a single revolution will be complete.

The data to be found on page 399 of the *Astronomical Observatory of Yale Transactions,* Vol. 2, give 0".021 ± 0".008 as the most probable parallax of this star, from which it follows that its distance from the sun is approximately 155 light years and that the linear separation of the two components of this visual pair is not less than 64,000,000,000 miles and it may be much greater than this value, depending on the angle between the line of sight and the line joining the two stars.

Basing her calculation on a parallax of 0".045, Miss Clerke has estimated that Mizar "sends abroad thirty-eight times more light than our sun".[2] Since there is good reason to believe that the parallax is not greater than half of the value used by Miss Clerke, it follows that the light giving power of this star is probably not less than one hundred and fifty times that of the sun.

Professor Stebbins gives 2.40 as the magnitude of ζ_1 Ursae Majoris. In the *Astrophysical Journal,* Vol. 39, page 475, he makes the following statement as a result of his investigation: "Observations throughout the spectroscopic period give no evidence of eclipses, nor of continuous variation. The light is constant within 0.02 or 0.03 mag."

It is an interesting fact that this second magnitude star should have been the first visual binary discovered and that the brighter of its two components, ζ_1 should have been the first known spectroscopic binary. In making a study of the plates taken with an objective-prism spectrograph at the Harvard College Observatory, Miss Maury observed that the K line of ζ_1 Ursae Majoris appeared as a double line on certain plates, while on many others it was a well defined single line. The separation of the K line was so evident on certain plates that its reality could not be doubted. This discovery was made in 1889 and was the beginning of a new line of most fruitful investigation in Astronomy. The Harvard plates, though numerous, had been taken at irregular intervals and, as a result, the period of this doubling of the lines was not evident. It was erroneously placed at 52 days, thus making the time of a complete revolution 104 days. This period of 104 days is approximately five times the true period, which was first derived by Vogel. The Harvard staff found difficulty in their effort to harmonize the period of 104 days with the doubling of the lines as shown by the plates. At times the lines appeared as sharply defined single lines, when the period of 104 days would have called for a wide separation. However, it was left for Vogel to discover the true time of revolution of this spectroscopic pair and to announce a period which would satisfy the observations.

This star, ζ Ursae Majoris, is of peculiar interest in that the fainter component, ζ_2, has been announced as showing variable radial velocity.

[1] A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of Michigan.

[2] Clerke, *Problems in Astrophysics,* page 290.

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Frost and Lee[3], of the Yerkes Observatory, have announced the variation as from —17 km. to +10 km., while Dr. Ludendorff[4] has announced that the range shown by his plates is from —17.3 km. to —7.3 km. Nearly a hundred spectrograms have been measured and reduced by the writer. A frequency curve based on the velocities obtained from these measures conforms closely to the probability curve, thus indicating that the differences are due, largely at least, to accidental errors rather than to real changes, so far as plates made with a single prism instrument are concerned.

It is the purpose of this paper to give the results of a thorough study of ζ_1 Ursae Majoris, the brighter of the two visual components. The results and conclusions will be based on velocities published by Dr. Ludendorff and those which were obtained from the measures of spectrograms[5] made at the Detroit Observatory, Ann Arbor, Michigan. It is the aim in this discussion to treat the observations separately; to make a complete and independent solution for each component of ζ_1 Ursae Majoris; and finally, to coordinate the results of the two sets of observations and of the two components, thus putting the subject upon a basis which will be as complete and definitive as possible.

HISTORICAL

Harvard Observations.[6] As previously stated ζ_1 Ursæ Majoris was the first known spectroscopic binary. The first plate to show the K line distinctly double was made in 1886, but this fact was not observed until 1889. More than 150 plates of this star were made at the Harvard College Observatory between the years 1885 and 1889. These spectrograms were used primarily in connection with the work of the observatory on the classification of stellar spectra. However, some consideration was given to the doubling of the lines; to the meaning of this apparent change in the spectrum; to the probable period; and to the velocity in the line of sight, which would correspond to the relative displacement of the lines as observed. There was considerable uncertainty as to the period, but it was thought to be approximately 104 days. Professor Pickering derived the relative velocity of the two components of this spectroscopic binary. Using the displacements of the K line, he found the relative velocity to be 94 miles per second, and from the magnesium line, λ 4481, he placed the value at 102 miles per second. In this connection he says it was not probable that the plates were made at the time of a maximum displacement, hence the maximum velocity is certainly not less than 100 miles per second.[7]

Potsdam Observations. During the months of March and April, 1901, Dr. Ludendorff and Dr. Eberhard secured a series of plates of ζ_1 Ursæ Majoris with Spectrograph IV of the 33 cm. refractor of the Potsdam Observatory. Vogel undertook the measurement and reduction of these plates. The results of his work are given in Vol. XIII, page 324, of the *Astrophysical Journal.* The results are based on the relative velocities of one component with respect to the other and not on the velocities of each component with respect to the center of mass of the system. The measures of only 25 plates are included in Vogel's publication. When Dr. Ludendorff, at a later time, discussed this series of plates, he rejected 13 of the 25 used by Vogel as being of uncertain value, owing to the very small relative displacement of the lines. In Dr. Ludendorff's paper he states that the time recorded is that of the middle of the plate's exposure. Since the time as given by Vogel for these same plates is from 20 to 30 minutes earlier, it seems to be a reasonable supposition that he gave the time for the beginning of the exposure. This would introduce an appreciable error, particularly in the steepest part of the velocity curve. In consideration of this fact and the fact that the good plates are included in the data published by Dr. Ludendorff, which will be discussed in detail, nothing further will be

[3] *Astronomische Nachrichten*, Vol. 177, page 171.

[4] *Astronomische Nachrichten*, Vol. 177, page 9.

[5] For complete description of the apparatus employed and the constants of the instruments see *Publications of the Detroit Observatory*, Vol. 1.

[6] *Monthly Notices*, Vol. 50, page 296.
Harvard College Observatory Circular, No. 11.
Harvard College Observatory Annals, Vol. 26, page xvii.
Astrophysical Journal, Vol. 8, page 174.

[7] *American Journal of Science*, Third Series, Vol. 39, page 46.

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done with these earlier results except to give the elements which Dr. Eberhard derived by the method of Lehmann-Filhés from Vogel's measures. The elements follow:

$T_0 = 1901$, March 28.60. (Relative motion in line of sight $= 0$),
$T = 1901$, March 28.88,
$\omega = 101°.3$,
$e = 0.502$,
$\log \mu = 9.4843$,
$\mu = 17°.476$,
$P = 20.6$ days,
$a \sin i = 35$ million km.,
$$m_1 + m_2 = \frac{4 \cdot \odot}{\sin^3 i}.$$

Dr. Ludendorff and Dr. Eberhard continued their observations in 1901 throughout the months of May and June. The work was taken up again in the years 1903, 1905, and 1906. In all, 118 spectrograms were secured. Of these 118 plates, the measures by Dr. Ludendorff of 59 are given in the *Astronomische Nachrichten*, Vol. 180, No. 4313, page 278. These measures have not been used for the determination of a complete set of elements. They were used for finding the center of mass velocity; the ratio of the masses; and for improving the period as announced by Vogel.

Since a full discussion of Dr. Ludendorff's measures will be taken up in this paper, the details of his work will be omitted and his final results stated. The period, he says, is correct to within a few thousandths of a day. The probable errors of the other quantities show what confidence may be placed in their determination.

$$P = 20.536 \text{ days},$$
$$\gamma = -12.6 \pm 0.49 \text{ km.},$$
$$\frac{m_1}{m_2} = 1.014 \pm 0.018,$$

where P is the period; γ the velocity of the center of mass of the system; and $\frac{m_1}{m_2}$ the ratio of the masses of the two components.

It is worthy of note at this point that the values of γ and $\frac{m_1}{m_2}$ are somewhat uncertain. The values given above were derived from the entire series of 59 plates. Dr. Ludendorff also divided the plates into two groups; the first, containing the plates made in 1901, the second, those of 1903, 1905, and 1906. The first group contained 26 plates and the second 33. When these two groups were considered separately, unexpected differences were found as shown below:

1901.	1903-6.
$\gamma = -13.2$ km.,	$\gamma = -12.2$ km.,
$\frac{m_1}{m_2} = 0.980$.	$\frac{m_1}{m_2} = 1.035$.

These differences will be discussed later, but it may be well to note at this point that such change in the ratio of the masses would not be possible, and that the change in center of mass velocity seems to be improbable, unless there is a third body in the system, a resisting medium, or other disturbing factors.

DERIVATION OF ELEMENTS.

Potsdam Measures. The velocities referred to above, as derived by Dr. Ludendorff, will now be discussed in full. Table I contains the data as given in the *Astronomische Nachrichten*. Column (1) gives the number of the plate; column (2) gives the Greenwich mean time for the middle of the observation; column (3) gives the phase as counted from 1912 March 10.000, using the final value of the period; the next three columns contain data for Component 1, the first gives the number of lines measured, the second gives the velocity in the line of sight, and the third gives the error in the sense, observed minus computed; the next three columns refer to Component 2, giving the same data as was given for Component 1; the last column gives the weight to be assigned. It is difficult for anyone, other than he who measures the plates, to determine the weights which should be assigned. Hence, the writer adopted Dr. Ludendorff's assignment of weights and used the same plan in the reduction of his own measures. It is as follows. Unless there be good reason for an increase or decrease in the weight to be assigned to a plate, it is made equal to one-half of the sum of the number of lines measured on the two components. There are other methods of assigning weights which are better, but it seemed desirable to treat both sets of data exactly alike. Hence, the plan adopted by the observer at Potsdam was used in both cases.

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TABLE I. VELOCITIES

POTSDAM OBSERVATIONS.

NO.	DATE, G. M. T.	PHASE.	COMPONENT 1.			COMPONENT 2.			WT.
			N.	V.	O–C.	N.	V.	O–C.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	1901	DAYS		KM.	KM.		KM.	KM.	
1	Mar. 24.383	0.989	3	+42	+68	5	−67	− 8.3	4
2	26.393	2.999	4	+42	− 3.7	3	−66	+ 7.6	3
3	29.293	5.899	0	..	....	1	+36	+ 1.5	0
4	30.333	6.939	4	−90	− 4.2	3	+64	+ 3.8	3
5	Apr. 2.313	9.919	0	..	....	4	+44	+ 2.5	0
6	16.333	3.402	11	+44	− 2.2	3	−78	− 2.7	5
7	17.333	4.402	4	+36	+ 2.3	2	− 59	+ 5.8	3
8	20.323	7.392	5	−88	− 9.7	3	+47	− 8.5	2
9	21.333	8.402	3	−76	− 2.7	2	+38	−11.5	2
10	22.333	9.402	3	−63	− 2.2	3	+46	+ 8.7	3
11	23.323	10.392	4	−52	− 3.2	2	+22	− 5.4	3
12	24.343	11.412	3	−40	− 0.2	1	+16	− 2.8	2
13	May 2.363	19.432	2	+23	+ 2.7	1	−48	− 5.8	1
14	3.353	20.422	2	+22	− 5.2	2	−53	− 3.2	2
15	5.363	1.869	7	+46	+ 8.7	6	−68	− 6.7	6
16	12.363	8.896	4	−67	− 0.5	3	+46	+ 4.5	3
17	13.343	9.876	1	−62	− 6.8	3	+34	+ 2.2	2
18	14.343	10.876	2	−54	− 8.8	4	+20	− 2.6	3
19	24.373	0.369	5	+35	+ 4.2	3	−60	− 6.2	4
20	31.353	7.349	7	−85	− 0.2	5	+56	− 2.8	6
21	June 1.353	8.349	7	−72	+ 1.8	5	+48	− 0.8	6
22	13.373	20.369	2	+38	+11.2	1	−47	+ 2.8	1
23	19.413	5.873	3	−65	−10.5	4	+41	+ 6.8	3
24	20.353	6.813	6	−87	− 1.5	2	+56	− 4.2	4
25	21.393	7.853	5	−82	− 1.7	8	+50	− 4.8	3
26	23.363	9.823	3	−60	− 4.2	4	+32	− 0.8	3
	1903								
27	Apr. 28.432	6.190	8	−69	+ 1.5	8	+51	+ 2.4	8
28	30.432	8.190	2	−75	+ 1.5	2	+58	+ 7.4	2
29	May 13.313	0.534	5	+40	+ 8.2	5	−48	+ 7.3	5
30	14.373	1.594	4	+46	+ 6.8	6	−56	+ 7.7	5
31	20.323	7.544	5	−75	+ 8.2	9	+50	− 7.2	7
32	22.323	9.544	4	−60	− 1.4	3	+39	+ 3.5	3
33	23.323	10.544	3	−57	− 8.4	2	+28	+ 2.2	2
	1905								
34	Apr. 20.353	10.335	4	−53	− 2.7	4	+25	− 2.8	4
35	30.333	20.315	2	+21	− 5.8	2	−47	+ 2.3	2
36	May 3.353	2.799	7	+40	− 5.0	7	−67	+ 5.8	7
37	4.363	3.809	8	+36	− 8.2	7	−78	− 3.2	7

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TABLE I. VELOCITIES.

POTSDAM OBSERVATIONS—CONT'D.

NO.	DATE, G. M. T.	PHASE.	COMPONENT 1.			COMPONENT 2.			WT.
			N.	V.	O–C.	N.	V.	O–C.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	1905	DAYS		KM.	KM.		KM.	KM.	
38	May 9.343	8.789	1	−70	− 1.6	1	+39	− 5.8	1*
39	25.343	4.252	5	+43	+ 5.5	5	−75	− 5.5	5
40	27.343	6.252	3	−70	+ 2.5	3	+44	− 5.6	3
41	28.343	7.252	4	−83	+ 2.8	5	+65	+ 6.7	4
42	29.343	8.252	6	−77	− 1.2	7	+55	+ 4.6	6
43	30.363	9.272	4	−60	+ 3.3	3	+39	+ 0.2	3
44	31.363	10.272	9	−49	+ 2.3	9	+26	− 2.6	9
45	June 19.423	8.796	3	−64	+ 4.6	5	+43	− 0.7	4
	1906								
46	Mar. 29.403	4.266	4	+41	+ 3.2	1	−36	+33.3	o†
47	Apr. 1.373	7.236	2	−83	+ 2.7	2	−62	+ 2.8	2
48	3.373	9.236	1	−73	− 9.8	1	+28	−10.6	1*
49	15.353	0.679	2	+33	+ 0.2	3	−63	− 6.7	2
50	17.303	2.629	3	+50	− 4.8	1	−75	− 3.6	2
51	21.363	6.689	3	−89	− 3.6	5	+58	− 2.0	4
52	23.313	8.639	3	−68	+ 2.5	6	+51	+ 5.0	4
53	24.343	9.669	4	−55	+ 2.5	5	+31	− 3.3	4
54	May 5.333	0.123	7	+20	− 9.3	11	−51	+ 0.8	9
55	6.323	1.113	8	+37	+ 1.2	6	−65	− 5.4	7
56	7.343	2.133	6	+41	− 1.2	5	−66	+ 2.2	5
57	8.323	3.113	3	+53	+ 6.8	3	−64	+10.2	3
58	13.333	8.123	6	−82	− 6.0	7	+59	+ 8.5	6
59	14.323	9.113	4	−59	+ 5.5	3	+44	+ 3.7	3

* Under exposure.

† Only 1 line on Comp. 2.

Normal places. The final velocities as given in Table I were combined into sixteen normal places with the phase as the basis of grouping. Care was taken that the limits of phase for each normal place be so small that the velocity curve over that interval would be essentially straight. The weight assigned to each place was determined by the number of good, measurable lines on all of the plates used in making up the normal place. The phases as given in column (2) have been reduced to the sun. Column (3) gives the limits within which the phases of the plates fall which were used in forming each normal place. The velocities of the two components are given; also the residuals in the sense, observed minus computed, based on the final elements.

The normal velocities given in Table II are plotted in Plate VI. The exact positions are shown by the centers of the small circles.

From the phases which had been reduced to the sun and the velocities corrected for curvature as given in Table II, velocity curves were plotted. From these curves preliminary elements for each component were derived by use of the Forty-five Degree Chordal Method.[8] The two components were carried entirely through, separately, Component 1 having been taken up first. But before

[8] *Astrophysical Journal*, Vol. 28, page 212.

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taking up either component it is desirable to settle the question of the period.

Period. In order to obtain an accurate value for the period, Ludendorff's velocities were plotted with P taken equal to 20.536 days which was known already to be a good value. The Ann Arbor velocities were similarly plotted and the curves transferred to tracing cloth which was then fitted, as nearly as could be, to the curves from Ludendorff's measures. With the curves in the best agreement, the time-axes were compared at six different, well defined points and the average of the six determinations gave $\Delta T = +0.07$ days.

The mean time for the Potsdam observations was taken as 1903.5 and for the Ann Arbor observations, 1912.6. The time interval between these two dates is 9.1 years which is approximately 161 periods. From this it follows that

$$\Delta P = \frac{+0.07}{161} = +0.00044 \text{ days.}$$

This correction is much smaller than might have been expected since Ludendorff stated that his determination, $P = 20.536$, was correct within a few thousandths of a day. By applying the correction which was found, we have $P = 20.53644$ days as the true period. From the method of its determination and the large time interval over which it extended, we shall adopt this as final and no further correction for this element will be sought in connection with the least square solution. The period having been accurately found, the value of μ, the mean daily motion, was easily derived.

$$\mu = 17^\circ.5298.$$

Since μ and P are mutually dependent, the values of these two quantities will be accepted without further correction. Since $\delta\mu$ will not be needed in the least-square formulæ, only five unknowns will occur and the computation will be greatly simplified.

TABLE II. NORMAL PLACES.

POTSDAM OBSERVATIONS.

NO.	PHASE.	LIMITS OF PHASE.	WT.	COMPONENT 1.		COMPONENT 2.	
				VELOCITY.	O–C.	VELOCITY.	O–C.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	6.135	5.87 to 6.26	0.56	−69.11	+0.29	+48.10	+1.36
2	6.802	6.68 to 6.94	0.44	−88.93	−3.46	+59.08	−0.95
3	7.439	7.23 to 7.56	1.00	−82.06	+2.14	+55.32	−2.80
4	8.250	8.12 to 8.41	0.88	−76.73	−1.35	+52.96	+2.96
5	8.768	8.63 to 8.90	0.48	−66.60	+2.30	+46.04	+1.87
6	9.260	9.11 to 9.41	0.40	−61.90	+0.96	+41.50	+2.76
7	9.711	9.54 to 9.92	0.48	−58.67	−1.11	+33.75	−0.18
8	10.336	10.27 to 10.55	0.72	−51.28	−0.64	+25.33	−2.29
9	11.144	10.87 to 11.42	0.16	−47.00	−4.56	+18.00	−2.02
10	20.212	19.43 to 20.43	0.24	+24.50	−1.37	−49.27	−0.98
11	20.828	20.65 to 21.07	0.72	+28.89	−1.27	−52.17	+0.98
12	21.543	21.21 to 21.65	0.52	+37.92	+2.82	−65.31	−6.37
13	22.412	22.13 to 22.67	0.64	+44.44	+3.70	−63.62	+2.40
14	23.415	23.16 to 23.65	0.60	+44.39	−1.08	−67.73	+5.47
15	24.175	23.93 to 24.35	0.48	+39.65	−5.29	−78.35	−3.31
16	24.844	24.79 to 24.94	0.32	+40.49	+4.43	−69.10	−1.55

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Component 1. Velocity curves having been drawn with as much accuracy as was possible, the following set of preliminary elements was derived:

PRELIMINARY ELEMENTS (COMPONENT 1).

$$P = 20.53644 \text{ days},$$
$$\mu = 17^\circ.5298,$$
$$T = 5.600 \text{ days from 1912 March 10.000.}$$
$$e = 0.5440,$$
$$\omega = 101^\circ\ 12'.2,$$
$$K = 68.0 \text{ km.},$$
$$V_0' = -18.0 \text{ km.},$$
$$\gamma' = -10.82 \text{ km.}$$

From these elements an ephemeris was computed and the residuals found in the sense, observed minus computed. The agreement being sufficiently close between observation and computation, a least square solution was undertaken. The sixteen normal places gave sixteen equations of condition. The usual method of treatment gave the five normal equations which follow. The notation is that of Schlesinger in his paper published in the *Allegheny Observatory Publications*, Vol. 1, No. 6.

NORMAL EQUATIONS.

$$\begin{aligned} 8.641\Gamma - 0.870\kappa - 3.248\pi - 0.712\epsilon - 0.271\tau + 8.794 &= 0, \\ + 5.730\kappa + 0.383\pi + 1.209\epsilon + 0.014\tau + 7.264 &= 0, \\ + 2.910\pi + 0.730\epsilon + 0.715\tau + 0.620 &= 0, \\ + 0.945\epsilon + 0.258\tau + 0.478 &= 0, \\ + 0.305\tau + 0.866 &= 0. \end{aligned}$$

The five normal equations were treated after the method given in Vol. 1 of Chauvenet's *Practical Astronomy*. A solution of these equations gave the following values:

$$\tau = +0.4844,$$
$$\epsilon = +2.6638,$$
$$\pi = -3.1655,$$
$$\kappa = -1.9489,$$
$$\Gamma = -2.1693.$$

To check the accuracy of the numerical work, the values of the unknowns were substituted in the normal equations. The agreement was found to be satisfactory, the largest of the residuals being 0.005 while the average value was 0.003. From these values of the unknowns the corrections to be applied to the elements were obtained. The corrections and the corrected elements follow:

CORRECTIONS.

$$\delta K = -1.949 \text{ km.},$$
$$\delta\omega = +2^\circ\ 40'.0,$$
$$\delta e = -0.01248,$$
$$\delta T = +0.00577 \text{ days},$$
$$\delta\gamma = -0.851 \text{ km.}$$

CORRECTED ELEMENTS.

$$T = 5.60577 \text{ days},$$
$$e = 0.53152,$$
$$\omega = 103^\circ\ 52'.2,$$
$$K = 66.051 \text{ km.},$$
$$\gamma = -11.671 \text{ km.}$$

The residuals recorded in column 6 of Table II were found by comparison of the observed velocities with an ephemeris computed from the corrected elements. The differences between these residuals and those found from the normal equations were so small that the second order terms neglected in making a least square solution were evidently inappreciable, hence a repetition of the least square solution was not necessary.

It is now possible to find the quantity, $a \sin i$, by use of the formula,

$$a \sin i = [4.13833]\ (1 - e^2)^{\frac{1}{2}} K \cdot P.$$

Substituting in the equation we obtain the following value:

$$a \sin i = 15{,}800{,}000 \text{ km.}$$

The probable errors given in connection with certain of the elements were computed by the usual method. The value for the mass, m_1, cannot be determined at this point in the solution, but is given here even though derived later. The

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final elements for Component 1 derived from Ludendorff's measures follow:

FINAL ELEMENTS. (COMPONENT 1).

$$P = 20.53644 \text{ days},$$
$$\mu = 17^\circ.5298,$$
$$T = 5.6058 \pm 0.0700 \text{ days from}$$
1912, March 10.000,
$$e = 0.53152 \pm 0.01024,$$
$$\omega = 103^\circ\ 52'.2 \pm 2^\circ\ 47'.9,$$
$$K = 66.05 \pm 0.77 \text{ km.},$$
$$\gamma = -11.67 \text{ km.},$$
$$a \sin i = 15{,}800{,}000 \text{ km.},$$
$$m_1 = \frac{1.57\odot}{\sin^3 i}.$$

The probable errors in ω and T indicate some uncertainty in the values of these quantities. This was to be expected since, in forming the elimination equations, the quantity [*ee.* 4] is very small. This quantity, [*ee.* 4], is a function of the coefficients found in the normal equations and since it nearly vanishes owing to the peculiarities of the normal places and the velocity curve, the value of τ and hence of δT are uncertain. The value of ω changes with the value of T, therefore, it is uncertain also.

The least square solution having been completed, the following results may be stated:

1. The sum of the weighted squares of the residuals for the sixteen normal places was reduced from 87.48 to 53.52.

2. The probable error of a normal place of weight unity was found to be ± 1.49 km.

3. The residuals recorded in column 6 of Table I were scaled from the final velocity curve, Plate VI. These residuals gave ± 3.5 km. as the probable error of the average plate and ± 2.3 km. as the probable error of the best plates of the series.

Next in order should come a like treatment of the measures of Component 2. While the discussion is more brief, the details of the process were identically the same as those used in Component 1.

Component 2. As in the preceding case, preliminary elements were derived and residuals found by a comparison of the observed and the computed velocities for each of the sixteen normal places.

PRELIMINARY ELEMENTS. (COMPONENT 2).

$$P = 20.53644 \text{ days},$$
$$\mu = 17^\circ.5298,$$
$$T = 5.56 \text{ days from}$$
1912, March 10.000,
$$e = 0.5410,$$
$$\omega = 282^\circ\ 6'.4,$$
$$K = 67.0 \text{ km.},$$
$$V_0' = -5.0 \text{ km.},$$
$$\gamma = -12.61 \text{ km.}$$

The normal equations derived from the sixteen equations of condition took the following form:

NORMAL EQUATIONS.

$$8.64\Gamma + 0.813\kappa + 2.459\pi + 0.836\epsilon + 0.297\tau + 4.712 = 0,$$
$$5.737\kappa + 0.489\pi + 1.107\epsilon + 0.063\tau - 0.912 = 0,$$
$$2.903\pi + 0.765\epsilon + 0.704\tau - 2.062 = 0,$$
$$0.794\epsilon + 0.247\tau + 1.459 = 0,$$
$$0.287\tau - 0.458 = 0.$$

As indicated in connection with Component 1, the elimination equations were formed from these normal equations. Again the value of [*ee.* 4] was small enough to indicate some uncertainty in the values of T and ω.

The five normal equations gave the following as the values of the unknowns:

$$\tau = +0.8046,$$
$$\epsilon = -5.2975,$$
$$\pi = +2.4510,$$
$$\kappa = +1.1694,$$
$$\Gamma = -0.8679.$$

The accuracy of the numerical work was shown by the small residuals which were obtained when the values of the unknowns were substituted in the normal equations, the average value for the five equations being 0.005, while the largest was only 0.007. The corrections and the corrected elements follow:

CORRECTIONS.

$$\delta K = +1.169 \text{ km.},$$
$$\delta\omega = -2^\circ\ 5'.8,$$
$$\delta e = +0.02530,$$
$$\delta T = +0.00983,$$
$$\delta\gamma = -0.0596 \text{ km.}$$

FIRST CORRECTED ELEMENTS.

$$K = 68.169 \text{ km.},$$
$$\omega = 280^\circ\ 0'.6,$$
$$e = 0.56630,$$
$$T = 5.56983 \text{ days from}$$
1912, March 10.000,
$$\gamma = -12.670 \text{ km}$$

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The residuals (observed minus computed velocities) were found from the normal equations and also from an ephemeris, the corrected elements being used. The differences for eight of the sixteen normal places exceeded 0.1 and ranged in value from 0.11 to 0.31. To make sure that the second order terms were not affecting the results, a second least square solution was made in which the corrected elements were taken as a beginning. The normal equations follow:

NORMAL EQUATIONS (SECOND SOLUTION).

$$\begin{aligned} 8.64\Gamma + 0.879\kappa + 3.596\pi + 0.658\epsilon + 0.318\tau + 1.958 &= 0, \\ 5.567\kappa + 0.228\pi + 1.272\epsilon + 0.002\tau - 0.876 &= 0, \\ 3.073\pi + 0.716\epsilon + 0.651\tau - 0.686 &= 0, \\ 0.936\epsilon + 0.238\tau - 1.217 &= 0, \\ 0.246\tau + 0.066 &= 0. \end{aligned}$$

This set of equations gave the following as the values of the unknown:

$$\begin{aligned} \tau &= -10.8148, \\ \epsilon &= +\ 2.8932, \\ \pi &= +\ 3.6645, \\ \kappa &= -\ 0.4082, \\ \Gamma &= -\ 1.5328. \end{aligned}$$

As in the other solution, the value of the quantity, [*ee.* 4], was small. To check the numerical computation, the values just derived were substituted in the normal equations. The average residual obtained from the five equations was 0.005, which showed complete agreement between the values of the unknowns and the equations.

The corrections and the corrected elements resulting from the second least square solution are given below.

CORRECTIONS.

$$\begin{aligned} \delta K &= -0.4082 \text{ km.}, \\ \delta\omega &= -3^\circ\ 4'.8, \\ \delta e &= -0.01304, \\ \delta T &= -0.11832 \text{ days}, \\ \delta\gamma &= +\ 0.706 \text{ km.} \end{aligned}$$

CORRECTED ELEMENTS.

$$\begin{aligned} K &= 67.761 \text{ km.}, \\ \omega &= 276^\circ\ 55'.8, \\ e &= 0.55326, \\ T &= 5.45151 \text{ days}, \\ \gamma &= -11.964 \text{ km.} \end{aligned}$$

The agreement between residuals by the normal equations and by ephemeris was satisfactory, thus indicating that second order terms were negligible. Hence, a third solution would yield unimportant changes.

As in the case of Component 1, the quantity $a \sin i$ was found. Its value follows:

$$a \sin i = 15{,}940{,}000 \text{ km.}$$

It is now possible to find the masses of the two components in terms of the mass of the sun and the angle i. The two values follow:

$$m_1 = \frac{1.57\odot}{\sin^3 i},$$

$$m_2 = \frac{1.46\odot}{\sin^3 i}.$$

The value of $\sin^3 i$ cannot be greater than unity and hence the numerators of these expressions represent the minimum values. However, in finding the ratio of the masses this indeterminate quantity cancels out and the following value is obtained:

$$\frac{m_1}{m_2} = 1.079.$$

As the final elements for Component 2, derived from the Ludendorff measures, we have the following:

FINAL ELEMENTS (COMPONENT 2).

$$\begin{aligned} P &= 20.53644 \text{ days}, \\ \mu &= 17^\circ\ 5298, \\ T &= 5.4515 \pm 0.0835 \text{ days from} \\ &\quad 1912, \text{ March } 10.000, \\ e &= 0.55326 \pm 0.01241, \\ \omega &= 276^\circ\ 55'.8 \pm 2^\circ\ 17'.5, \\ K &= 67.76 \pm 0.98 \text{ km.}, \\ \gamma &= -11.96 \text{ km.}, \\ a \sin i &= 15{,}940{,}000 \text{ km.}, \\ m_2 &= \frac{1.46\odot}{\sin^3 i}, \end{aligned}$$

$$\frac{m_1}{m_2} = 1.079.$$

The uncertainty in the determination of T and ω is again evident from the probable errors.

Having arrived at a set of elements which give satisfactory agreement between observed and

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computed velocities, the following points of interest should be noted:

1. The sum of the weighted squares of the residuals was reduced from 99.66 to 83.12 by the first least square solution and to 75.83 by the second.

2. The probable error of a normal place of weight unity was found to be ± 1.77 km.

3. The residuals recorded in column 9 of Table I were scaled from the final velocity curve, Plate VI. From these residuals, the probable error of an average plate was found to be ± 3.7 km. and for the best plates of the series it was found to be ± 2.4 km.

Velocity Curves. The velocity curves shown in Plate VI were plotted from an ephemeris computed from the final elements for each of the two components as derived from the Potsdam measures. Points were located at frequent intervals throughout the entire length of the curves so that they might give an accurate representation of the velocities in all parts of the orbits. Beginning at the left border of the plate, the upper curve belongs to Component 1 and the lower one to Component 2.

This completes the discussion of Dr. Ludendorff's measures with the exception of certain questions arising in connection with the ratio of the masses and the center of mass velocity. These two points will be taken up and discussed later in this paper.

ANN ARBOR OBSERVATIONS

Observations of ζ_1 Ursæ Majoris were begun at the Detroit Observatory March 10, 1912, and were continued until August 27, of the same year. During the following year, a few plates were made in order to strengthen the weak parts of the velocity curve. In all 137 spectrograms were made. Of this number, 128 were measurable, 90 plates showing the lines measurably separated and 38 showing single lines. In general, lantern slide plates were used. Of the 128 measurable plates, 122 were of this variety, while 6 were Seed 23. The average time of exposure for the lantern slide plates was 9.1 minutes for 108 plates. Fourteen lantern slide plates were made with a very narrow slit with an average time of exposure equal to 35 minutes. The 6 Seed 23 plates required an exposure of 5.4 minutes on the average. While much time might have been saved by using the more rapid plates, the coarser grain made measuring much more difficult and in some cases, impossible. The lantern slide plates should be used on this and similar binaries in preference to more rapid plates. It is an interesting fact that the spectrograms made with the one prism instrument of the Detroit Observatory gave points on the velocity curve almost as near the cross-points of the curves as did the plates made with the three prism instrument at Potsdam.

Standard of Dispersion. The writer has previously measured and reduced a hundred plates of γ Lyræ. For use in that piece of work, a standard was obtained by taking the average screw reading for each of twenty-eight well defined titanium lines of the comparison spectrum as measured on ten of the best plates of the series. Three good lines, well distributed over the region of the spectrum to be used, were selected for the determination of the constants of the corrected Hartmann interpolation formula. This set of constants, and hence the same standard of dispersion, were used in the reduction of the plates of ζ_1 Ursæ Majoris.

Measures. The spectrum of ζ_1 Ursæ Majoris is usually classified as belonging to Class Ap, meaning that the star is of division A of the Harvard classification with certain peculiarities. The measures have been considered as difficult by each astronomer who has worked on the plates of this star. Vogel says, "When the spectra are more strongly displaced with reference to one another, most of the lines which appear double, become so weak that the measurement of their separation is rendered very difficult. On some plates, in fact, it was found possible to measure only the Mg line, λ 4481. It is to be noted that the fine lines of the spectra of Class Ia2 are only obtained when the exposure time is exactly correct, and the plate correctly developed. In general, the measures are to be classed as difficult, either on account of their excessive fineness or, in case of the Mg and H lines, on account of the too great width and diffuseness of the lines." All other measurers have experienced the difficulties set forth in the quotation from Vogel. The uncertainty of the measures is shown by the

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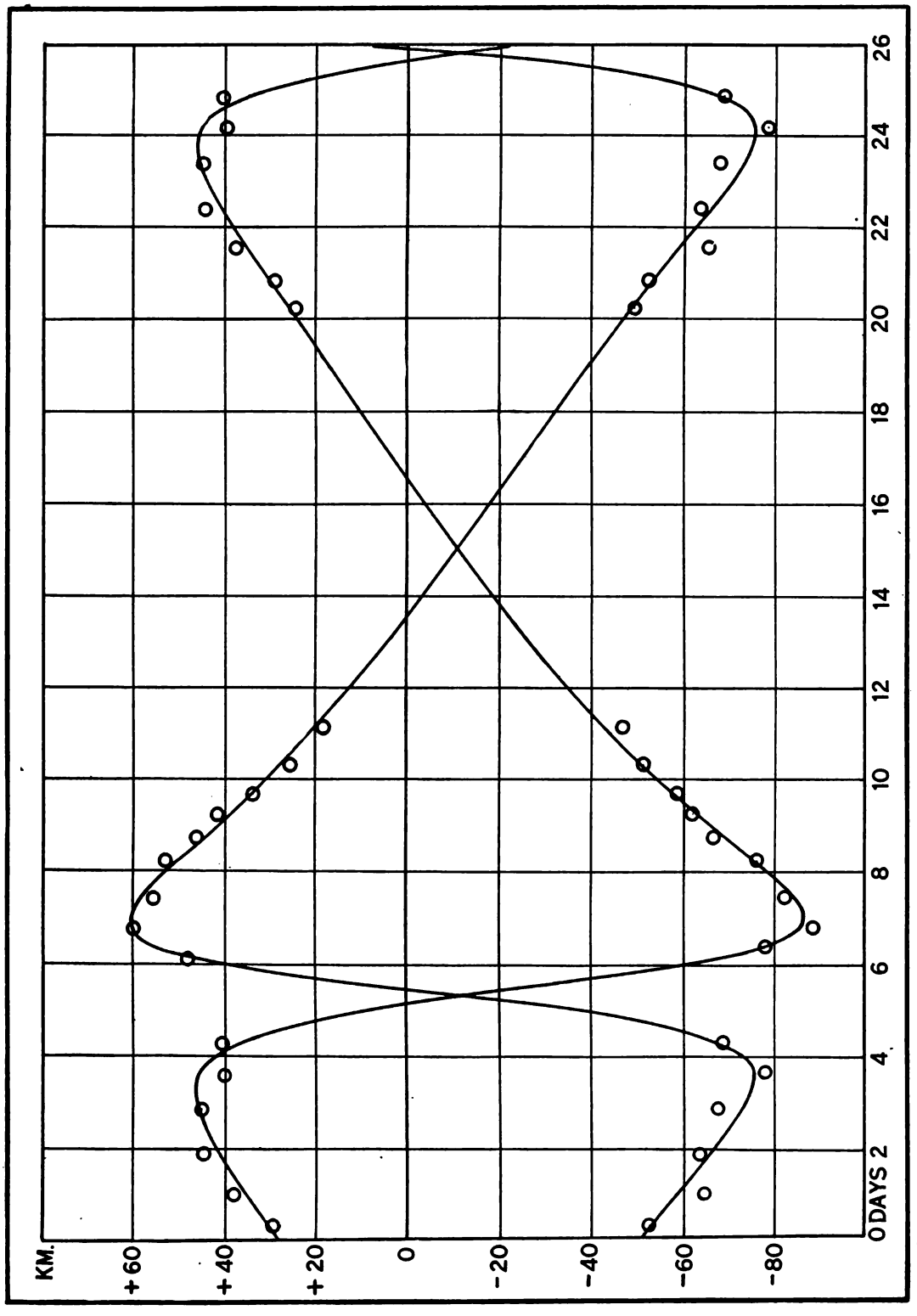

PLATE VI. VELOCITY CURVES OF ζ_1 URSÆ MAJORIS. VELOCITIES BY LUDENDORFF

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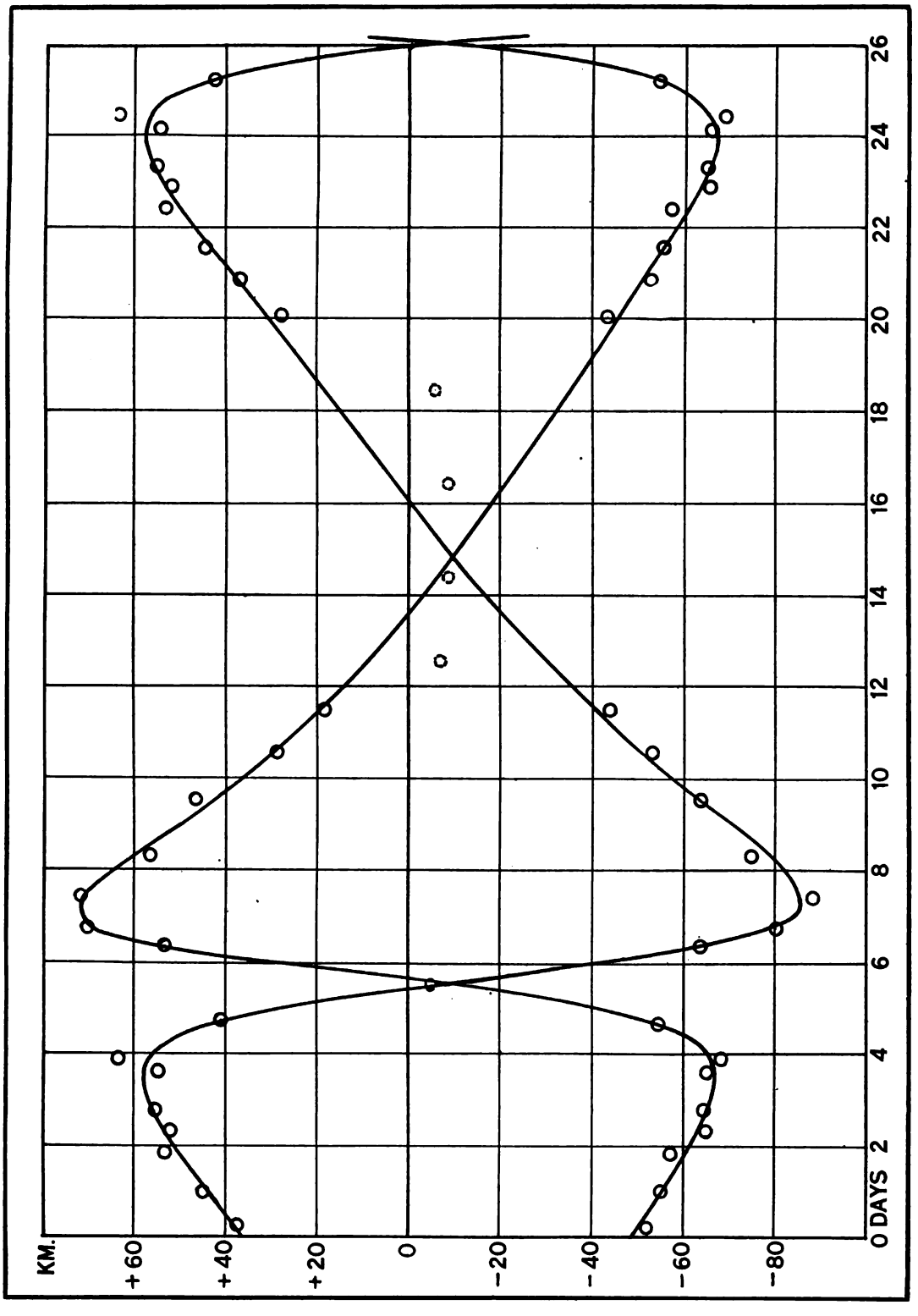

PLATE VII. VELOCITY CURVES OF ζ_1 URSAE MAJORIS. ANN ARBOR VELOCITIES

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size of the probable error of a single plate and was also very evident from the lack of agreement in the measures of the displacement of the various lines on a single plate.

Identification of lines. Of the 30 lines used in this series of plates, all except 5 were identified with certainty so that the corresponding wave-lengths were obtainable from the tables giving such data. The wave-lengths for the five unidentified lines were found as will be explained later.

Preliminary velocities. Having identified as many lines as possible, the next process was the determination of the velocities resulting from the displacements shown by the known lines. The wave-lengths adopted were taken from recognized authorities as will be seen by referring to Table III. Using the corrected Hartmann interpolation formula and the constants previously found, the screw reading for zero displacement of each star line was computed and also the corresponding factor necessary to transform displacements in terms of screw readings into velocities in km. per second. The velocity, as given from the displacement of each known measured line on every plate, was then found. The arithmetical mean of the velocities as shown by the lines of each plate was taken as the velocity for that plate unless the lines were clearly of unequal weight. By this means, a complete set of preliminary velocities was determined.

Unknown lines. These preliminary velocities and the proportionality factors of the unknown lines for the purpose of changing screw readings into velocities having been determined, it was not difficult to reverse the process and find the wave-lengths of the lines so that they would give the same velocity as that shown by the known lines of the plate. This was done for each unknown line for all of the plates on which it was measured and a weighted mean of the results taken as the wave-length of that particular unidentified line. The results are shown in Table III. The accuracy of these wave-lengths and confidence to be placed in their determination is shown by their probable errors and by the number of measures.

Use of the newly determined lines. The probable errors in the values of the wave-lengths, which were found, indicate such accuracy as would lead to a better determination of the velocities from the measures of the separate plates if these new lines were included. Hence, the velocities were computed, use being made of the displacements of these new lines. That the values were improved was very evident from the closer agreement between the velocities as determined from successive plates made on the same night.

Final velocities. One other correction was necessary before the velocities could be accepted as final. The residuals for each line on all of the plates, which showed measurable separation, were found. These residuals came from a comparison of the velocity as given by the line itself with the velocity as given by the plate. A correction to be applied to each line was then found which would reduce to zero the algebraic sum of the residuals of that line for all of the plates on which it was measured. These corrections produced noticeably better agreement between observed velocities and those computed from the final elements.

Table III. This table contains data concerning the lines used in this investigation. The first column gives the assumed wave-length of the line. The second column gives the authority. R stands for Rowland, K for Kayser, L for Lockyer, RHC for Professor Ralph H. Curtiss of this observatory in his paper on β Lyræ, L H for the value as determined by the writer of this paper. The third column gives the intensity on a scale of 1 to 20. The fourth column gives the element, if it is known. The fifth column gives the wave-length in ζ_1 Ursæ Majoris, from which the final velocities were obtained. Columns 6, 7, and 8 give the number of times the lines were measured on plates showing separation of the components.

Following the table is to be found the computed wave-length for each of the unknown lines, the probable error, and the number of measures on which the determination depended. The probable errors do not, however, take into account the very slight inaccuracies affecting the determination of the wave-lengths by the corrected Hartmann interpolation formula.

Ann Arbor velocities. The results of the measures of the plates made at this Observatory are given in Tables IV and V. Table IV contains

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TABLE III. WAVE-LENGTHS.

ASSUMED WAVE-LENGTH. (1)	AUTHORITY. (2)	INTENSITY. (3)	ELEMENT. (4)	WAVE-LENGTH IN ζ_1 URSAE MAJORIS. (5)	NUMBER OF MEASURES. COMP. I (6)	COMP. 2 (7)	TOTAL. (8)
3905.670	R	2	Si	3905.724	9	10	19
3913.640	K	2	Ti	3913.605	15	13	28
3933.825	R	10	Ca	3933.839	27	27	54
4005.730	RHC	2	?	4005.627	21	22	43
4045.975	R	3	Fe	4045.989	60	58	118
4063.770*	LH[1]	1	?	4063.773	42	43	85
4067.30	L	1	Enh. Ni.	4067.397	21	19	40
4071.908	R	1	Fe	4071.983	18	18	36
4077.89	L	2	Enh. Sr.	4077.928	41	40	81
4101.927	R	20	Hδ	4101.946	1	1	2
4128.204	RHC	1	Si	4128.336	16	13	29
4132.139	K	1	Van.	4132.455	10	12	22
4136.869*	LH[2]	1	?	4136.826	9	8	17
4144.038	R	1	Fe	4143.992	31	28	59
4163.882*	LH[3]	1	?	4163.877	6	6	12
4173.765	RHC	1	?	4173.748	36	38	74
4179.070	RHC	1	?	4179.146	7	5	12
4215.680	K	1	Sr	4215.725	43	42	85
4233.543*	LH[4]	1	?	4233.505	39	39	78
4290.382	K	1	Ti	4290.327	22	26	48
4308.072*	LH[5]	1	?	4308.114	17	15	32
4325.939	K	2	Fe	4325.799	10	10	20
4340.634	R	20	Hγ	4340.662	3	3	6
4352.42†	K	2	Sb	.....	..	..	..
4443.99	L	1	Enh. Ti	4444.031	11	11	22
4481.397	RHC	6	Mg	4481.462	46	45	91
4534.171	L	2	Enh. Ti	4534.220	3	0	3
4549.808†	R	2	Ti & Co	.....	..	..	..
4572.473†	K	1	Ce	.....	..	..	..
4861.527†	R	20	Hβ	.....	..	..	..

* LH[1] λ 4063.770 $\pm$ 0.012, 71 measures.
* LH[2] λ 4136.869 $\pm$ 0.040, 10 measures.
* LH[3] λ 4163.882 $\pm$ 0.035, 10 measures.
* LH[4] λ 4233.543 $\pm$ 0.019, 54 measures.
* LH[5] λ 4308.072 $\pm$ 0.039, 24 measures.
† This line was measured only on plates showing lines unresolved.

data coming from the plates which showed the lines of the two spectra measurably separated and Table V, the data coming from the plates in which the lines were not resolved and hence had to be measured as single lines. The different columns of these tables give the same data as the corresponding columns in the table of Ludendorff's measures. The initials in column (2) are those of the observer at the telescope; Prof. Curtiss, Mr. Mellor, and the writer. All of the measures were made by the writer. The measuring engine was carefully tested for screw error, which was found to be inappreciable. Hence, no correction was needed for that source of error.

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TABLE IV. VELOCITIES.

ANN ARBOR OBSERVATIONS.

PLA. NO.	OBS.	DATE G. M. T.	PHASE.	COMPONENT 1			COMPONENT 2			WT.
				N	V	O–C	N	V	O–C	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1912								
319	H	March 10.760	0.760	5	+53.2	+10.2	5	−52.0	+ 1.5	5
321	H	10.786	0.786	3	+49.8	+ 6.6	3	−59.9	− 6.0	3
333	M	12.792	2.792	8	+64.3	+ 8.0	7	−49.7	+15.3	7
334	M	12.808	2.808	11	+48.5	− 7.8	8	−76.4	−11.4	9
358	M	16.847	6.847	9	−73.8	+ 7.5	9	+76.6	+ 6.1	9
359	H	16.912	6.912	11	−75.9	+ 6.1	11	+79.8	+ 9.3	11
412	H	29.809	19.809	5	+31.5	+ 1.9	4	−31.1	+12.4	4
413	M	29.819	19.819	1	+21.0	− 8.6	1	−44.5	− 1.0	1
421	M	Apr. 5.800	6.264	6	−51.0	+ 7.0	5	+54.9	+ 7.9	5
422	H	5.811	6.275	5	−61.2	− 3.2	3	+29.7	−17.3	4
435	M	7.754	8.218	9	−65.9	+14.1	10	+52.5	+ 1.0	9
436	M	7.764	8.228	3	−84.5	− 4.5	3	+54.7	+ 3.2	3
460	M	23.750	3.677	8	+61.2	+ 3.8	8	−55.1	+11.0	8
461	H	23.756	3.683	7	+54.6	− 2.8	7	−69.5	− 3.4	7
472	C	26.708	6.635	8	−70.2	+ 4.8	8	+72.6	+ 6.4	8
473	M	26.712	6.639	7	−78.8	− 3.8	7	+60.6	− 5.6	7
475	M	26.738	6.665	9	−88.1	−13.1	9	+58.6	− 7.6	9
531	M	May 10.631	0.022	7	+38.2	+ 2.7	7	−43.1	+ 5.5	7
533	M	10.671	0.062	4	+44.5	+ 7.5	3	−47.6	+ 1.6	3
535	M	14.589	3.980	7	+53.4	− 2.4	7	−73.2	− 8.2	7
536	M	14.594	3.985+	8	+66.1	+10.3	8	−73.6	− 8.6	8
555	M	17.578	6.969	5	−91.4	− 8.4	4	+59.7	−11.5	4
556	M	17.585	6.976	3	−95.3	−12.3	3	+72.7	+ 1.5	3
568	C	19.672	9.063	7	−68.1	+ 1.4	6	+57.9	+ 9.8	6
607	C	30.617	20.008	2	+20.4	− 0.9	2	−45.2	− 0.2	2
608	C	30.624	20.015	2	+21.1	− 0.2	3	−51.3	− 6.3	2
620	M	31.600	0.454	11	+30.0	− 3.2	7	−52.7	− 1.1	9
621	M	31.606	0.460	6	+37.7	− 1.5	4	−65.6	−14.0	5
635	C	June 2.591	2.445	8	+35.1	−18.8	8	−63.2	+ 0.5	8
636	C	2.596	2.450	5	+46.0	− 7.9	7	−64.6	− 1.1	6
657	M	4.627	4.481	5	+39.2	− 8.8	6	−72.0	−13.0	5
658	M	4.634	4.488	5	+30.9	−17.1	5	−66.0	− 7.0	5
668	H	6.580	6.434	10	−66.9	+ 0.6	8	+53.0	− 5.0	9
669	H	6.594	6.448	4	−63.9	+ 3.6	4	+55.1	− 3.0	4
670	H	6.610	6.464	9	−65.1	+ 2.4	11	+58.2	+ 0.2	10
681	M	7.603	7.457	18	−88.2	− 3.0	18	+73.1	+ 2.3	18
682	M	7.609	7.463	21	−89.4	− 4.2	20	+71.6	+ 0.8	20
688	M	8.594	8.448	15	−77.7	− 0.2	14	+54.6	− 3.6	14
689	M	8.601	8.455	15	−72.6	+ 4.9	15	+61.5	+ 3.3	15
706	H	9.593	9.447	8	−58.8	+ 5.4	7	+43.6	+ 0.2	7

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TABLE IV. VELOCITIES.

ANN ARBOR OBSERVATIONS—CONT'D.

PLA. NO.	OBS.	DATE G. M. T.	PHASE.	COMPONENT 1			COMPONENT 2			WT.
				N	V	O–C	N	V	O–C	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1912								
707	H	June 9.606	9.460	6	−61.2	+ 3.0	6	+44.8	+ 1.4	6
718	C	10.617	10.471	5	−49.7	+ 1.6	5	+35.5	+ 5.3	5
719	C	10.639	10.493	7	−58.0	− 6.7	7	+24.3	− 5.9	7
731	M	11.617	11.471	6	−45.1	− 5.1	6	+ 9.2	−10.8	3
732	M	11.631	11.485	3	−42.7	− 2.8	3	+28.0	+ 7.8	3
749	M	21.669	0.987	6	+34.6	+ 9.1	5	−51.7	+ 3.6	5
750	M	21.677	0.995	6	+43.2	− 0.5	5	−66.1	−10.8	5
758	M	22.588	1.906	1	+46.1	− 4.2	1	−64.9	− 4.5	1
759	M	22.597	1.915	6	+47.7	− 2.6	7	−49.3	+11.1	6
773	M	28.619	7.937	2	−80.8	+ 2.2	2	+58.7	+ 3.9	2
774	M	28.624	7.942	4	−77.2	+ 5.0	4	+52.5	− 2.3	4
782	H	30.594	9.912	5	−64.4	+ 7.6	7	+45.3	+ 4.3	6
783	H	30.603	9.921	5	−64.2	+ 7.8	5	+43.5	+ 2.5	5
845	C	July 11.613	0.394	9	+33.1	− 6.1	10	−48.0	+ 3.5	9
846	C	11.625	0.406	7	+38.1	− 1.1	6	−62.2	−10.7	6
853	H	13.594	2.375	7	+62.0	+ 8.3	7	−69.9	− 6.7	4
854	H	13.608	2.389	5	+63.2	+ 9.5	5	−65.4	− 2.2	5
863	H	14.593	3.374	6	+37.7	−20.3	6	−73.2	− 5.7	6
864	H	14.606	3.387	8	+53.2	− 4.8	8	−69.8	− 3.2	8
873	H	17.638	6.419	14	−60.5	− 3.0	14	+61.0	+ 3.9	14
884	H	18.588	7.369	10	−80.9	+ 4.3	10	+76.1	+ 4.1	10
887	C	18.648	7.429	4	−93.9	− 8.7	4	+56.5	−15.5	4
926	H	31.577	20.359	3	+31.6	− 2.9	3	−48.4	− 0.7	3
927	H	31.636	20.417	3	+35.1	+ 0.6	3	−43.9	+ 3.8	3
936	C	Aug. 1.600	0.845	7	+43.1	+ 0.6	7	−48.7	+ 5.6	7
939	C	2.591	1.836	8	+42.8	− 7.0	7	−60.5	− 0.3	7
940	H	2.501	1.846	11	+60.0	+10.2	11	−52.3	+ 7.9	11
941	H	2.623	1.868	9	+58.9	+ 9.1	9	−63.7	− 3.5	9
949	H	4.580	3.825	8	+70.5	+13.5	9	−58.8	+ 8.9	8
950	H	4.588	3.833	5	+66.5	+ 9.5	5	−66.9	+ 0.8	5
951	H	4.597	3.842	7	+67.2	+10.2	8	−64.8	+ 2.9	7
952	H	4.603	3.848	8	+58.7	+ 1.7	8	−72.7	− 5.0	8
959	H	11.584	10.829	4	−47.5	+ 0.2	5	+29.4	+ 2.6	4
999	H	21.565	0.273	5	+32.2	− 6.5	6	−53.8	− 2.8	5
1000	H	21.586	0.294	5	+45.0	+ 6.3	6	−47.5	+ 3.5	5
1009	C	22.623	1.331	3	+40.1	− 5.1	4	−62.7	− 6.7	3
1010	C	22.647	1.355	6	+50.0	+ 4.8	6	−52.6	+ 3.4	6
1015	C	23.565	2.273	10	+51.6	− 1.8	10	−67.6	− 4.6	10
1016	H	23.590	2.298	9	+60.9	+ 7.5	9	−65.8	− 2.8	9
1017	H	23.619	2.327	11	+52.6	− 0.8	11	−61.7	+ 1.3	11

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TABLE IV. VELOCITIES.

ANN ARBOR OBSERVATIONS—CONT'D.

PLA. NO.	OBS.	DATE G. M. T.	PHASE.	COMPONENT 1			COMPONENT 2			WT.
				N	V	O–C	N	V	O–C	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1912								
1030	H	Aug. 27.567	6.275	6	−55.0	+ 5.2	6	+48.2	− 2.1	6
1031	H	27.600	6.308	7	−58 2	+ 2.0	7	+46.4	− 3.9	7
		1913								
2272	H	Aug. 29.597	3.650	17	+51.1	− 6.1	18	−64.0	+ 2.0	17
2273	H	29.657	3.710	11	+60.5	+ 3.3	12	−66.1	− 0.1	11
2274	H	29.662	3.715	11	+58.4	+ 1.2	10	−64.3	+ 1.7	10
2278	H	30.613	4.666	16	+44.3	+ 0.3	16	−54.5	+ 0.5	16
2279	H	30.639	4.692	14	+50.1	+ 6.6	15	−61.9	− 6.9	14
2280	H	30.663	4.716	18	+47.7	+ 4.7	19	−58.5	− 4.0	18
2281	H	30.675	4.728	14	+38.8	− 3.7	14	−49.0	+ 5.0	14
2282	H	30.688	4.741	8	+31.9	−10.1	6	−41.0	+12.0	7

Normal Places. The data derived from plates showing the lines measurably separated will be discussed first. The observations were combined into sixteen normal places with phase as the basis of grouping. Care was taken that the limits of phase for each place be small enough for the velocity curves to be essentially straight over that interval. However, the correction for curvature was applied even though very small. The weight assigned was based on the number of measurable lines on all of the plates used in making up the normal place. The phases given in column (2) of Table VI are based on the final period and have been reduced to the sun. The values given in column (3) are the limits of phase within which all of the plates of that normal place fall. The velocities of the two components are given; also, the errors in the sense, observed minus computed, the final elements being used in determining the computed velocities.

The normal velocities, given in Table VI, are shown by the small circles as plotted in Plate VII.

Component 1. From the data of Table VI, a velocity curve was plotted and the following set of preliminary elements found.

PRELIMINARY ELEMENTS (COMPONENT 1).

$P = 20.53644$ days,
$\mu = 17°.5298$,
$T = 5.83$ days from 1912, March 10.000,
$e = 0.5224$,
$\omega = 103° 18'.2$,
$K = 72.85$ km.,
$V_0' = -13.35$ km.,
$\gamma = -4.1$ km.

An ephemeris having been computed and from it the residuals obtained, a least square solution was undertaken. The usual method of treatment gave the following

NORMAL EQUATIONS.

$$8.101\Gamma + 1.536_K - 0.485\pi - 0.185e + 0.659\tau + 5.332 = 0,$$
$$+ 6.101_K - 0.297\pi - 0.096e - 0.109\tau + 8.715 = 0,$$
$$+ 1.999\pi - 0.053e + 0.833\tau - 5.362 = 0,$$
$$+ 0.894e + 0.037\tau - 1.020 = 0,$$
$$+ 0.511\tau - 2.126 = 0.$$

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TABLE V. VELOCITIES.

ANN ARBOR OBSERVATIONS.

PLATE NO.	OBS.	DATE G. M. T.	PHASE.	SINGLE LINES.			
				N	V	RESIDUALS.	WT.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		1912					
377	C	Mar. 22.827	12.827	16	— 3.5	+ 3.8	16
378	C	22.833	12.833	21	— 3.5	+ 3.8	21
452	H	Apr. 15.724	16.188	21	— 8.4	— 1.1	21
453	H	15.733	16.197	23	— 5.9	+ 1.4	23
495	H	May 2.655	12.582	26	— 3.0	+ 4.3	26
496	H	2.661	12.588	23	— 8.1	— 0.8	23
512	M	7.681	17.608	17	+ 2.7	+ 10.0	17
513	M	7.687	17.614	11	— 5.9	+ 1.4	11
517	H	8.669	18.596	24	— 2.6	+ 4.7	24
518	H	8.678	18.605	19	— 13.4	— 6.1	19
573	H	22.666	12.057	23	— 9.1	— 1.8	23
574	H	22.676	12.067	27	— 16.7	— 9.4	27
580	M	24.680	14.071	24	— 12.5	— 5.2	24
581	M	24.686	14.077	22	— 11.2	— 3.9	22
595	M	25.742	15.133	16	— 1.4	+ 5.9	16
596	M	25.749	15.140	25	— 5.5	+ 1.8	25
602	C	27.642	17.033	23	— 8.5	— 1.2	23
603	C	27.652	17.043	23	— 7.9	— 0.6	23
734	H	June 12.646	12.500	15	— 4.4	+ 2.9	15
735	H	12.653	12.507	17	— 5.9	+ 1.4	17
763	M	25.588	4.906	5	— 8.8	— 1.5	5
807	H	July 3.649	12.967	18	— 10.3	— 3.0	18
808	H	3.677	12.995	22	— 1.8	+ 5.5	22
811	M	6.653	15.971	19	— 13.3	— 6.0	19
812	M	6.675	15.993	17	— 14.5	— 7.2	17
825	M	9.625	18.943	6	— 2.6	+ 4.7	6
826	M	9.635	18.953	5	— 4.5	+ 2.8	5
902	H	25.598	14.379	6	— 3.6	+ 3.7	6
903	H	25.628	14.409	11	— 12.7	— 5.4	11
911	M	27.610	16.391	16	— 6.2	+ 1.1	16
912	M	27.622	16.403	16	— 6.1	+ 1.2	16
918	C	29.624	18.405	7	— 6.0	+ 1.3	7
919	C	29.638	18.419	8	— 3.9	+ 3.4	8
972	H	Aug. 14.580	13.825	13	— 3.3	+ 4.0	13
973	H	14.603	13.848	14	— 13.6	— 6.3	14
1020	H	26.580	5.288	7	— 7.9	— 0.6	7
1021	H	26.611	5.319	7	— 2.3	+ 5.0	7
1022	H	26.647	5.355	6	— 0.5	+ 6.8	6

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These equations gave the following values of the unknowns:

$$\tau = +1.048,$$
$$\epsilon = +1.009,$$
$$\pi = +2.004,$$
$$\kappa = -1.202,$$
$$\Gamma = +0.372.$$

That these results are correct was shown by the small residuals obtained when the values of the unknowns were substituted in the normal equations. The largest of the residuals was 0.005 and the average of the five was 0.003.

The normal equations having been satisfied, the corrections to the elements and the corrected elements were found to be:

CORRECTIONS.

$$\delta K = -1.202 \text{ km.},$$
$$\delta\omega = -1^\circ\ 34'.6,$$
$$\delta e = -0.00456,$$
$$\delta T = +0.01258 \text{ days},$$
$$\delta\gamma = -1.611 \text{ km.}$$

CORRECTED ELEMENTS (COMPONENT 1).

$$K = 71.648 \text{ km.},$$
$$\omega = 101^\circ\ 43'.6,$$
$$e = 0.51784,$$
$$T = 5.84258 \text{ days from 1912, March 10.000},$$
$$\gamma = -6.221 \text{ km.}$$

The residuals as found from the normal equations and from the ephemeris as computed from these corrected elements were in satisfactory agreement. In no case was the difference larger than 0.08 km., while the average value was slightly less than 0.04 km. From this, it was evident that the second order terms neglected in the least square formulæ were negligible and hence a second least square solution was unnecessary.

As in the preceding cases, the formula for $a \sin i$ was applied. The value follows:

$$a \sin i = 17{,}310{,}000 \text{ km.}$$

When Component 2 had been carried through to this point, it was possible to find the mass of

TABLE VI. NORMAL PLACES.

ANN ARBOR OBSERVATIONS.

NO.	PHASE.	LIMITS OF PHASE.	WT.	COMPONENT I		COMPONENT 2	
				VELOCITY.	O–C.	VELOCITY.	O–C.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	6.380	6.26 to 6.47	0.75	−63.40	+1.05	+53.61	−0.97
2	6.784	6.63 to 6.98	0.65	−80.06	−0.73	+70.33	+1.72
3	7.440	7.36 to 7.47	0.66	−88.03	−2.68	+71.84	+0.99
4	8.327	7.93 to 8.46	0.60	−74.41	+3.84	+56.43	−3.38
5	9.545	9.06 to 9.93	0.38	−63.58	−0.66	+46.94	+4.68
6	10.570	10.47 to 10.83	0.20	−52.78	−2.24	+29.08	−0.37
7	11.479	11.47 to 11.49	0.04	−43.90	−3.35	+18.60	−0.94
8	20.050	19.80 to 20.42	0.08	+27.95	−3.79	−42.80	+2.66
9	20.853	20.55 to 21.00	0.62	+37.27	−0.86	−52.21	−1.43
10	21.539	21.29 to 21.90	0.43	+44.89	+1.32	−55.10	+0.14
11	22.400	22.37 to 22.45	0.43	+53.59	+3.45	−56.85	+3.69
12	22.889	22.80 to 22.99	0.67	+52.14	−1.35	−65.03	−1.83
13	23.337	23.32 to 23.35	0.20	+55.41	−0.62	−64.72	+0.45
14	24.156	23.91 to 24.26	0.85	+54.43	−3.32	−65.35	+0.94
15	24.427	24.36 to 24.53	0.54	+63.70	+7.01	−68.43	−3.14
16	25.194	24.91 to 25.28	1.00	+42.94	−0.99	−53.94	+0.15

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each component in terms of i and the mass of the sun. For Component 1 we have

$$m_1 = \frac{1.83\odot}{\sin^3 i}.$$

Bringing together the data which have been derived, we have the following

FINAL ELEMENTS (COMPONENT 1).

$P = 20.53644$ days,
$\mu = 17^\circ.5298$,
$T = 5.84258 \pm 0.07721$ days from 1912, March 10.000,
$e = 0.51784 \pm 0.00812$,
$\omega = 101^\circ\ 43'.6 \pm 2^\circ\ 24'.9$,
$K = 71.65 \pm 0.69$ km.,
$\gamma = -6.22$ km.,
$a \sin i = 17{,}310{,}000$ km.,
$m_1 = \frac{1.83\odot}{\sin^3 i}$.

The probable errors of ω and T show about the same degree of uncertainty in the determination of these quantities as was found in the reduction of Ludendorff's measures. This was to have been expected since the quantity [*cc.* 4] almost vanished as before.

Points to be noted in connection with this curve and the corresponding elements are the following:

1. The sum of the weighted squares of the residuals was reduced by the least square solution from 90.86 to 62.04.

2. The probable error of a normal place of weight unity was found to be $\pm$ 1.60 km.

3. The residuals recorded in column (7) of Table IV were scaled from the velocity curve which was plotted from an ephemeris based on the final elements. See Plate VII. From these residuals, the probable error of an average plate was found to be $\pm$ 4.6 km. and that of the best plates of the series was found to be $\pm$ 2.8 km.

Component 2. From the data of Table VI, a velocity curve was plotted and the following set of preliminary elements found for Component 2.

PRELIMINARY ELEMENTS (COMPONENT 2)

$P = 20.53644$ days,
$\mu = 17^\circ.5298$,
$T = 5.88$ days from 1912, March 10.000,
$e = 0.5420$,
$\omega = 287^\circ\ 14'.6$,
$K = 70.00$ km.,
$V_0' = +2.60$ km.,
$\gamma = -8.62$ km.

By means of an ephemeris from these elements, the residuals were found for use in a least square solution. The sixteen equations of condition gave the following normal equations:

NORMAL EQUATIONS.

$$\begin{aligned} 8.101\Gamma - 1.630\kappa + 0.681\pi + 0.390\epsilon - 0.538\tau + 2.287 &= 0, \\ + 6.131\kappa - 0.831\pi - 0.028\epsilon - 0.071\tau + 3.942 &= 0, \\ + 1.969\pi + 0.037\epsilon + 0.746\tau - 1.458 &= 0, \\ + 0.864\epsilon + 0.054\tau + 0.196 &= 0, \\ + 0.434\tau - 1.102 &= 0. \end{aligned}$$

A solution of these equations gave the values of the unknowns:

$\tau = +5.672$,
$\epsilon = -0.587$,
$\pi = -1.770$,
$\kappa = -0.791$,
$\Gamma = +0.113$.

The usual test for accuracy in the computation was applied and satisfactory agreement found, the largest residual being 0.008 and the average of the five, 0.004. The corrections and the corrected elements follow:

CORRECTIONS.

$\delta K = -0.791$ km.,
$\delta\omega = +1^\circ\ 27'.9$,
$\delta e = +0.00268$,
$\delta T = +0.06611$ days,
$\delta\gamma = -0.732$ km.

CORRECTED ELEMENTS (COMPONENT 2).

$K = 69.209$ km.,
$\omega = 288^\circ\ 42'.5$,
$e = 0.54468$,
$T = 5.94611$ days from 1912, March 10.000,
$\gamma = -9.352$ km.

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In only one case did the difference between the residuals as given by the normal equations and by the ephemeris exceed 0.1 km. and the value in that case was only 0.12 km. The average value of the sixteen differences was less than 0.04. A second least square solution was considered unnecessary, since it would yield no changes of importance.

As in the preceding cases, the value of the quantity $a \sin i$ was found.

$$a \sin i = 16{,}390{,}000 \text{ km.}$$

The formulæ for finding the masses were applied with the following results:

$$m_1 = \frac{1.83\,\odot}{\sin^3 i},$$

$$m_2 = \frac{1.79\,\odot}{\sin^3 i}.$$

A comparison of the two masses gives the following value for their ratio:

$$\frac{m_1}{m_2} = 1.025.$$

Bringing together the data concerning Component 2, we have the following

FINAL ELEMENTS (COMPONENT 2).

$P = 20.53644$ days,
$\mu = 17°.5298$,
$T = 5.94611 \pm 0.05825$ days from 1912, March 10.000,
$e = 0.54468 \pm 0.00681$,
$\omega = 288° 42'.5 \pm 1° 58'.6$,
$K = 69.21 \pm 0.56$ km.,
$\gamma = -9.352$ km.,
$a \sin i = 16{,}390{,}000$ km.,

$$m_2 = \frac{1.79\,\odot}{\sin^3 i},$$

$$\frac{m_1}{m_2} = 1.025.$$

In connection with these elements and the corresponding velocity curve, the following points are of interest:

1. The sum of the weighted squares of the residuals was reduced from 41.41 to 40.92. The amount of this reduction is inconsiderable, but the fact should be noted that the sum of the weighted squares from the preliminary elements was less than the final sum after the application of the method of least squares in each of the other cases, indicating that the preliminary elements in this case satisfied the observed velocities exceptionally well.

2. The probable error of a normal place of weight unity was found to be $\pm$ 1.30 km.

3. The residuals recorded in column (10) of Table IV were scaled from the final velocity curve as plotted from the corrected elements. See Plate VII. From these residuals the probable error of the average plate was found to be $\pm$ 4.2 km., while that of the best plates of the series was $\pm$ 2.5 km.

Velocity curves. The velocity curves shown in Plate VII were plotted from an ephemeris computed from the final elements for each of the two components as derived from the Ann Arbor measures. Points were located at frequent intervals throughout the entire length of the curves so that they might give an accurate representation of the velocities in all parts of the orbits. Beginning at the left border of Plate VII, the upper curve belongs to Component 1 and the lower curve to Component 2.

Ratio of the masses and the center of mass velocity. Since there was an apparent lack of agreement between the values determined for each of these quantities and since Ludendorff found a difference in the values as determined at different epochs, it seemed desirable to give these elements further consideration. A formula is easily derived which gives a mathematical relation between the following quantities:

V_1 = a velocity for component 1,
V_2 = the corresponding velocity for component 2,
γ' = an assumed value for the center of mass velocity,
$\Delta\gamma$ = correction to be applied to the assumed center of mass velocity,
$\gamma = \gamma' + \Delta\gamma$,
$\frac{m_1}{m_2} = 1 + x$, defines the quantity, x.

If the assumed value of the center of mass velocity be near the true value and if the ratio of the masses be not far from unity, then $\Delta\gamma$ and x

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are both small quantities. These conditions were both satisfied in the case under consideration. The formula follows and is exact except for the omission of the second order term, $\frac{1}{2} x \cdot \Delta\gamma$ which may be neglected without appreciable error.

$$\Delta\gamma - \tfrac{1}{2}(V_1 - \gamma') \cdot x = \tfrac{1}{2}(V_1 + V_2) - \gamma'.*$$

Ann Arbor velocities. γ' having been assumed as —7.50 km., each pair of corresponding velocities when substituted in the equation gave an equation of condition. By the usual process these sixteen equations of condition were reduced to two normal equations with $\Delta\gamma$ and x as the unknowns. From a solution of the normal equations, the following values were found:

$$x = +0.032 \pm 0.011,$$
$$\Delta\gamma = +1.44 \pm 0.35 \text{ km.}$$

From which were obtained the ratio of the masses and the center of mass velocity:

$$\gamma = -6.06 \pm 0.35 \text{ km.},$$

$$\frac{m_1}{m_2} = 1.032 \pm 0.011.$$

The equation given above is not symmetrical in V_1 and V_2 as is evident from the left member. The analogous equation was derived and applied, giving results in agreement with the values just found.

At this point, it should be noted that this value of γ is less than 1 km. different from the mean of the velocities obtained from nearly a hundred spectrograms of ζ_2 Ursæ Majoris, as measured and reduced by the writer.

Potsdam velocities. A like solution was made using the normal velocities derived from the Ludendorff measures. The following results were obtained which are in close agreement with Ludendorff's determination of these same quantities as previously given in this paper:

$$\gamma = -12.55 \pm 0.58 \text{ km.},$$

$$\frac{m_1}{m_2} = 1.001 \pm 0.021.$$

This value of $\frac{m_1}{m_2}$ and its probable error throw some light on the fact that Ludendorff found from a part of his observations that the relative order of magnitude of this pair seemed to be reversed.

At this point it should be noted that there is less than 0.1 km. difference between this value of γ and the mean of the velocities obtained by Ludendorff for ζ_2 Ursæ Majoris.

Measures of plates showing single lines. These plates were measured and reduced with the same care as the plates showing the lines measurably separated. There were thirty-eight such spectrograms. In cases of this kind the measurement is that of the blends of the two sets of spectral lines. In case one spectrum had been decidedly stronger than the other, the fact should have revealed itself by an inclination of the line connecting the normal places formed from these velocities. No such inclination is evident, indicating that the spectra are of practically the same intensity. The apparent difference in the relative intensity of the spectral lines of the two components as noted by Miss Maury in the *Astrophysical Journal,* Vol. 8, page 174, was not observed in the measurement of the spectrograms made at this Observatory. The plates secured for the cross point at the steepest part of the curves were too few to give a good determination. However, the weights assigned to each of the other four normal places show the determinations to be of considerable importance in that they should give, theoretically, the center of mass velocity. The weighted mean of the last four normal places is —7.26 km.

The measures from the single line plates as combined into normal places are given below. The table shows the phase, limits of phase, weight, velocity and residual. These normal velocities are shown by the small dotted circles in Plate VII.

SUMMARY OF RESULTS

That a comparison of the elements may be made and the differences which occur may be clearly evident, the data concerning each component as found from both sets of measures are brought together. Rather unexpected differences in the values of some of the elements are found. These apparent discrepancies will be discussed later.

* *Publications of the Allegheny Observatory,* Vol. 1, No. 22, page 175.

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TABLE VII. VELOCITIES.

(FROM MEASURES OF SINGLE LINES.)

ANN ARBOR OBSERVATIONS

NO.	PHASE.	LIMITS OF PHASE.	WT.	SINGLE LINES.	
				VELOCITY.	RESIDUAL.
(1)	(2)	(3)	(4)	(5)	(6)
1	5.523	4.90 to 5.36	10	−4.92	+2.34
2	12.564	12.05 to 13.00	82	−6.99	+0.27
3	14.388	13.82 to 15.14	52	−8.56	−1.30
4	16.458	15.97 to 17.05	58	−8.56	−1.30
5	18.458	17.60 to 18.95	33	−5.48	+1.78

CORRECTED ELEMENTS OF COMPONENT 1.

	LUDENDORFF.	HADLEY.
P	$= 20.53644$ days,	20.53644 days,
μ	$= 17°.5298$,	$17°.5298$,
T	$= 5.6058 \pm 0.0700$ days,*	5.8426 ± 0.0772 days,*
e	$= 0.5315 \pm 0.0102$,	0.5178 ± 0.0081,
ω	$= 103° 52'.2 \pm 2° 47'.9$,	$101° 43'.6 \pm 2° 24'.9$,
K	$= 66.05 \pm 0.77$ km.,	71.65 ± 0.69 km.,
γ	$= -11.67$ km.,	-6.22 km.,
$a \sin i$	$= 15{,}800{,}000$ km.,	$17{,}310{,}000$ km.,
m_1	$= \frac{1.57\odot}{\sin^3 i}$.	$\frac{1.83\odot}{\sin^3 i}$.

* From 1912, March 10.000.

CORRECTED ELEMENTS OF COMPONENT 2.

P	$= 20.53644$ days,	20.53644 days,
μ	$= 17°.5298$,	$17°.5298$,
T	$= 5.4515 \pm 0.0835$ days,*	5.9461 ± 0.0582 days,*
e	$= 0.5533 \pm 0.0124$,	0.5447 ± 0.0068 days,
ω	$= 276° 55'.8 \pm 2° 17'.5$,	$288° 42'.5 \pm 1°58'.6$,
K	$= 67.76 \pm 0.98$ km.,	69.21 ± 0.56 km.,
γ	$= -11.96$ km.,	-9.35 km.,
$a \sin i$	$= 15{,}940{,}000$ km.,	$16{,}390{,}000$ km.,
m_2	$= \frac{1.46\odot}{\sin^3 i}$,	$\frac{1.79\odot}{\sin^3 i}$,
$\frac{m_1}{m_2}$	$= 1.079$.	1.025.

* From 1912, March 10.000.

A comparison of the four sets of elements is both interesting and instructive. The differences between the values of certain of the quantities is greater than the probable errors would lead one to expect. It seemed advisable to derive the elements in this way so that such differences as do occur may be clearly evident. These differences in no way discount the accuracy of the elements of such an orbit in so far as they are to represent the observations, but they do raise the question as to whether the observations give the exact conditions of motion as they exist in the binary system. However, as has been pointed out, the measures in this case were difficult.

The discrepancies between the four sets of elements, which have been tabulated, may be divided into two classes: those between the elements for the two components at the same epoch and those between the elements of the same component at different epochs.

Relatively large differences of both kinds were to be expected in the case of this star on account of the limitations imposed by the character of the spectra and by the existence of two components for each line when relative motion was sufficiently great to produce separation. Vogel has called attention to the poor quality of the lines from the standpoint of measurability. However, his conclusions are not surprising. In general, when resolved, each line of the star's spectrum falls upon the continuous spectrum of the other star and thus cannot present a clearly defined interruption. In other cases, a line of one spectrum might also coincide with a very faint line of the other spectrum and thus render the measure defective. Further, during about half of each orbital cycle the relative motion of the two components was too small to cause complete separa-

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tion of the lines, hence the measures under those conditions were of uncertain value. From a consideration of these facts it becomes very evident that the available data for the determination of elements is unavoidably limited in this case.

In a system of this character, with high orbital eccentricity and relatively close approach of the two bodies at periastron, there would be strong probability that librations and tidal disturbances would arise to such an extent that they would produce irregularities in the velocity curves. These irregularities in the velocity curves would in turn give rise to spurious differences in the elements of the two orbits. Irregularities, which are probably of this nature, have been observed in highly eccentric orbits with relatively short periods, such as θ Aquilæ and ψ Orionis. The great variation in the rate of anomalistic motion as the bodies pass from apastron to periastron in very eccentric orbits suggests at once a cause for at least a part of the apparent lack of consistency in the results.

An examination of the original papers discussing binaries with highly eccentric orbits shows clearly that the difficulties which have been encountered in the study of this star are not peculiar to it alone. The writer has examined with some care the papers dealing with thirty-three orbits whose eccentricities exceed 0.3, the average value being 0.498. While it is difficult, if not impossible, to determine what precision may be expected in such work, it is the opinion of the writer that not less than two-thirds of the cases examined show some peculiarity. In six cases secondary oscillations had been assumed in order to reduce the differences between observed and computed velocities. In many other cases a like treatment would undoubtedly have reduced the residuals. At least half of the cases examined show more or less marked indications of a secondary oscillation. Other cases showed unexpectedly large residuals. It seems more reasonable to attribute these irregularities in the observed velocities either directly or indirectly to the high eccentricity than it does to assume the presence of a small third body. There is no reason to believe that the presence of a small third body tends to produce a highly eccentric orbit. Visual observations have rarely revealed three-body systems, indicating that they are few in number. It seems to be a reasonable supposition that the same is true of systems less widely separated which can be revealed only by spectroscopic methods of observing. Hence, it seems more to the point to attribute slight departures from elliptic motion to tidal disturbances present in highly eccentric orbits.

In this connection, attention should be called to two sets of elements as derived from θ Aquilæ by Baker and Harper. A discussion of the elements of the orbit of this binary is to be found in the *Publications of the Royal Astronomical Society of Canada,* Vol. III, page 95. The difference in time for the two sets of observations is too small for the elements to have suffered appreciable change. The lack of agreement between the values as determined for certain elements is of the same order as has been found in the study of ζ_1 Ursæ Majoris. In both cases the probable errors indicate that the elements as derived satisfy the observations well, but the differences between the two sets of elements for the same star seem to show that the actual conditions of motion in the orbit are not obtained with as great precision as the probable errors of a single set of elements indicate.

A careful examination of the two plates showing the velocity curves for ζ_1 Ursæ Majoris reveals irregularities which can hardly be attributed to accidental errors. The many cases in which the observed velocities for the two sets of measures at approximately the same phase are either both larger or both smaller than the corresponding computed velocities is striking. The frequency with which this occurs is convincing evidence of its reality even though the real cause can not be definitely stated.

Apparently real irregularities existing in the velocity curves of stars of certain classes may place a limit on the accuracy of our determination of their orbital elements. The discrepancies observed in this paper furnish some evidence as to the uncertainty which may come from such irregularities and in that connection are highly instructive.

An effort was made to combine the four sets of elements, first, using the elements from the

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two components at the same epoch, and then using these elements in securing the final values.

Elements from the Potsdam Observations. In bringing together the values of the elements as derived from the two components and also from the two sets of observations, weights will be assigned which are inversely proportional to the squares of the probable errors of the different determinations.

Nothing further need be said concerning the values of P and μ.

Evidently, the two values of T cannot be different since the two stars of the system must both have the same time for periastron passage; hence, the weighted mean will be taken as the best value for this quantity.

In connection with the value of e, it is of interest to note that the eccentricity of the orbit of Component 2 is the larger for both sets of measures. The weighted mean will be accepted as the best determination of this quantity since the two orbits cannot differ in this respect.

The two values of ω should differ by 180°. The best value of ω for Component 1 is probably given by taking the weighted mean of the two after 180° has been subtracted from the value of ω for Component 2.

The values of K for the two components are not very different. Only a small difference would be expected for a system in which $\frac{m_1}{m_2}$ is nearly equal to unity, as it is in this case.

Values for the ratio of the masses and for the center of mass velocity were obtained from the least square solution along with the other elements. The values of these two quantities alone were determined from other considerations by a least square solution. The means of these values will be taken as the most probable values of γ and $\frac{m_1}{m_2}$.

The two values for $a \sin i$ should not be the same unless the ratio of the masses be unity, which it is not. However, the difference should be small in the case of this star.

Elements from Ann Arbor Observations. The treatment will be like that of the Potsdam observations, the one exception being in connection with the center of mass velocity. A determination of the velocity of the center of mass was obtained from the plates showing single lines as has been mentioned already. The precision of this method is less than that of the other methods, hence, the weight assigned to the result will be only half as great as that of the other determinations.

It should be noted at this point that the value of $a \sin i$ is larger for Component 1 than for Component 2, whereas m_1 is greater than m_2. The explanation is to be found in the fact that the two velocity curves as plotted from the normal velocities gave different values for the eccentricity. In the case of the Potsdam elements, the value of $a \sin i$ for Component 2 is the larger; however, it is only slightly larger, the difference being less than the value of $\frac{m_1}{m_2}$ would lead one to expect.

The elements of the two components for each set of observations having been combined in the manner set forth, the following sets of elements are given as the result of that process:

POTSDAM OBSERVATIONS.

$$P = 20.53644 \text{ days},$$
$$\mu = 17°.5298,$$
$$T = 5.5356 \pm 0.0546 \text{ days},^*$$
$$e = 0.54058 \pm 0.00805,$$
$$\omega = 99° 39'.3 \pm 1° 50'.9,$$
$$K_1 = 66.05 \pm 0.77 \text{ km.},$$
$$K_2 = 67.76 \pm 0.98 \text{ km.},$$
$$\gamma = -12.06 \text{ km.},$$
$$m_1 = \frac{1.57 \odot}{\sin^3 i},$$
$$m_2 = \frac{1.46 \odot}{\sin^3 i},$$
$$\frac{m_1}{m_2} = 1.038,$$
$$a_1 \sin i = 15{,}800{,}000 \text{ km.}$$
$$a_2 \sin i = 15{,}940{,}000 \text{ km.}$$

* From 1912, March 10.000 G. M. T.

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ANN ARBOR OBSERVATIONS.

$P = 20.53644$ days,
$\mu = 17°.5298$,
$T = 5.9088 \pm 0.0483$ days,*
$e = 0.53248 \pm 0.00530$,
$\omega = 105°35'.9 \pm 1°33'.8$,
$K_1 = 71.65 \pm 0.69$ km.,
$K_2 = 69.21 \pm 0.56$ km.,
$\gamma = -7.22$ km.,

$$m_1 = \frac{1.83\odot}{\sin^3 i},$$

$$m_2 = \frac{1.79\odot}{\sin^3 i},$$

$$\frac{m_1}{m_2} = 1.027,$$

$a_1 \sin i = 17{,}310{,}000$ km.,
$a_2 \sin i = 16{,}390{,}000$ km.

* From 1912, March 10.000, G. M. T.

CONCLUSIONS.

By combining the two sets of elements in the same manner as were the elements of the two components for one set of observations, we obtain the following as the most probable values and announce them as the definitive elements of ζ_1 Ursæ Majoris:

DEFINITIVE ELEMENTS.

$P = 20.53644$ days,
$\mu = 17°.5298$,
$T = 1912$, March. 15.7440 ± 0.0364 days, G. M. T.,
$e = 0.53476 \pm 0.00482$,
$\omega = 103°\ 57'.5 \pm 1°\ 12'.4$,
$K_1 = 69.22 \pm 0.52$ km.,
$K_2 = 68.83 \pm 0.56$ km.,
$\gamma = -9.64$ km.,

$$m_1 = \frac{1.70\odot}{\sin^3 i},$$

$$m_2 = \frac{1.62\odot}{\sin^3 i},$$

$$\frac{m_1}{m_2} = 1.032,$$

$a_1 \sin i = 16.4$ million km.,
$a_2 \sin i = 16.4$ million km.

There has been no marked change in the spectrum of the star so far as the writer was able to judge.

The small difference in the position of the line of nodes is not thought to be real. It is probably to be attributed to the peculiarities of the velocity curve, the positions of the normal places, and to personal equation.

An appreciable change in the eccentricity is improbable and the difference for the two components cannot be real so far as the actual orbits are concerned.

The values of the K's for the two sets of observations are in fair agreement. No real change in these quantities is probable in so brief a period of years. Personal equation and choice of spectral lines would account for a part, if not all, of the difference. The half-amplitude of the velocity curve for Component 1 is slightly larger than it is for Component 2, whereas m_1 is greater than m_2. This is due to peculiarities of the curves and the close approach of the ratio of the masses to unity.

The velocity of the center of mass of ζ_1 Ursæ Majoris and the mean velocities of ζ_2 Ursæ Majoris as determined from the Potsdam measures are in close agreement. The same is true of the Ann Arbor observations. It follows that the difference between the Potsdam and Ann Arbor values of γ for ζ_1 Ursæ Majoris is approximately equal to the difference in the mean velocity of ζ_2 Ursæ Majoris as determined at these two observatories. That each of the two stars should have a variable center of mass velocity and that the changes should be almost the same in magnitude and of the same sign seems to be an unreasonable assumption. Hence the writer has been led to the conclusion that the apparent change in γ for ζ_1 Ursæ Majoris is not real, but that it is due to systematic differences attributable to personal equation and other causes. The difference in this quantity noticed by Ludendorff as mentioned early in this paper may be explained by the fact that the parts of the spectrum used by Ludendorff in the two groups of plates were not identical.

The masses of the two components are nearly equal, but a summary of the data bearing on this

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point indicates that one component is approximately three per cent. larger than the other.

The determinations of $a \sin i$ for the two components are somewhat uncertain, but are of nearly equal value. Probably 16.4 million km. for each component is as definite as the data warrant.

The positions of the normal places as shown in connection with the velocity curves plotted from the final elements indicate the presence of a disturbing factor. The reality of these irregularities is shown by the fact that in many cases the residuals for the normal places of practically the same phase for the two components are of the same sign. A secondary oscillation suggests itself. However, the unavoidable lack of data in this case covering nearly half of the length of the curves makes the study of such an oscillation very difficult. The examination of many orbits of high eccentricity has led the writer to the conclusion that irregularities are the rule in such cases rather than the exception, and that they are attributable directly or indirectly to the high eccentricity.

While the probable errors seem to show the precision with which the derived elements represent the observed velocities, they do not show the precision with which the elements represent the actual conditions of motion of the star in its orbit.

The generally accepted theory as to the mutual increase of eccentricity and period seems to be sustained by the values of these two quantities as found in connection with this spectroscopic pair.

If the views as expressed by Wicksell in his paper entitled "Contributions to the Statistics of Spectroscopic Binary Stars" be correct, then this star would seem to belong to the class consisting of those binaries which have originated through fission rather than through the "capture process". The length of the period and the high eccentricity indicate that the existence of this star as a binary has been of long duration even though the spectral types of the two components show that they have not progressed very far in their evolutionary course.

The writer wishes to express his thanks to Professor Ralph H. Curtiss whose counsel, direction, and assistance have made this investigation possible. A word of appreciation is due Mr. Lewis Mellor of this Observatory for assistance in making plates.

Ann Arbor, Michigan, May, 1915.

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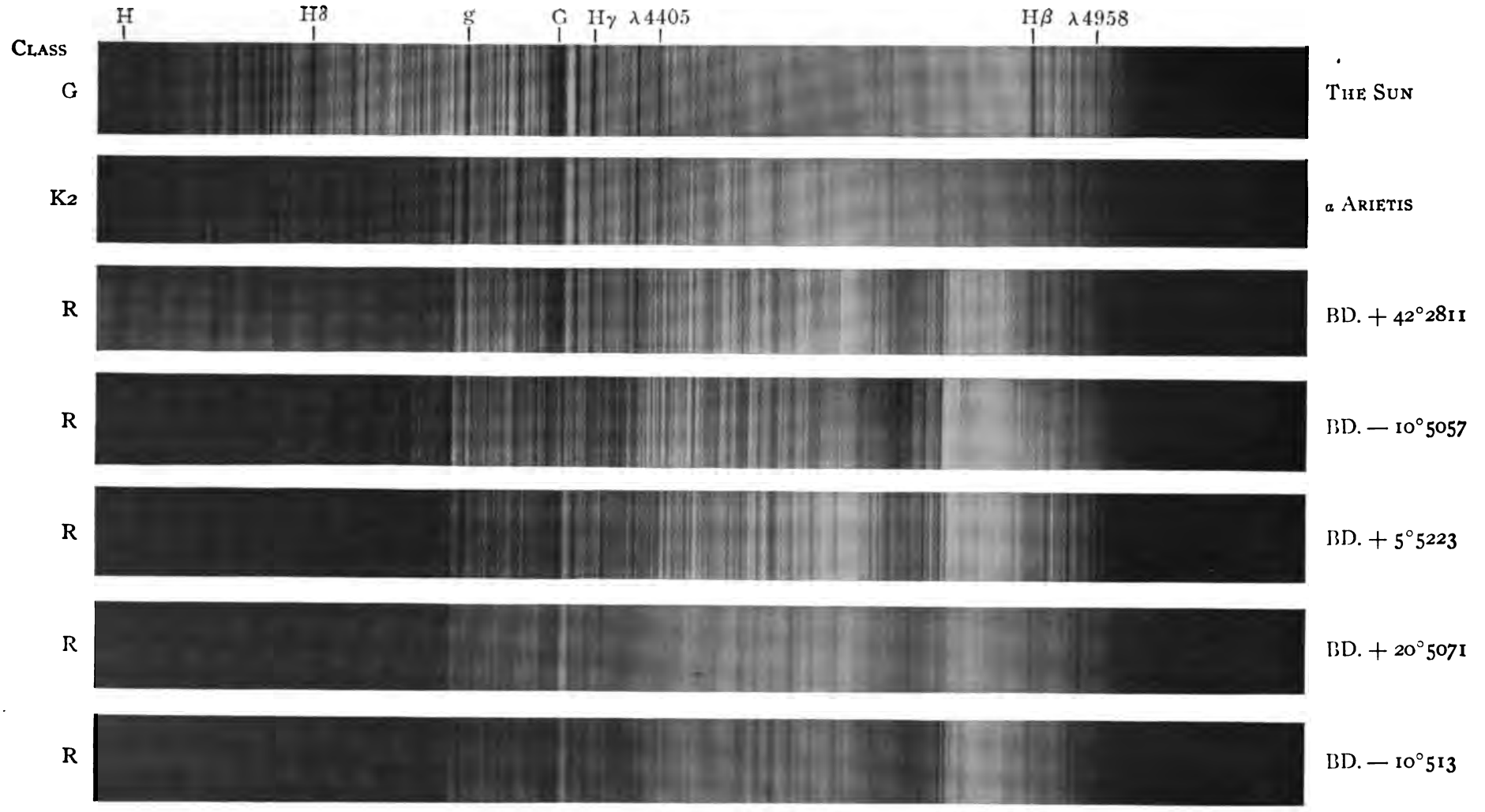

PLATE G. CLASS G, CLASS K2 AND EARLIER CLASS R STELLAR SPECTRA COMPARED

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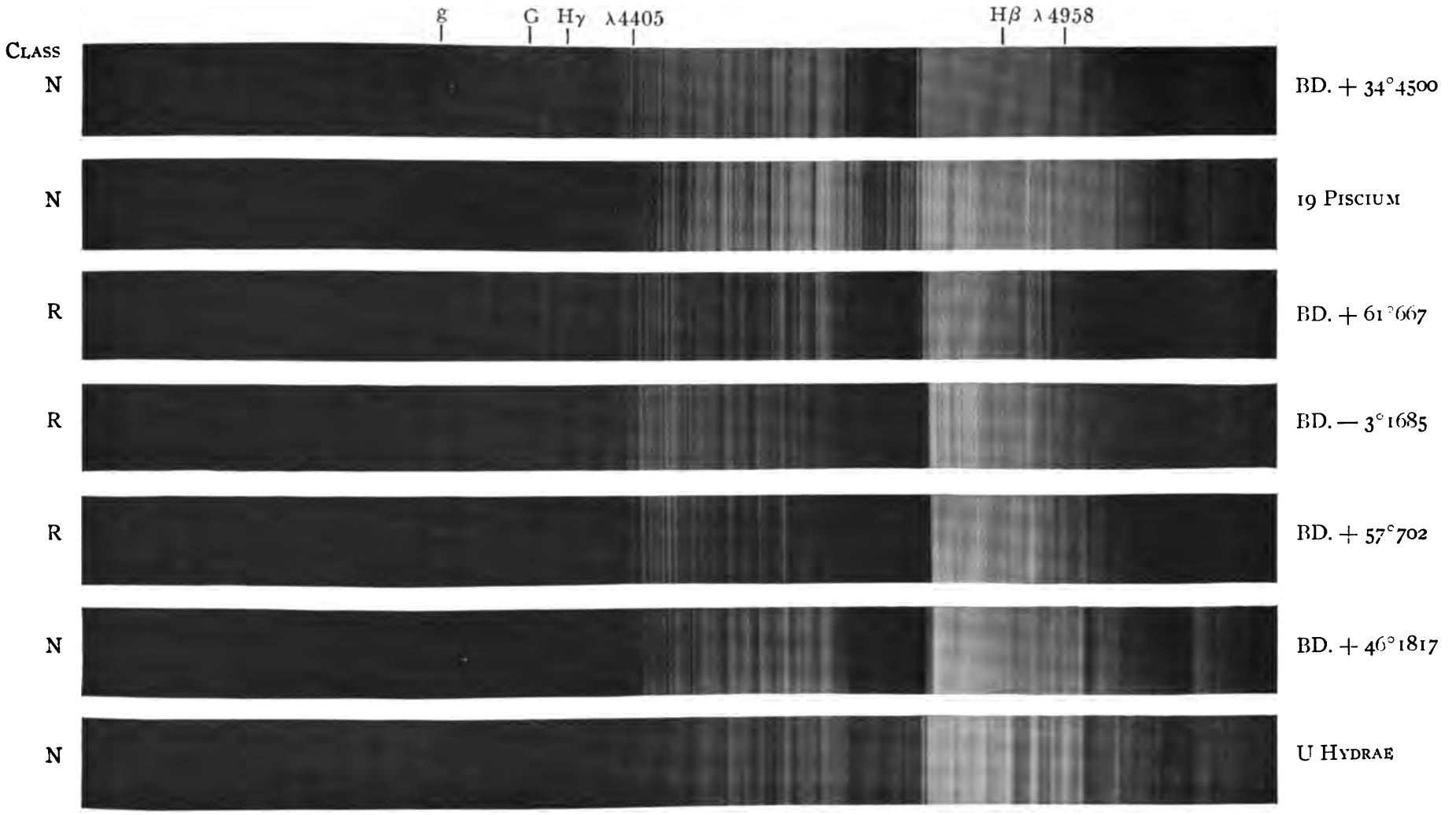

Plate H. Class N and Late Class R Stellar Spectra in the Photographic Region

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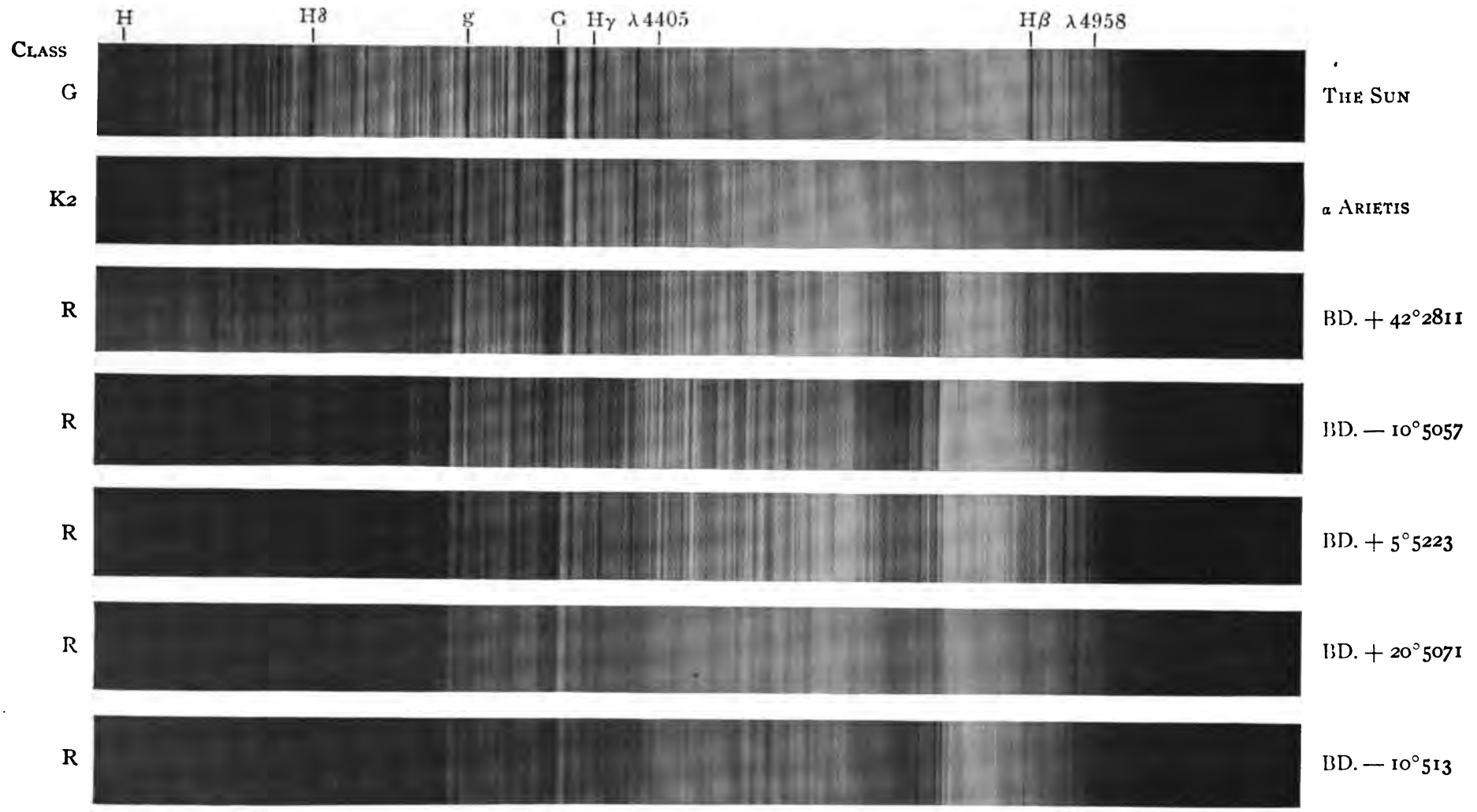

Plate G. Class G, Class K2 and Earlier Class R Stellar Spectra Compared

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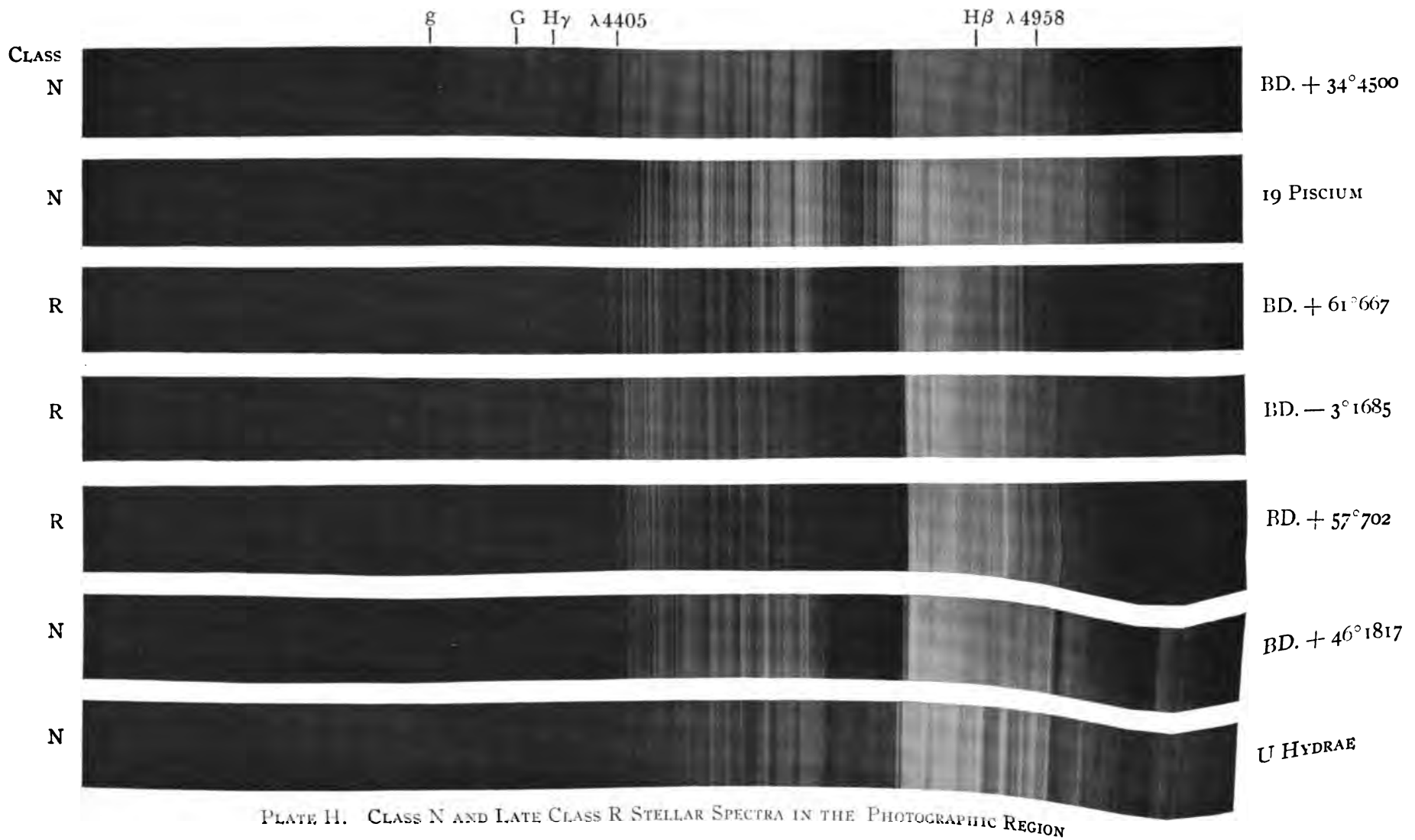

Plate H. Class N and Late Class R Stellar Spectra in the Photographic Region

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AN INVESTIGATION OF THE SPECTRA OF STARS BELONGING TO CLASS R OF THE DRAPER CLASSIFICATION[1]

By W. CARL RUFUS

INTRODUCTION

The genesis of Class R of the Draper Classification of stellar spectra may be traced from a list of "Stars Having Peculiar Spectra," published in *Harvard College Observatory Circular*, No. 9, July 9, 1896. Following the table is the statement: "Of the seven stars whose spectra are here announced as of Type IV the first, second, and seventh[2] (fourth), are normal. The spectra of others contain rays of much shorter wave-length than ordinary fourth type stars." This is a characteristic feature of the spectra of stars later designated as Class R and is here attributed to four stars, DM. $-38°$ 12843, $+85°$ 332, $-12°$ 5755, and $+5°$5223. Two other stars of the list, DM. $-31°$ 15954 and $-29°$ 15574, whose spectra were described as peculiar, but not associated with the four previously mentioned, have also been assigned to this class. These six stars constitute the nucleus of Class R.

Later announcements of stars having peculiar spectra increased the number possessing the common characteristics, "rays of shorter wave-lengths than the ordinary fourth type stars," and "a strong band extending from about λ 461 $\mu\mu$ to λ 471 $\mu\mu$." These stars were frequently referred to as belonging to "the same class as Z.C. 10^h 2112 described above", or "the same type as C.D.M. $-47°$ 6614, described in *Circular*, No. 76." A more convenient method of reference seemed to warrant the addition of a new class, which was suggested by Professor Pickering in *Harvard Circular*, No. 145, "A Sixth Type of Stellar Spectra," published December 1, 1908. A table of "Stars Having Spectra of Class R," containing sixty-one stars is given in *Harvard Annals*, Volume 56, page 220. A few others are known. In order to complete the list to date Table I is appended. The successive columns give the star's designation, right ascension and declination for 1900, Durchmusterung magnitude, Harvard photometric magnitude, galactic longitude and latitude, date of discovery, name of discoverer, and reference concerning announcement.

PURPOSE OF THIS INVESTIGATION

1. We propose to study the relationship between the spectra of stars of Class R and stars of Class N.

That a close relationship exists is evident from the fact that many Class R stars were announced

[1] A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of Michigan.

[2] In a letter dated February 11, 1915, Professor E. C. Pickering says: "In *Harvard Circular* 9, the word 'fourth' should be substituted for 'seventh' in the passage you quote."

TABLE I. STARS WITH SPECTRA OF CLASS R NOT INCLUDED IN TABLE XI, PAGE 220, VOLUME 56, ANNALS OF THE HARVARD COLLEGE OBSERVATORY.

STAR.	R. A. 1900.	DECL. 1900.	MAGNITUDE.		GALACTIC		DATE.	DISCOVERER	REFERENCE.
			D. M.	H. P.	LONG.	LAT.			
$+23°123$	$0^h48^m.9$	$+23°32'$	8.3	8.8	91°	$-32°$	1914	Cannon	H. C. 184
$+14$ 1598	7 7 .2	$+14$ 46	Var.	9.2	171	$+12$	1914	Cannon	H. C. 184
-14 4371	16 7 .6	-14 57	9.5	...	326	$+24$	1910	Fleming	H. C. 158
$+42$ 2811	17 10 .4	$+42$ 15	7.3	7.74	37	$+34$	1914	Cannon	H. C. 184
Uncertain	17 33 .5	-57 52	...	...	302	-15	1901	Fleming	H. C. 60 H. A. 56, 223
-3 5751	23 57 .0	-3 23	9.2	9.9	63	-64	1914	Cannon	H. C. 184

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as having a spectrum of Type IV (N), or resembling Type IV with peculiarities (N_p). Parkhurst expresses this condition by stating, "No sharp line can be drawn between Classes N and R."[3]

2. We propose also to study the relationship between stars of Classes R and N on the one hand and stars of the Harvard sequence (B, A, F, G, K, M) on the other.

Concerning stars of the fourth type Miss Clerke says:[4] "They have indeed traceable relationships; but the genealogy obscurely indicated by them needs authentication." Vogel in 1874 proposed that Secchi's Types III (Harvard K, M) and IV (Harvard R?, N) be considered in one class and designated as IIIa and IIIb. Hale, Ellerman, and Parkhurst in a valuable treatise on "The Spectra of Stars of Secchi's Fourth Type"[5] reached the conclusion: "Stars of the third and fourth types should therefore be classed together, as coordinate branches leading back to stars like the sun."

Pickering, in the letter previously referred to, says: "We have not been able to establish the sequence between classes M and R. Owing to the intensity of the blue light and the presence of lines H and K in stars of Class R, it seems more probable that Class R should fall between M and N."

In particular the thesis is proposed: Stars belonging to Class R of the Draper Classification of stellar spectra form the connecting links between stars of the solar type (G) and stars of Class N; and stars of the two classes, R and N, form a branch of the sequence arranged in order of stellar evolution coordinate with the branch consisting of stars of Classes K and M.

Furthermore, it is hoped that the quantitative and qualitative data obtained during this investigation will contribute in some measure to the broader problem of stellar evolution in general.

GENERAL CHARACTERISTICS

The table of Class R stars contains 66 members, none of which is brighter than the seventh visual magnitude. Among the number whose magnitude is given there are:

VISUAL MAGNITUDE.	TOTAL NUMBER.	NORTH OF — 20°.
7.0 to 7.9	7	5
8.0 to 8.9	13	8
9.0 to 9.9	30	13
10.0 to 10.9	6	0

Five are marked as having a variable magnitude. Only twenty are in the northern hemisphere. Not as great preference seems to be given for the galactic region as in the case of Classes N, B, and O.

The following table, showing the distribution of stars of various spectral classes with reference to the Galaxy, is based upon the work at Harvard[6] as summarized by Russell[7] with the data for Class R stars added by the writer. The count for Class R was made for the region + 30° to — 30° galactic latitude. The Harvard count for the other classes varied from these limits in different regions on account of the irregularity of the Galaxy.

Class	O	B	A	F	G	K	M	R	N
Percentage in Galactic Region	100	82	66	57	58	56	54	63	87

The color of Class R stars has been referred to as "probably yellow like the stars of the second type."[8] This characteristic is suggested as one of the features distinguishing stars of Class R from those of Class N:[9] "Stars having spectra of the fourth type are commonly regarded as red stars;" which indicates that the Harvard observers considered that, in general, stars of Class R are not red. Miss Clerke referred to two of the number as "white stars"[10] — 10° 5057 and — 10° 513.

[3] The Spectra and Colors of Red Stars of Harvard Classes N and R. *Astrophysical Journal*, Vol. 35, p. 125.

[4] *Problems in Astrophysics*, p. 215.

[5] *Publications of the Yerkes Observatory*, Vol. 2 p. 385, 1903.

[6] *Harvard Annals*, Vol. 64, p. 134.

[7] *Publications American Astronomical Society*, Sixteenth Meeting, p. 26.

[8] *Harvard Annals*, Vol. 56, p. 219.

[9] *Harvard Circular*, 145, p. 3.

[10] *Problems in Astrophysics*, p. 221.

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Parkhurst has determined the color index of five stars of the list as follows:

STAR.	COLOR INDEX.
−10°5057	1.09
+85 332	1.56
+53 66	1.66
+20 5071	1.82
+ 6 3898	2.37

Compared with his results for the color index of nine stars classed as N and Na (1.94 to 3.26) it appears that in general stars of Class R are not as red as those of Class N. He reaches the conclusion, however, that the expression "Fourth-Type Stars not Red," seems inappropriate;[11] and he apparently includes Class R stars in the expression, Fourth-Type Stars.

The difference in color between stars of these classes (N and R) as seen by visual observers may be tested by referring to any catalogue of red stars. Birmingham-Espin's catalogue of 766 red stars contains only two out of twenty stars of Class R from the seventh to the ninth magnitude, DM. +61° 667 and −3° 1685; while it catalogues seventy per cent. of the stars of Class N within the same range of magnitude. The difference in color between the two classes was clearly discernible during our observations, forming a gradual transition from Class R to Class N with increasing redness.

The following table is inserted to show the change in color index with spectral type. It gives the mean of the values obtained by different observers and tabulated by Russell.[12] The value for Class R has been added by the writer.

The general characteristics of the spectra of stars of Class R are given in the various circulars previously referred to and include:

1. Rays of much shorter wave-length than ordinary fourth-type stars.

2. The blue end is no longer cut off but extends to as short a wave-length as in spectra of Class K.

3. The lines H and K are well shown.

4. One or more dark bands, resembling the spectrum of the fifth type reversed on a continuous spectrum.

5. Two well-marked absorption bands, one of which has a center near the calcium line λ 4227, the other extending from λ 4640 to λ 4750.

COLOR INDICES OF STARS BY SPECTRAL TYPES.

CLASS OF SPECTRUM.	AVERAGE COLOR INDEX.
B 0	−0.32
B 5	−0.19
A 0	0.00
A 5	0.00
F 0	0.38
F 5	0.58
G 0	0.80
G 5	1.02
K 0	1.27
K 5	1.64
M	1.65
R	1.7
N	2.5

Further indication of the relation of these spectra to others is found in the following remarks:

1. "It appears probable that stars can be found forming a continuous sequence from Class N to Class R, like that connecting Class B and Class M." *Harvard Annals,* Vol. 56, p. 220.

2. The designation N5R is applied to three stars of intermediate type listed in "Stars of Class N." *Harvard Annals,* Vol 56, p. 219, Remark 5.

The order of the sequence *from* Class N *to* Class R suggested above and applied in the notation N5R appears to be inconsistent with the order of the sequence connecting Class B and Class M with which it is compared. The well established sequence, B A F G K M, is usually assumed to represent the order of stellar evolution. Now, if Classes N and R are closely related and are considered to be late types according to Hale, Pickering and other authorities, the larger color index or increasing redness of the stars of Class N, due to the gradual weakening in intensity of rays of shorter wave-length, demands a reversal of the order suggested and the adoption of the order from Class R to Class N and the notation R5N. This is apparent in any classifica-

[11] *Astrophysical Journal,* Vol. 35, p. 132, 1912.

[12] *Publications American Astronomical Society,* Sixteenth Meeting p. 27.

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tion that places Class N near the end of the list. In addition to the system mentioned above, Lockyer's classification based upon his meteoritic hypothesis also makes Class N a late type. Russell,[13] however, prefers to assign this class a position near the beginning of the process of stellar development.

OBSERVATIONS

The original program of observations included all known stars of Class R down to the ninth visual magnitude observable in the latitude of Ann Arbor, also a sufficient number of stars of Classes N and O for the purpose of comparison and contrast. The suggestion that the spectrum of stars belonging to Class R resembles the spectrum of the fifth type reversed on a continuous spectrum prompted the inclusion of a number of stars of Class O. It was soon discovered that the resemblance was one of a very general nature, better illustrated by photographs taken with an objective prism spectrograph than with a slit spectrograph; the details brought out by the latter detract from the resemblance, and apparently give no additional clue to a physical relationship between these classes of stars.

All these spectrograms were made with the one-prism spectrograph attached to the 37½-inch Reflector of the Detroit Observatory. Since this spectrograph has been fully described by Professor Curtiss,[14] we will mention only the following features.

Focal length of collimator	686 mm.
Aperture of collimator	36.6 mm.
Refracting angle of prism	64°.5
Focal length of camera	420 mm.
Linear dispersion at λ 4500	47.7 angstroms per mm.

The focal plane of the camera is sensibly flat for the range of spectrum of this investigation λ 4000 to λ5000. The minimum deviation setting is for Hγ rays.

Seed plates were used exclusively, 23's and 27's for stars of Class O, 27's and Graflex for stars of Classes N and R. The coarseness of the silver grains of the Graflex plates and a tendency to form bubbles in the film during the process of development renders them much less satisfactory than the 27's excepting for the faintest stars, where accuracy of measurement was sacrificed for speed during exposure.

A determination of the slit width for the setting 36.0 based upon the measurement of thirty comparison lines on plates 3063A, 3039B, and 3055A, and the ratio of the focal length of the collimator to the focal length of the camera gave the approximate value 0.075 mm.[15] A titanium spark has been used for comparison on all the plates.

The accompanying table of observations, Table II, only partially reveals the arduousness of the observational work. The visual faintness of the stars of Class R and the relatively small amount of blue light combined to render the photography of their spectra difficult under the most favorable conditions, and the success of the work is a testimony to the efficiency of the apparatus used. Frequently an exposure begun under apparently favorable conditions and continued for two or three hours was interrupted before completion by clouds or haze and the exposure time was entirely lost or the spectrogram was so faint that its weight was small for radial velocity and wave-length determination. The work begun at Mount Wilson on stars of Classes N and R with the 60-inch Reflector has been temporarily discontinued, because it was found that the exposures require so much time. Mr. Van Maanen writes, January 23, 1915: "We are waiting for the time when there will be more time available or when we will be able to use a more powerful instrument."

In Table II, C.S.T. refers to Central Standard Time. The seeing (S) and transparency (T) are estimated on a scale 5. The temperature at the beginning (B) and end of exposures (E) are recorded in degrees Centigrade. Slit width (W) for setting 36.0 is approximately 0.075 mm.; the pitch of the screw is one-half millimeter and the head has 100 divisions. The length (L) of the slit is approximately 0.34 mm. for setting 0.8 and 0.46 mm. for setting 1.5, giving a spectrum approximately 0.20 to 0.27 mm. in width respectively.

[13] *Publications American Astronomical Society*, Sixteenth Meeting.

[14] *Publications of the Astronomical Observatory of the University of Michigan*, Vol. 1, p. 37.

[15] Plaskett has found that the exposure time in average seeing is almost inversely proportional to slit-width until this reaches at least 0.075 mm. *Astrophysical Jour.*, Vol. 28, p. 259, 1908. With this slit-width the fainter lines begin to disappear.

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TABLE II. JOURNAL OF OBSERVATIONS. CLASS R.

PLATE.	DATE.	EXPOSURE. C. S. T.	PLATE	S.	T.	DOME TEMPERATURE.		INSIDE TEMPERATURE.		SLIT.		REMARKS.
						B.	E.	B.	E.	W.	L.	
DM. — 10° 5057. R.A. 19h 17m.6; Decl. — 10° 54′. Mag. : DM. 7.0, H.P. 7.04.												
2882 A	1914 July 1	9:50 to 12:40	27	2	2.5	17.4	16.0	18.60	18.52	35.0	1.5	Stopped by clouds.
2887 A	July 3	10:00 to 13:20	27	2.5	3	18.3	18.0	20.93	20.70	35.0	1.5	
2914 A	July 15	10:14 to 13:44	27	2	2.5	23.0	22.0	24.80	24.60	35.0	1.5	Fog near end of exposure.
2915 A	July 17	9:17 to 12:47	27	1.5	4	21.3	20.0	25.55	25.32	35.5	1.3	
2923 A	July 21	11:32 to 15:02	27	2	3	25.0	21.8	25.75	25.38	36.5	1.0	
2958 A	Aug. 12	10:30 to 13:00	27	2	3	20.1	18.2	20.95	20.84	37.0	1.0	Hazy toward end.
DM. + 20° 5071. R.A. 21h 59m.7; Decl. + 20° 34′. Mag. : DM. 8.7.												
2942 A	1914 Aug. 3	8:50 to 14:20	27	1	3	17.8	14.8	25.25	24.98	36.5	1.0	Trace only.
2966 C	Sept. 4	8:20 to 16:00	27	2	3	16.1	11.6	17.78	17.45	36.0	0.8	
2967 A	Sept. 6	8:15 to 16:15	27	2	3	19.4	13.8	19.80	19.56	36.0	0.8	
2968 A	Sept. 8	7:05 to 16:15	27	2	2.5	12.8	8.0	14.91	14.57	36.0	0.8	Fleecy clouds, 14:00 to 15:00.
DM. + 5° 5223. R.A. 23h 44m; Decl. + 5° 50′. Mag. : DM. 8.7.												
2988 A	1914 Sept. 17	8:45 to 16:15	27	2	2	18.9	15.7	19.69	19.62	36.0	0.8	Clouds, 10:45 to 11:45.
2989 B	Sept. 18	9:20 to 16:20	27	2.5	3	19.7	14.0	20.27	19.95	36.0	0.8	Hazy, 2:00 to 2:30.
DM. + 57° 702. R.A. 3h 3m.8; Decl. + 57° 31′. Mag., DM. 7.9, H.P. 8.06.												
3044 A	1914 Oct. 31	12:05 to 17:25	Graf.	2	2	10.5	8.2	11.12	10.97	36.0	1.0	Light clouds, 1:00 to 3:00.
3079 A	1915 Jan. 2	7.10 to 13:10	Graf.	2	3	−8.6	−12.2	−5.40	−5.54	36.0	0.8	
DM. — 10° 513. R.A. 2h 30m; Decl. — 9° 53′. Mag., DM. 8.0, H.P. 8.26.												
3038 C	1914 Oct. 25	8:50 to 10:30	Graf.	1	1	7.8	....	7.82	.....	36.0	1.0	Incomplete.
3039 B	Oct. 30	8:30 to 13:00	Graf.	2	3	8.1	6.0	10.43	10.33	36.0	1.0	
3055 A	Nov. 11	9:55 to 11:55	27	2	2	2.8	2.2	5.21	5.18	36.0	0.8	Hazy after 11:30.
3063 A	Nov. 23	9:00 to 13:20	27	1.5	3	−3.3	−4.6	−3.23	−3.28	36.0	0.8	Hazy after 1:00.

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TABLE II. JOURNAL OF OBSERVATIONS. CLASS R. Continued.

PLATE.	DATE.	EXPOSURE. C. S. T.	PLATE	S.	T.	DOME TEMPERATURE. B.	DOME TEMPERATURE. E.	INSIDE TEMPERATURE. B.	INSIDE TEMPERATURE. E.	SLIT. W.	SLIT. L.	REMARKS.
DM. + 61° 667. R.A. 3h 57m.2; Decl. + 61° 31′. Mag., DM. 7.5, H.P. 7.92.												
3051 A	1914 Nov. 6	11:15 to 13:00	Graf.	2	2	3.8	4.2	6.44	6.40	36.0	0.8	Cloudy at 12:30.
3054 C	Nov. 10	12:15 to 14:45	Graf.	1.5	2	4.8	3.1	5.02	5.00	36.0	0.8	Stopped by clouds.
3064 C	Nov. 23	13:45 to 15:00	Graf.	1.5	2	−4.6	−4.2	−3.30	−3.29	36.0	1.0	Stopped by haze.
3073 A	Dec. 21	10.05 to 15.05	Graf.	1.5	3	−12.4	−14.4	−7.66	−7.78	36.0	0.8	Interrupted by clouds.
3088 A	1915 Jan. 23	7:53 to 14:03	27	1	3	−10.6	−16.0	−7.54	−7.61	36.0	1.0	Incomplete.
3089 A	Jan. 26	6:50 to 13:20	Graf.	2	3	−7.0	−9.5	−7.44	−7.55	36.0	1.0	
DM. — 3° 1685. R.A. 6h 56m; Decl. − 3° 7′. Mag., DM. 7.7, H.P. 7.06.												
3087 A	1915 Jan. 20	10:05 to 12:30	Graf.	1	3	−7.2	−10.0	−3.07	−2.60	36.0	1.0	Stopped by clouds.
3094 D	Feb. 8	7:05 to 12:35	27	1.5	4	−6.4	−9.5	−3.42	−3.64	36.0	1.0	
3097 A	Feb. 17	7:00 to 11:30	Graf.	2.4	4	−1.8	−3.8	−0.06	−0.08	36.0	1.0	
DM. + 34° 1929. R.A. 8h 53m.6; Decl. + 34° 9′. Mag., DM. 8.9.												
3138 B	1915 Mar. 11	7:40 to 14:00	Graf.	2	4	0.0	....	1.25	1.10	36.0	0.7	
3143 A	Mar. 16	7:15 to 13:15	Graf.	1.5	4	−1.2	−4.5	2.05	1.87	36.0	0.8	
DM. + 14° 2048. R.A. 9h 8m.3; Decl. + 14° 37′. Mag. DM. 8.8. H.P. 8.68.												
3126 A	1915 Mar. 2	7:35 to 10:35	Graf.	1.5	2.5	−3.0	−7.1	−1.04	−1.12	36.0	0.8	Stopped by clouds.
3127 A	Mar. 3	7:55 to 12:55	Graf.	2	2	−4.6	−6.4	−4.78	−4.72	36.0	0.8	Hazy after 12:00.
3134 A	Mar. 8	7:30 to 12:30	Graf.	2	3	−2.4	−4.4	−2.60	−2.48	36.0	0.8	
DM. + 42° 2811. R.A. 17h 10m.4; Decl. + 42° 15′. Mag., DM. 7.3, H.P. 7.74.												
3128 B	1915 Mar. 3	13:05 to 14:15	Graf.	2	2	−7.0	−7.5	−4.75	−4.75	36.0	0.8	Stopped by clouds.
3135 B	Mar 8	12:45 to 15:45	27	1.5	3	−4.6	−5.7	−2.52	−2.55	36.0	0.8	
3144 B	Mar. 16	13:30 to 17:10	27	2	4	−4.8	−5.7	1.85	1.81	36.0	0.8	
3169 A	Apr. 3	11:50 to 16:50	27	2	3	−1.4	−3.0	1.46	1.40	36.0	0.8	Continued until near dawn.

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TABLE II. JOURNAL OF OBSERVATIONS. CLASS N.

PLATE.	DATE.	EXPOSURE. C. S. T.	PLATE	S.	T.	DOME TEMPERATURE. B.	DOME TEMPERATURE. E.	INSIDE TEMPERATURE. B.	INSIDE TEMPERATURE. E.	SLIT. W.	SLIT. L.	REMARKS.
DM. — 5° 4858. V Aquilæ. R.A. 18h 59m.1; Decl. — 5° 50′. Mag. DM. 7.0, H.P. Var.												
2902 D	1914 July 6	10:33 to 14:33	27	2	3	2.07	1.76	24.22	23.92	35.0	1.5	Trace only.
DM. + 76°734. R.A. 19h 25m.1; Decl. + 76° 22′.1. Mag., DM. 6.5, H.P. Var.												
2883 B	1914 July 1	12:59 to 14:49	27	2.5	3	15.8	15.8	18.52	18.50	35.0	1.5	
2921 C	July 20	9:40 to 12:10	27	2	3.5	23.0	22.0	24.11	23.95	35.5	1.5	
19 Piscium. R.A. 23h 41m.3; Decl. + 2° 56′. Mag., DM. 6.2, H.P. Var.												
2965 B	1914 Sept. 2	10:25 to 13:30	27	1	3	15.4	15.5	22.45	22.39	35.0	1.5	Clouds 25 min.
2977 B	Sept. 11	8:55 to 11:25	27	1.5	3	11.2	10.3	13.21	13.20	35.0	1.5	
2978 D	Sept. 11	11:48 to 14:18	27	2	3	10.1	8.5	10.19	13.01	34.0	1.0	
DM. + 34°4500. R.A. 21h 37m.8; Decl. + 35° 3′.1. Mag., DM. 6.2, H.P. Var.												
2916 D	1914 July 17	13:02 to 15:22	27	2	4	20.0	18.5	25.32	25.30	35.5	1.3	
2922 B	July 20	12:28 to 14:48	27	2.5	3	22.0	18.8	23.99	23.85	35.5	1.5	
U Hydræ. R.A. 10h 32m.6; Decl. — 12° 52′. Mag., DM. Var., H.P. Var.												
3084 A	1915 Jan. 14	12:37 to 15:07	Graf.	1.5	3	−1.5	−2.5	−0.61	−0.63	36.0	1.0	
3086 B	Jan. 15	13:05 to 15:15	Graf.	2	1	−3.5	−3.6	−1.60	−1.61	36.0	1.0	
3090 D	Jan. 26	13:45 to 16:30	Graf.	1.5	3	−9.5	−9.7	−7.60	−7.62	36.0	1.0	
3098 B	Feb. 17	11:45 to 14:00	27	2	4	−3.9	−4.3	−0.09	−0.10	36.0	1.0	
3168 C	Apr. 3	8:15 to 10:45	27	2	4	+0.5	−1.0	1.49	1.47	36.0	0.8	
DM. + 46° 1817. R.A. 12h 40m.4; Decl. + 45° 58′. Mag., DM. Var., H.P. Var.												
3085 B	1915 Jan. 14	15:22 to 17:22	Graf.	2	3	−2.4	−3.6	−0.65	−0.65	36.0	1.0	

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TABLE II. JOURNAL OF OBSERVATIONS. CLASS O.

PLATE.	DATE.	EXPOSURE. C. S. T.	PLATE	S.	T.	DOME TEMPERATURE. B.	DOME TEMPERATURE. E.	INSIDE TEMPERATURE. B.	INSIDE TEMPERATURE. E.	SLIT. W.	SLIT. L.	REMARKS.
DM. + 35° 3953. R.A. 20h 2m.2; Decl. + 35° 31′. Mag., DM. 7.0, H.P. 7.01.												
2938 A	1914 July 29	12:00 to 2:15	27	1.5	4	15.5	14.6	19.05	18.76	35.5	1.0	
2950 A	Aug. 5	11:40 to 12:40	27	2	3	20.6	19.0	22.96	22.85	35.5	1.0	
DM. + 43° 3571. R.A. 20h 17m.1; Decl. + 43° 32′. Mag., DM. 7.5, H.P. 6.83.												
2982 D	1914 Sept. 14	12:43 to 14:38	27	1	3	16.8	16.2	17.57	17.53			Cloudy near end.
3012 B	Oct. 1	11:08 to 12:58	27	3	3.5	11.0	10.0	13.37	13.32	34.0	1.8	
DM. + 37° 3821. R.A. 20h 8m.5; Decl. + 38° 3′. Mag., DM. 7.1, H.P. 7.44.												
2981 A	1914 Sept. 13	11:35 to 13:35	27	2	2	14.1	13.8	15.10	15.09	34.0	1.0	Interrupted by clouds.
2990 A	Sept. 19	12:15 to 14:00	27	2.5	3	19.2	19.2	21.06	21.07	34.0	1.5	Spoiled in developing.
3011 A	Oct. 1	9:03 to 10:53	27	3	4	12.8	11.1	13.40	13.37	34.0	1.8	
λ Cephei. R.A. 22h 8m.1; Decl. + 58° 56′. Mag., DM. 5.6, H.P. 5.19.												
2933 A	1914 July 27	12:27 to 12:57	27	2	3	19.6	19.1	25.67	25.58	34.5	2.0	
2934 B	July 27	13:03 to 14:23	23	2	3	19.1	19.0	25.58	25.55	34.5	2.0	
2952 C	Aug. 5	14:08 to 14:23	23	2	3	18.9	18.9	21.84	21.84	34.5	2.0	
2953 B	Aug. 5	14.55 to 15:20	23	2	2	18.9	18.9	21.83	21.83	34.5	2.0	
2979 A	Sept. 11	14:40 to 15:40	23	2	3	8.5	8.0	12.90	12.92	34.0	1.0	
2983 B	Sept. 14	14:57 to 15:27	23	1.5	2.5	16.2	16.1	17.53	17.50	34.0	1.5	
2984 C	Sept. 14	15:37 to 16:07	23	1.5	2.5	16.0	15.8	17.50	17.48	33.0	2.0	

THE SPECTROGRAMS

The spectra of stars of Class R are marked by strong absorption bands, numerous dark lines, and a few bright lines. Only plates of the stars + 42° 2811 and — 10° 5057 gave a spectrum above λ 4188 strong enough for the measurement of lines; sufficient continuous spectrum is visible, however, beyond this limit on plates of other stars to indicate the presence of violet rays. The H and K lines are clearly seen on plates of + 42° 2811. Some plates show a sharp drop in intensity at λ 4216, others fade away gradually. The calcium line λ 4227 is strong. The broad G group, extending in some cases from λ 4295 to λ 4315, is the most prominent feature of this region, showing almost complete absorption on some of the plates. The line λ 4384 is conspicuous. Hγ is present, but is not prominent. On plates of the stars showing the strongest ordinary bands, there is also strong absorption from λ 4395 toward the violet rendering the spectrum very weak as far as Hγ or even as far as G in some cases. The

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most prominent feature of the whole spectrum under consideration, from the violet end to λ 5000, are the strong absorption bands with head at λ 4737 when the bands are weaker and at λ 4752 when they are stronger. These bands are sharply defined toward the red but gradually fade away toward the violet, usually to about λ 4630-40. The continuous spectrum on the red side of these bands is much stronger than it is on the other side, the relative intensity differing greatly in the different stars; in general, the spectrum showing the stronger absorption bands suffers the greater loss in intensity on the violet side. An absorption line varying in intensity in the different stars occupies the position of Hβ, but a companion line and the low dispersion in this region renders its identification difficult. On some plates it is very weak, on others not discernible. Hβ does not clearly appear as a bright line on any plate. The intensity of the spectrum, which is a maximum in the broad bright zone adjoining the head of the strong absorption band, gradually grows less toward the red limit of visibility of the photographic plate; at λ 5000 the spectrum is much fainter for Class R stars as a rule than for Class N stars taken under the same conditions. Prominent lines varying in intensity from star to star are found at λλ 4876, 4886, 4921, 4958, and 4985.

The features here mentioned will be discussed in the section on qualitative results.

MEASUREMENT AND REDUCTION

Measurement. The spectrograms were measured on Measuring Engines No. 1 and No. 3 of this Observatory. The pitch of the screws is one-half millimeter and the least reading 0.0005 mm. Determinations of the periodic error for different sections indicate that no corrections to the micrometer readings are necessary. Low magnifying power gave the best results on account of the coarseness of the silver grains of the photograph plates used; for the Graflex plates power 7 to 8 was used, for 27's power 12 to 15. All the available star lines were measured for wavelength determination. About 25 comparison lines were measured on each plate. The average of three settings on a star line was taken. The mean of two readings on the inner tip of the upper comparison line was averaged with the mean of two readings on the inner tip of the lower. This has a double advantage: in the first place, if the point is fairly symmetrical its bisection gives a better result than the bisection of a broad line; and in the second place, the effect due to the curvature of the slit image on the plate is minimized. In our work the curvature correction was not appreciable. All the plates were measured direct and reversed to eliminate personal equation as far as possible.

Radial Velocity Determination. The method of reduction proposed by Professor Curtiss[16] has been followed using the moon as the standard velocity source. For the standard table about forty lines were selected, that were found to be more or less common to stars of Class R and the solar spectrum, and about 25 comparison lines of average intensity well distributed throughout the region λ 3900 to λ 5100.

The first standard table was made from three moon plates with titanium comparison. During the course of the observations the spectrograph was readjusted, which resulted in a change of dispersion sufficient to necessitate a corresponding change in the standard table. Accordingly, the second standard table was prepared by changing the dispersion of the old table to the new by means of a graphical method, plotting the micrometer readings as abscissae and the differences in readings between the old and the new as ordinates. A smooth curve was then formed which together with the scale correction gave the means of conversion. The micrometer readings of the comparison lines in the second standard table are based upon a larger number of measurements than in the first standard table; also a few comparison lines were rejected and others substituted, and a few additional moon lines were included, which were found to be common to Class R stars.

The method of making the lines homogeneous was applied in the case of star — 10° 5057, for which five plates were available. The maximum correction to the computed velocity for a single plate due to this somewhat laborious process was 0.38 km. for plate 2887 A. The small number of plates available for each star and the degree of precision required did not warrant the general adoption of this refinement.

[16] *Astrophysical Jour.*, Vol. 20, p. 149, 1904.

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Wave-length Determination. Wave-lengths were determined by means of the Hartmann interpolation formula,

$$\lambda = \lambda_0 + \frac{c}{R_0 - R}.$$

In the determination of the constants of the formula the titanium lines λ 4078.632, λ 4338.081, and λ 4981.916, expressed in Rowland's scale, were used as standard lines in the first standard table. These lines are among the ones selected by Mr. Mellor as standards for "A Study of the Titanium Spark as a Comparison Spectrum in the Single-Prism Spectrograph." The line λ 4163.829 of his list was substituted for λ 4078.632 in the second standard table.

The constants for the first standard table are

$$R_0 = 185.108,$$
$$\lambda_0 = 2,204.293,$$
$$c = 224,289.5.$$

For the second standard table the constants are

$$R_0 = 186.184,$$
$$\lambda_0 = 2,197.682,$$
$$c = 227,282.66.$$

The residuals, observed wave-length minus computed wave-length, which form the ordinates of the correction curves, are based upon the list of titanium lines given by Mr. Mellor, in these *Publications,* Vol. 1, p. 140. The correction curve to accompany the use of the first set of constants is not well determined; but it is more symmetrical than the second, due to the use of standard lines separated by intervals more nearly equal.

Before application of the formulæ

$$\lambda = 2,204.293 + \frac{224,289.5}{185.108 - R}, \qquad (1)$$

and

$$\lambda = 2,197.682 + \frac{227,282.66}{186.184 - R}, \qquad (2)$$

the micrometer reading, R was corrected for plate velocity. This correction was obtained by multiplying the plate velocity by $\frac{dR}{dv}$, and is applied with the sign changed, since a positive velocity indicates a displacement toward the red or larger wave-length and the micrometer readings increase in the same direction. After R was corrected for all the plates of a single star the mean value for each line was found, due consideration being given to the quality of the line on the individual plates. This value of R was substituted in the Hartmann formula, the application of which was greatly facilitated by the use of the Millionaire computing machine. The resulting value of the wave-length was corrected by means of the ordinate of the correction curve corresponding to the micrometer reading of the line, which gave the final value of the wave-length. After the wave-lengths of stars — 10° 5057 and + 57° 702 had been determined by this method they were used as standards and the difference in R for the other stars was changed into difference in λ by the factor $\frac{d\lambda}{dR}$ tabulated in the standard table. The application of this difference to the computed wave-length in the standard star gave the value of λ directly, saving much time in computation. These values of the wave-lengths are tabulated for each star in the *Table of Mean Wave-Lengths.*

Degree of Precision. The visual faintness of the stars of Class R and their greater photographic faintness required the use of fast plates, the coarse silver grains of which necessitated a low magnifying power and interfered with precise measurement. The wideness of the slit, usually 0.075 mm., combined with the relatively low dispersion, tended to produce broad lines and blends instead of sharp well-defined slit images. The change in intensity of the lines passing from the moon plates to those of Class R and Class N stars introduce displacements of unknown magnitude. While the shift of ordinary lines with spectral type may not be large between the solar spectrum and spectra of Class R stars, there appears to be a marked change in the position of the center of many measured lines, probably due to the presence of new components or to the unequal change in the relative intensity of the components passing from the solar type to types VI and IV. A few cases that will be pointed out later seem to indicate a systematic shift from star to star passing along the sequence of stars arranged in the order of the intensity of the absorption band. The presence of bright lines also tends to

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shift the center of mass of adjacent absorption lines. Error due to this cause was avoided in radial velocity determination by omitting as far as practicable the use of disturbed lines.

The precision of radial velocity determinations is indicated by the tabulated probable errors accompanying the plate velocities, which are based upon the agreement of the velocities given by the lines of the plate. The average for a plate velocity is ± 2.06 km. for the ten Class R stars. The small number of lines on some plates available for comparison with the lines of the standard table accounts for the large average probable error in such cases. The average number of lines used for each plate was 17, giving an average probable error for a single line of 7.18 km., or in wave-lengths

At λ 4000, ± 0.11 Å,
At λ 4500, ± 0.12 Å,
At λ 5000, ± 0.14 Å.

On this basis the probable error of the mean wave-length of a line measured on ten plates is about ± 0.04 Å, and for a line measured on five plates the probable error is about ± 0.06 Å.

After the lines were made homogeneous in the case of star — 10° 5057, for which five plates were available, the average probable error of the wave-length of a single line was found to be ± 0.050 Å.

MEAN WAVE-LENGTHS

In the table of mean wave-lengths the stars at the head of the columns are arranged in the order of the intensity of the absorption band with head at λ 4737. Under each star are three columns giving respectively the quality, intensity, and computed wave-length of the line. G, F, and P stand for good, fair, and poor, respectively. Occasionally a line is designated as wide (W), very wide (VW), sharp (S), diffuse (Dif.), or nebulous (N or Neb.). Max. stands for the position of maximum intensity in a broad line or band, Str. for the strongest of a group of lines, and Bl. for blend. Br. indicates an emission line. The intensity is estimated on a scale of 10; special difficulties, however, render these estimates approximations only. The last column but one of the table gives the mean wave-lengths of lines common to two or more stars. In general a single star line occupies a single horizontal line of the table, but owing to uncertainties of identification this may not always be the case. The lines of the table are numbered consecutively for convenience of reference.

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TABLE III. MEAN WAVE-LENGTHS. CLASS R.

NO.	+ 42° 2811.			— 10° 5057.			+ 5° 5223.			+ 20° 5071.			+ 34° 1929.			— 10° 513.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
1	V	5	4188.68	F	5	4187.77	P	3	4187.75	..	..	.....	..	..	.....	..	..	.....
2	..	..	.....	?	2	4191.44	..	..	.....	F	2	4191.96	..	..	.....	..	..	.....
3	P	1	4195.07	W	2	4195.71	P	2	4195.68	..	..	.....	..	..	.....	F	2	4196.16
4	?	2	4196.57	?	3	4196.72	..	..	.....	..	..	.....	..	..	.....	..	..	.....
5	..	..	.....	?	3	4201.83	..	..	.....	..	..	.....	..	..	.....	..	..	.....
6	..	..	.....	..	..	.....	..	..	.....	F	3	4203.00	..	..	.....	..	..	.....
7	W	5	4207.96	W	3	4205.27	F	4	4208.69	?	1	4206.46	..	..	.....	P	2	4207.29
8*	G	10	4215.67	G	10	4215.46	..	..	.....	..	..	.....	..	..	.....	F	7	4215.64
9	Head		4216.59	Head		4216.23	..	..	.....	..	..	.....	..	..	.....	..	..	.....
10	?	3	4218.90	..	..	.....	..	..	.....	P	3	4218.93	..	..	.....	P	3	4218.40
11	..	..	.....	?	3	4220.29	..	..	.....	..	..	.....	P	3	4220.63	..	..	.....
12	P	2	4223.19	?	2	4223.16	..	..	.....	P	2	4223.00	..	..	.....	F	3	4223.28
13	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
14	..	..	.....	..	..	.....	..	..	.....	F	2	4225.89	..	..	.....	..	..	.....
15*	G	10	4227.12	G	7	4227.32	F	8	4227.11	F	7	4227.22	F	7	4227.12	F	7	4227.02
16	Edge		4227.94	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
17	F	2	4230.19	F	1	4230.50	..	..	.....	..	..	.....	..	..	.....	F	2	4229.85
18*	F	4	4233.51	G	6	4233.30	..	..	.....	W	4	4232.92	F	3	4233.91	F	3	4233.34
19*	F	5	4236.51	F	3	4236.31	..	..	.....	..	..	.....	W	4	4236.68	P	3	4236.63
20	..	..	.....	F	4	4239.14	..	..	.....	G	5	4238.58	..	..	.....	..	..	.....
21	Edge		4240.56	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
22	..	..	.....	?	2	4242.60	..	..	.....	..	..	.....	..	..	.....	..	..	.....
23*	F	5	4243.32	..	..	.....	F	3	4243.11	..	..	.....	W	8	4242.99	Bl.	5	4243.47
24	..	..	.....	..	..	.....	..	..	.....	W	5	4244.68	..	..	.....	..	..	.....
25	..	..	.....	?	5	4247.24	..	..	.....	..	..	.....	..	..	.....	..	..	.....
26			4248.02	..	..	.....			4248.01	..	..	.....			4247.87			4247.86
27*	F	3	4250.44	F	3	4250.46	F	3	4250.76	F	5	4250.49	W	4	4250.86			4250.86
28	..	..	.....	F	2	4252.65	..	..	.....	..	..	.....	..	..	.....	..	..	.....
29	..	..	.....	..	..	.....	W	3	4254.62	F	3	4254.43	..	..	.....	P	3	4254.56
30	F	4	4255.72	..	..	.....	..	..	.....	..	..	.....	F	2	4255.29	..	..	.....
31	..	..	.....	?	2	4256.48	..	..	.....	..	..	.....	..	..	.....	..	..	.....
32	..	..	.....	..	..	.....	F	3	4257.61	P	2	4257.13	..	..	.....	..	..	.....
33			4258.04	F	2	4258.47	F	2	4258.70	..	..	.....	..	..	.....	F	3	4259.09
34*	Center		4260.63	F	4	4260.74	..	..	.....	F	3	4260.20	P	5	4260.70	..	..	.....
35	..	..	.....	..	..	.....	F	4	4261.42	Br.	2	4261.46	..	..	.....	..	..	.....
36			4262.29	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
37	..	..	.....	..	..	.....	..	..	.....	F	2	4263.03	..	..	.....	..	..	.....
38	F	3	4264.83	..	..	.....	..	..	.....	..	..	.....	F	2	4264.38	F	3	4264.64
39	..	..	.....	..	..	.....	..	..	.....	W	5	4266.83	..	..	.....	..	..	.....
40	F	3	4268.28	W	4	4269.12	F	4	4268.29	..	..	.....			4267.88	F	4	4267.67
41	..	..	.....	..	..	.....	..	..	.....	P	2	4270.44	..	..	.....	..	..	.....
42*	F	5	4271.98	G	4	4271.96	F	3	4271.67	..	..	.....	F	4	4272.02	F	4	4271.79
43	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	F	2	4274.65
44	F	3	4275.21	F	4	4275.26	F	4	4275.01	G	4	4275.86	F	4	4275.48	..	..	.....
45	F	1	4277.09	F	2	4278.07	..	..	.....	..	..	.....	..	..	.....	..	..	.....

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TABLE III. MEAN WAVE-LENGTHS. CLASS R.

NO.	+61° 667. CHARACTER, INTENSITY, AND WAVE-LENGTH.	−3° 1685. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+57° 702. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+14° 2048. CHARACTER, INTENSITY, AND WAVE-LENGTH.	MEAN AND IDENTIFICATION. ANGSTROMS.	ELEMENT.
1	.. ..	.. ..	.. ..	.. ..	4187.8	Fe.
2	.. ..	.. ..	.. ..	.. ..	4191.7	Fe.
3	.. ..	.. ..	.. ..	.. ..	4195.8	Fe.
4	.. ..	.. ..	.. ..	.. ..	4196.7	Cy.
5	.. ..	.. ..	.. ..	.. ..	4201.8	Fe.
6	.. ..	.. ..	F 4 4203.01	.. ..	4203.0	
7	.. ..	.. ..	.. ..	.. ..	.....	
8*	.. ..	F 5 4215.43	F 8 4215.69	F 7 4215.81	4215.6	Cy. Fe.?
9	.. ..	.. ..	Head 4216.16	Head 4216.43	4216.4	
10	.. ..	.. ..	.. ..	.. ..	4218.8	Zr.
11	.. ..	P 3 4220.06	.. ..	Dr. ? 2 4220.38	4220.3	Fe.
12	.. ..	.. ..	.. ..	P 2 4223.01	4223.2	
13	.. ..	Bl. 5 4224.21	.. ..	.. ..	.....	
14	.. ..	.. ..	.. ..	.. ..	.....	
15*	.. ..	N 5 4227.35	N 7 4227.01	W 10 4227.29	4227.2	Ca. Fe.
16	.. ..	.. ..	.. ..	Edge 4228.11	4228.0	
17	.. ..	F 2 4229.99	.. ..	F 1 4230.45	4230.2	Fe.
18	.. ..	F 3 4233.71	F 3 4233.36	3 4233.89	4233.5	Fe.
19	.. ..	P 3 4236.62	F 4 4237.08	F 4 4237.08	4236.7	
20	.. ..	.. ..	.. ..	.. ..	.....	
21	.. ..	.. ..	.. ..	.. ..	.....	
22	.. ..	.. ..	.. ..	.. ..	.....	
23*	P 3 4243.32	G 4 4243.27	F 4 4243.34	G 5 4243.56	4243.3	Fe.
24	.. ..	.. ..	.. ..	.. ..	.....	
25	.. ..	.. ..	.. ..	.. ..	.....	
26	.. ..	.. ..	4247.48	4248.01	.....	
27*	.. ..	W 3 4250.66	4250.62	W 5 4250.66	4250.6	Fe, blend.
28	.. ..	.. ..	.. ..	Br. ? 1 4252.78	.....	
29	P 3 4254.62	W 8 4254.95	P 5 4254.90	.. ..	4254.7	Cr.
30	.. ..	N 4 4255.70	.. ..	G 10 4255.46	4255.5	Fe. Cr.
31	.. ..	4256.56	.. ..	.. ..	.....	
32	.. ..	.. ..	.. ..	Br. ? 2 4257.82	.....	
33	.. ..	.. ..	.. ..	4258.61	4258.8	Fe.
34	P 4 4260.37	W 6 4260.72	P 3 4260.57	.. ..	4260.6	
35	.. ..	4261.34	.. ..	Str. 4261.07	.....	
36	.. ..	.. ..	.. ..	4262.44	.....	Fe.
37	.. ..	.. ..	.. ..	.. ..	.....	
38	.. ..	.. ..	.. ..	F 2 4265.13	4264.7	Fe.
39	.. ..	.. ..	.. ..	.. ..	.....	
40	F 3 4267.90	F 3 4268.28	.. ..	4268.66	4268.0 ?	Fe. C.
41	.. ..	.. ..	.. ..	.. ..	.....	
42*	F 3 4271.49	F 3 4271.85	F 4 4271.85	F 3 4272.24	4271.9	Fe.
43	.. ..	.. ..	.. ..	.. ..	.....	
44	P 3 4275.14	F 4 4275.16	.. ..	F 4 4275.33	4275.3	Cr.
45	.. ..	.. ..	.. ..	F 1 4278.28	4278.1	

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 5057. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 5° 5223. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 20° 5071. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 34° 1929. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 513. CHARACTER, INTENSITY, AND WAVE-LENGTH.
46*	G 6 4280.56	F 3 4280.74	G 4 4280.55	.. ..	Bl. 5 4280.58	G 5 4280.42
47	.. ..	.. ..	.. ..	G 6 4281.38	.. ..	.. ..
48	F 1 4283.06	F 2 4282.91	.. ..	.. ..	.. ..	.. ..
49	F 4 4286.09	F 2 4286.18	F 3 4286.02	F 5 4286.62	N 3 4286.00	F 4 4285.90
50*	F 3 4289.88	F 6 4290.11	F 3 4289.83	4290.01	F 4 4290.11	F 3 4289.67
51	F 3 4291.74	.. ..	P 2 4291.97	.. ..	.. ..	.. ..
52	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
53	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
54	.. ..	.. ..	.. ..	.. ..	.. ..	F 2 4294.44
55	S 2 4294.94	F 3 4295.31	F 3 4295.22	.. ..	.. ..	.. ..
56*	F 5 4300.08	G 8 4299.98	F 5 4300.02	.. ..	.. ..	.. ..
57	P 1 4303.04	F 2 4302.93	.. ..	.. ..	.. ..	W 4 4303.52
58	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
59	.. ..	F 3 4305.73	.. ..	.. ..	.. ..	.. ..
60	.. ..	.. ..	.. ..	.. ..	F 5 4306.62	.. ..
61	P 2 4308.41	W 5 4308.72	W 4 4308.50	.. ..	.. ..	.. ..
62	.. ..	.. ..	.. ..	4310.98	.. ..	.. ..
63*	F 4 4314.47	G 6 4314.69	4314.62	.. ..	F 6 4314.77	F 2 4314.63
64	.. ..	.. ..	.. ..	.. ..	.. ..	F 2 4317.57
65	.. ..	.. ..	.. ..	.. ..	4318.55	.. ..
66	F 1 4319.24	.. ..	.. ..	W 6 4319.39	.. ..	.. ..
67	.. ..	.. ..	F 2 4320.16	.. ..	.. .. 4320.68	.. ..
68	F 1 4321.31	F 4 4321.20	.. ..	.. ..	.. ..	4321.93
69	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
70	4324.16	.. ..	.. ..	.. ..	.. ..	.. ..
71*	G Max. 4325.62	G 7 4325.59	G 10 4324.62	F 3 4324.79	W 6 4325.42	W 8 4324.68
72	4327.29	.. ..	.. ..	.. ..	.. ..	4326.65
73	.. ..	.. ..	.. ..	F 3 4329.05	.. ..	.. ..
74	F 2 4330.92	F 4 4330.77	F 3 4330.63	.. ..	.. ..	F 3 4330.01
75	F 2 4334.42	F 3 4334.12	F 4 4333.93	F 5 4333.33	P 2 4334.21	F 2 4333.44
76*	F 4 4337.91	4337.72	G 6 4337.88	.. ..	F 4 4337.89	F 3 4337.51
77	.. ..	F 2 4339.99	.. ..	F 2 4339.02	.. ..	.. ..
78*	F 6 4340.42	.. ..	F 3 4340.90	.. ..	F 3 4340.54	4340.34
79	.. ..	.. ..	.. ..	F 3 4342.91	.. ..	.. ..
80	F 3 4344.26	F 4 4344.40	F 3 4343.97	.. ..	F 2 4343.88	P 3 4344.04
81*	F 4 4347.69	F 3 4347.60	F 3 4347.58	G 5 4347.43	F 3 4347.61	G 6 4347.79
82	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
83	.. ..	.. ..	.. ..	F 3 4351.22	.. ..	.. ..
84*	G 7 4352.11	W 5 4351.67	G 6 4352.01	.. ..	F 7 4351.94	F 5 4352.32
85	.. ..	.. ..	.. ..	F 2 4354.71	.. ..	.. ..
86	F 3 4355.87	F 3 4355.17	F 2 4355.56	.. ..	F 4 4355.61	.. ..
87	.. ..	.. ..	.. ..	F 2 4358.98	.. ..	.. ..
88	F 4 4359.78	P 5 4359.41	F 3 4359.76	.. ..	F 3 4359.70	.. ..
89	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
90	.. ..	.. ..	.. ..	F 3 4362.93	.. ..	.. ..

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667. CHARACTER, INTENSITY, AND WAVE-LENGTH.	−3° 1685. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+57° 702. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+14° 2048. CHARACTER, INTENSITY, AND WAVE-LENGTH.	MEAN AND IDENTIFICATION. ANGSTROMS.	ELEMENT.
46*	F 3 4280.63	P 3 4280.60	P 4 4280.92	F 4 4280.81	4280.6	Cr.
47	.. ..	.. ..	.. ..	.. ..	.....	
48	.. ..	.. ..	.. ..	F 3 4283.08	4283.0	Ti. Ca.
49	.. ..	F 3 4286.31	.. ..	F 4 4285.65	4286.2	Ti.
50*	.. ..	P 3 4290.02	F 3 4289.90	Bl. 4 4289.98	4289.9	Cr. Ca.
51	.. ..	F 3 4291.85	.. ..	F 3 4291.85	4291.8	Fe. Cr.
52	.. ..	.. ..	F 1 4292.43	.. ..	.....	
53	.. ..	.. ..	.. ..	4293.95	.....	
54	.. ..	.. ..	.. ..	.. ..	.....	
55	.. ..	.. ..	F 1 4295.09	.. ..	.....	
56*	.. ..	F 2 4300.02	.. ..	F 4 4300.13	4300.0	G, Fe, Ti.
57	.. ..	.. ..	.. ..	.. ..	.....	
58	.. ..	.. ..	F 3 4304.91	.. ..	.....	
59	.. ..	F 3 4305.86	.. ..	.. ..	.....	
60	.. ..	.. ..	.. ..	.. ..	.....	
61	.. ..	.. ..	.. ..	.. ..	.....	
62	.. ..	.. ..	.. ..	.. ..	.....	
63*	G 3 4314.82	F 3 4314.17	Edge 4315.50	Edge 4315.09	4314.7	Ti. ?
64	.. ..	.. ..	.. ..	.. ..	.....	
65	.. ..	.. ..	.. ..	.. ..	.....	
66	.. ..	.. ..	.. ..	.. ..	.....	
67	.. ..	.. ..	.. ..	.. ..	.....	
68	.. ..	.. ..	.. ..	F 2 4321.67	4321.5	
69	4323.55	.. ..	.. ..	.. ..	.....	
70	.. ..	.. ..	P 3 4324.18	F 3 4324.12	.....	Ti. Cr.
71*	.. ..	.. ..	F 3 4326.00	.. ..	.....	Fe.
72	4326.38	.. ..	4327.37	.. ..	.....	
73	.. ..	.. ..	F 3 4329.59	.. ..	.....	
74	F 2 4329.89	F 3 4330.37	.. ..	F 4 4330.13	4330.1	V.
75	P 3 4333.89	W 5 4334.73	.. ..	F 3 4334.65	4334.1	
76*	F 3 4337.81	4337.63	P 3 4337.73	.. ..	4337.8	Cr.
77	.. ..	.. ..	.. ..	.. ..	.....	
78	P 2 4340.92	4340.50	P 2 4340.16	P 2 4341.05	4340.5	Hγ.
79	.. ..	.. ..	F 1 4342.85	.. ..	4342.9	
80	.. ..	.. ..	.. ..	F 3 4343.92	4344.1	Cr. Blend.
81*	W 4 4347.58	.. ..	F 3 4347.77	.. ..	4347.6	
82	.. ..	.. ..	.. ..	F 2 4348.26	.....	
83	.. ..	.. ..	.. ..	.. ..	.....	
84	F 6 4352.08	.. ..	F 4 4352.28	.. ..	4352.0	Cr. Mg.
85	.. ..	.. ..	.. ..	.. ..	.....	
86	F 4 4355.26	.. ..	.. ..	.. ..	4355.5	Ca.
87	.. ..	.. ..	.. ..	.. ..	.....	
88	.. ..	.. ..	F 3 4359.62	.. ..	4359.7	Cr.
89	P 2 4360.49	.. ..	.. ..	.. ..	.....	
90	.. ..	.. ..	.. ..	.. ..	.....	

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 5057. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 5° 5223. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 20° 5071. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 34° 1929. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 513. CHARACTER, INTENSITY, AND WAVE-LENGTH
91	W 4 4363.88	W Max. 4363.67	F 4 4363.83	.. ..	F 3 4363.49	F 4 4363.31
92	4366.61	.. ..	P 2 4366.86	P 3 4366.98	.. ..	.. ..
93*	P 3 4367.41	P 3 4367.70	.. ..	.. ..	F 3 4367.90	F 3 4367.17
94	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
95	P 1 4370.39	.. ..	.. ..	.. ..	.. ..	.. ..
96	.. ..	F 4 4371.37	.. ..	.. ..	.. ..	F 4 4371.41
97	.. ..	.. ..	P 2 4373.25	.. ..	.. ..	.. ..
98	W 5 4374.76	F 6 4374.86	.. ..	.. ..	.. ..	.. ..
99	.. ..	.. ..	F 3 4375.93	.. ..	F 4 4375.54	F 4 4376.13
100	P 2 4380.33	P 2 4380.09	.. ..	.. ..	.. ..	.. ..
101	4382.95	.. ..	.. ..	.. ..	.. ..	.. ..
102	G 10 4384.06	.. ..	VW 8 4383.84	.. ..	G 9 4384.00	F 4 4383.96
103	4385.69	4385.28	.. ..	.. ..	.. ..	.. ..
104	.. ..	.. ..	.. ..	W 3 4386.20	.. ..	.. ..
105	.. ..	F 1 4388.02	F 3 4388.74	.. ..	F 2 4388.43	.. ..
106	W 4 4389.97	.. ..	.. ..	.. ..	.. ..	.. ..
107	.. ..	F 4 4390.99	F 3 4390.82	.. ..	.. ..	F 3 4390.26
108	.. ..	.. ..	.. ..	.. ..	4392.25	.. ..
109*	F 4 4395.24	G 5 4395.22	G 3 4395.12	F 3 4395.03	F 3 4395.46	G 6 4395.04
110	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
111	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
112*	F 5 4400.82	G 8 4400.52	F 4 4400.47	F 3 4400.69	F 4 4400.77	F 5 4400.37
113	Br. 2 4403.19	.. ..	.. ..	.. ..	.. ..	.. ..
114*	G 5 4404.89	F 4 4404.67	G 7 4404.87	F 2 4404.79	G 6 4404.84	G 4 4404.78
115	F 4 4408.96	G 7 4408.83	G 6 4408.63	.. ..	F 3 4408.98	F 3 4408.63
116	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
117	.. ..	P 2 4411.89	.. ..	.. ..	.. ..	.. ..
118	Br. 2 4413.04	.. ..	.. ..	.. ..	.. ..	.. ..
119	.. ..	.. ..	.. ..	Br. 2 4414.24	.. ..	.. ..
120*	F 6 4415.41	F 4 4415.09	G 6 4415.27	.. ..	G 6 4415.56	F 5 4414.97
121	.. ..	.. ..	.. ..	F 3 4416.76	.. ..	.. ..
122	Edge 4418.22	F 5 4417.94	.. ..	.. ..	.. ..	.. ..
123	.. ..	.. ..	.. ..	F 1 4420.17	.. ..	.. ..
124	F 4 4422.92	F 3 4422.88	F 4 4422.52	.. ..	W 4 4422.47	F 3 4422.39
125	.. ..	.. ..	.. ..	W 4 4425.46	N 3 4426.29	.. ..
126*	F 3 4430.63	F 4 4430.25	G 4 4430.50	F 5 4430.78	P 3 4430.59	F 4 4430.76
127*	W 8 4435.23	F 6 4435.08	W 4 4435.27	W 1	W 4 4435.25	G 4 4435.28
128	.. ..	.. ..	.. ..	P 4 4437.15	F 1 4437.49	.. ..
129	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
130	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
131	.. ..	.. ..	.. ..	.. ..	.. ..	F 4 4442.00
132*	W 10 4442.99	G 10 4443.37	G 8 4442.66	F 2 4442.80	W 8 4442.97	.. ..
133	.. ..	.. ..	.. ..	F 3 4445.16	4444.77	.. ..
134	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
135	F 1 4447.46	F 2 4447.37	F 3 4446.92	.. ..	.. ..	.. ..

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667. CHARACTER, INTENSITY, AND WAVE-LENGTH.	−3° 1685. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+57° 702. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+14° 2048. CHARACTER, INTENSITY, AND WAVE-LENGTH.	MEAN AND IDENTIFICATION. ANGSTROMS.	ELEMENT.
91	F 3 4363.53	.. ..	.. ..	.. ..	4363.6	
92	.. ..	.. ..	.. ..	.. ..	4366.9	
93*	G 1 4367.12	.. ..	.. ..	.. ..	4367.6	Fe.
94	W 2 4369.05	.. ..	.. ..	.. ..	.....	
95	.. ..	.. ..	.. ..	.. ..	.....	
96	.. ..	.. ..	.. ..	.. ..	4371.4	Cy. Cr.
97	.. ..	.. ..	.. ..	.. ..	.....	
98	.. ..	.. ..	.. ..	.. ..	4374.8	
99	W 4 4375.36	.. ..	F 3 4375.38	.. ..	4375.7	Cr. Fe.
100	.. ..	.. ..	.. ..	.. ..	.....	
101	.. ..	.. ..	.. ..	.. ..	.....	
102	W 6 4383.98	F 3 4383.85	P 3 4384.08	F 3 4384.39	4384.1	Fe.
103	.. ..	.. ..	.. ..	.. ..	.....	
104	.. ..	.. ..	.. ..	.. ..	.....	
105	.. ..	.. ..	.. ..	.. ..	.....	
106	.. ..	.. ..	.. ..	P 4 4389.59	.....	
107	F 3 4391.02	.. ..	.. ..	F 3 4391.15	4390.8	V. ?
108	.. ..	F 1 4392.40	.. ..	.. ..	.....	
109*	G 9 4395.01	F 7 4395.07	W 6 4394.67	G 9 4394.73	4395.1	V.
110	Edge 4395.63	Edge 4396.01	Edge 4395.78	F 4 4395.61	4395.7	Ti.
111	.. ..	.. ..	.. ..	Edge 4395.97	.....	
112*	F 6 4400.24	F 5 4400.71	F 4 4400.71	G 6 4400.71	4400.6	
113	.. ..	Br. 1 4402.57	.. ..	Br. 2 4403.21	4403.0	
114*	G 9 4404.95	G Max. 4404.78	G 8 4405.06	[F 7 4405.47]	4404.8	Fe.
115	F 6 4408.66	.. ..	F 6 4408.70	F 6 4408.85	4408.8	V. ?
116	.. ..	Edge 4409.58	Edge 4409.00	Edge 4409.47	4409.4	
117	.. ..	.. ..	.. ..	.. ..	.....	
118	Rev. 1 4412.15	Rev. 1 4412.86	.. ..	F 2 4412.72	.....	Cr. O in sun.
119	.. ..	.. ..	.. ..	.. ..	.....	
120*	G 8 4415.28	G 5 4415.52	F 6 4415.43	G 5 4415.82	4415.4	Fe.
121	.. ..	.. ..	.. ..	.. ..	.....	
122	.. ..	G 6 4418.04	.. ..	.. ..	4418.1	Fe. Ti.
123	.. ..	.. ..	.. ..	.. ..	.....	
124	F 4 4422.40	W 4 4422.60	.. ..	F 3 4422.71	4422.6	Fe.
125	.. ..	F 1 4426.76	.. ..	P 2 4426.10	4426.2	V.
126*	F 3 4430.35	F 4 4430.70	F 3 4430.72	[F 4 4431.13]	4430.6	Fe.
127*	G 10 4435.68	W Max. 4435.43	G 8 4435.47	G 8 4435.82	4435.4	Ca.
128	.. ..	.. ..	.. ..	.. ..	4437.4	
129	Rev. 1 4438.30	.. ..	.. ..	.. ..	.....	
130	.. ..	Br. 2 4439.65	Br. 1 4439.12	Br. 2 4439.21	4439.3	
131	.. ..	.. ..	.. ..	.. ..	.....	
132*	G 10 4442.77	F 8 4442.80	F 5 4442.79	G 8 4443.26	4443.0	
133	.. ..	.. ..	.. ..	.. ..	.....	
134	Br. 2 4445.46	Br. 1 4445.46	Br. 2 4445.79	Br. 2 4446.10	4445.7	
135	F 3 4447.04	.. ..	F 2 4447.42	F 3 4447.55	4447.3	

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 5057. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 5° 5223. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 20° 5071. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 34° 1929. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 513. CHARACTER, INTENSITY, AND WAVE-LENGTH.
136	P 2 4450.79	G 6 4450.23	F 3 4450.43	F 3 4450.75	F 3 4449.84	.. ..
137	W 4 4455.17	P 2 4454.69	F 3 4455.81	.. ..	.. ..	F 4 4454.49
138	.. ..	P 2 4457.69	.. ..	P 2 4457.55	.. ..	.. ..
139	.. ..	.. ..	.. ..	.. ..	F 3 4459.55	.. ..
140	.. ..	.. ..	.. ..	Br. 2 4461.37	.. ..	.. ..
141*	W 5 4461.87	F 4 4461.41	F 4 4461.76	.. ..	P 4 4462.12	F 5 4461.74
142	.. ..	.. ..	.. ..	F 3 4463.96	.. ..	.. ..
143	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
144	.. ..	P 3 4464.96	.. ..	W 5 4465.14	.. ..	.. ..
145	P 2 4466.20	.. ..	.. ..	P 2 4466.06	.. ..	.. ..
146	F 1 4468.85	F 5 4468.65	.. ..	.. ..	.. ..	.. ..
147	F 2 4469.80	.. ..	.. ..	P 3 4469.46	P 4 4469.57	.. ..
148	F 1 4470.76	.. ..	P 2 4470.19	.. ..	.. ..	F 6 4470.75
149	.. ..	P 2 4471.78	.. ..	.. ..	.. ..	.. ..
150	F 2 4473.58	.. ..	.. ..	.. ..	P 2 4474.82	N 2 4475.86
151*	F 6 4481.39	F 8 4481.11	G 6 4481.08	P 2 4481.73	F 4 4481.30	F 4 4481.48
152	.. ..	F 3 4484.35	.. ..	.. ..	.. ..	.. ..
153*	F 4 4489.54	F 6 4489.67	P 4 4489.54	F 4 4489.64	F 3 4489.52	.. ..
154	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
155	.. ..	.. ..	.. ..	F 2 4491.39	.. ..	.. ..
156	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
157	.. ..	P 3 4494.00	.. ..	.. ..	.. ..	.. ..
158	W 4 4495.04	.. ..	G 4 4494.67	.. ..	W 3 4495.14	F 3 4494.49
159	.. ..	P 3 4497.08	.. ..	F 3 4496.70	.. ..	.. ..
160	.. ..	.. ..	.. ..	.. ..	F 4 4498.70	F 3 4498.72
161*	F 4 4501.41	G 9 4501.21	F 4 4501.27	F 2 4501.57	G 4 4501.34	F 4 4501.09
162	.. ..	Br. ? 4506.92	P 3 4506.45	.. ..	.. ..	.. ..
163	F 3 4507.77	F 3 4508.10	.. ..	.. ..	.. ..	.. ..
164	.. ..	.. ..	.. ..	4512.35	.. ..	.. ..
165	N 6 4513.97	F 8 4514.19	W 8 4513.79	.. ..	F 7 4513.74	G 6 4514.47
166	.. ..	.. ..	.. ..	.. ..	P 3 4516.04	.. ..
167	.. ..	.. ..	.. ..	F 2 4518.38	.. ..	F 3 4518.28
168	4519.87	F 5 4519.86	F 3 4519.20	.. ..	.. ..	.. ..
169	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
170	.. ..	.. ..	.. ..	.. ..	.. ..	F 3 4522.29
171	F 2 4523.15	F 4 4522.86	4523.36	.. ..	.. ..	.. ..
172	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
173	4525.50	.. ..	.. ..	.. ..	.. ..	.. ..
174*	F 3 4526.67	P 4 4527.89	.. ..	F 4 4526.20	F 3 4526.71	F 4 4526.66
175	Str. 4530.72	P 5 4530.83	W 8 4530.52	.. ..	.. ..	F 5 4530.82
176	.. ..	P 5 4534.36	.. ..	.. ..	.. ..	.. ..
177	4536.43	.. ..	4536.03	.. ..	4536.02	F 3 4536.75
178	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
179	.. ..	Bl. 3 4540.65	W 4 4540.74	.. ..	.. ..	.. ..
180	.. ..	.. ..	.. ..	.. ..	.. ..	F 4 4541.23

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667. CHARACTER, INTENSITY, AND WAVE-LENGTH.	−3° 1685. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+57° 702. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+14° 2048. CHARACTER, INTENSITY, AND WAVE-LENGTH.	MEAN AND IDENTIFICATION. ANGSTROMS.	ELEMENT.
136	F 3 4449.83	F 3 4450.41	F 3 4450.21	F 3 4450.61	4450.3	Ti.
137	.. ..	N 4 4455.14	W 3 4455.43	F 4 4455.54	4455.2	Ca.
138	.. ..	.. ..	.. ..	.. ..	.....	
139	.. ..	.. ..	.. ..	.. ..	.....	
140	.. ..	.. ..	.. ..	.. ..	.....	
141*	F 4 4461.79	G 7 4461.74	G 7 4461.74	F 6 4462.10	4461.8	Fe.
142	.. ..	.. ..	.. ..	.. ..	.....	
143	Br. 2 4463.40	Br. 2 4464.17	Br. 2 4464.33	Br. 1 4464.01	4464.0	Ti.
144	4464.47	.. ..	.. ..	.. ..	.....	
145	F 3 4466.18	.. ..	F 3 4466.09	F 2 4466.66	4466.2	
146	.. ..	.. ..	.. ..	.. ..	4468.7	Ti.
147	.. ..	.. ..	F 3 4469.28	N 5 4469.94	4469.6	Cr. Fe.
148	.. ..	.. ..	.. ..	.. ..	4470.7	Ni.
149	4472.02	F 3 4471.21	.. ..	4471.50	4471.6	
150	F 4 4475.27	F 3 4475.88	F 4 4476.02	F 3 4476.29	.....	Fe.
151*	P 5 4481.16	N 3 4481.46	W 3 4481.39	N 4 4482.12	4481.4	Ti. Mag.
152	F 3 4484.74	.. ..	.. ..	.. ..	4484.5	Fe.
153*	F 3 4489.82	.. ..	.. ..	.. ..	4489.7	Cr. Fe.
154	.. ..	P 3 4490.24	.. ..	.. ..	.....	
155	.. ..	.. ..	.. ..	F 3 4491.41	4491.4	
156	4493.16	.. ..	.. ..	.. ..	.....	
157	.. ..	4494.03	.. ..	.. ..	.....	
158	.. ..	.. ..	W 5 4495.09	W 5 4495.34	4495.0	Fe.
159	4496.81	4496.20	.. ..	.. ..	.....	N. V.
160	.. ..	.. ..	.. ..	.. ..	.....	
161*	F 3 4501.51	F 4 4501.32	F 4 4501.39	F 5 4501.88	4501.4	Cr.
162	F 4 4506.67	F 3 4507.28	F 3 4506.83	F 3 4506.96	4506.8	
163	.. ..	.. ..	.. ..	.. ..	4507.9	
164	4511.08	4511.03	P 3 4511.24	4512.37	.....	
165	.. ..	W 8 4514.07	W 6 4514.42	W 9 4514.09	4514.1	Cy.
166	4515.08	4516.05	4515.20	4516.66	.....	
167	.. ..	.. ..	.. ..	F 2 4518.33	.....	Ti.
168	.. ..	.. ..	.. ..	.. ..	.....	
169	.. ..	.. ..	.. ..	F 2 4520.31	.....	
170	.. ..	.. ..	.. ..	.. ..	.....	
171	F 3 4523.20	F 3 4523.11	F 3 4522.76	F 2 4523.27	4523.1	Ti.
172	.. ..	Rev. 1 4524.78	Br. 2 4524.55	.. ..	4524.7	V.
173	.. ..	.. ..	S 1 4525.95	.. ..	.....	Cr.
174*	F 6 4526.67	F 3 4526.92	4527.75	F 4 4527.52	4527.0	Ti. Ca.
175	.. ..	.. ..	F 3 4531.71	F 5 4531.42	4531.0	Cr. Fe.
176	F 4 4534.49	.. ..	.. ..	.. ..	.....	Ti.
177	4536.05	4536.44	4537.08	4537.06	4536.6	Ti.
178	Br. 1 4537.73	Br. 2 4538.20	.. ..	Br. 3 4538.72	4538.2	
179	.. ..	.. ..	.. ..	.. ..	.....	
180	.. ..	.. ..	.. ..	.. ..	.....	

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811.			— 10° 5057.			+ 5° 5223.			+ 20° 5071.			+ 34° 1929.			— 10° 513.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
181	W	4	4542.20	F	3	4542.18	..	..	.....	..	..	.....	F	3	4541.55	..	..	.....
182	..	..	.....	..	..	.....	..	..	.....	F	4	4544.24	..	..	.....	..	..	.....
183	F	3	4545.20	F	3	4545.44	P	3	4545.54	..	..	.....	F	2	4545.25	F	3	4545.40
184	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
185	..	..	.....	..	..	.....			4546.30	..	..	.....	..	..	.....			4548.41
186	..	..	.....	..	..	.....	..	..	.....	W	5	4548.95	..	..	.....	..	..	.....
187*	F	3	4549.86	F	4	4549.61	F	3	4550.09	..	..	.....	F	3	4549.82	P	4	4549.74
188	..	..	.....	..	..	.....			4551.96	..	..	.....	Edge		4553.69	F	5	4552.54
189	F	3	4553.47	F	3	4553.48	..	..	.....	..	..	.....	..	..	.....	..	..	.....
190	..	..	.....	..	..	.....	..	..	.....	F	2	4555.27	..	..	.....	..	..	.....
191	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
192	F	2	4560.77	P	3	4559.96	F	4	4560.38	..	..	.....	P	3	4560.52	F	3	4560.69
193	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
194	..	..	.....	..	..	.....	..	..	.....	Br.	1	4563.82	..	..	.....	..	..	.....
195*	Var.		4564.93	F	6	4564.65	F	3	4564.68	..	..	.....	W	5	4564.78	..	..	.....
196	..	..	.....	..	..	.....	..	..	.....	F	3	4566.56	..	..	.....	..	..	.....
197	..	..	.....	..	..	.....	..	..	.....	..	..	.....	Br.	1	4567.69	..	..	.....
198	..	..	.....	..	..	.....	..	..	.....	..	..	.....	F	3	4568.95	..	..	.....
199	S	2	4571.85	F	4	4571.81	F	3	4572.26	F	3	4572.23	..	..	.....	P	3	4571.85
200	F	3	4576.73	F	3	4576.85	F	3	4577.02	..	..	.....	W	6	4576.83	F	6	4576.93
201	P	2	4580.20	P	1	4580.82	F	2	4579.86	..	..	.....	F	3	4580.50	F	3	4580.72
202	..	..	.....	F	4	4584.19	F	3	4585.51	..	..	.....	F	5	4585.48	F	3	4586.73
203	..	..	.....	P	3	4587.23	..	..	.....	..	..	.....	..	..	.....	F	2	4588.37
204	..	..	.....	..	..	.....	Br.	2	4589.33	..	..	.....	Br.	1	4590.51	..	..	.....
205	P	3	4592.20	F	2	4592.64	..	..	.....	..	..	.....	..	..	.....	..	..	.....
206	..	..	.....	P	2	4594.80	P	2	4594.33	..	..	.....	..	..	.....	F	2	4593.80
207	F	3	4596.42	..	..	.....	..	..	.....	..	..	.....			4596.21	..	..	.....
208	..	..	.....	P	2	4597.03	..	..	.....	F	2	4597.58	..	..	.....	F	2	4597.19
209	..	..	.....	..	..	.....	..	..	.....	Br. ?	2	4599.37	..	..	.....	..	..	.....
210	..	..	.....	F	3	4601.07	..	..	.....	G	4	4601.21	..	..	.....	F	4	4601.53
211	..	..	.....	P	2	4603.34	P	3	4603.21	..	..	.....	..	..	.....	..	..	.....
212	F	4	4605.51	F	5	4605.78	F	4	4605.52	..	..	.....	..	..	.....	F	3	4605.81
213	..	..	.....	..	..	.....	..	..	.....	..	..	.....			4606.18	..	..	.....
214	..	..	.....	F	4	4612.67	..	..	.....	..	..	.....	..	..	.....	..	..	.....
215	Dif.	3	4613.35	..	..	.....	P	3	4613.25	..	..	.....	..	..	.....	F	3	4613.24
216	..	..	.....	P	3	4614.97	..	..	.....	..	..	.....	..	..	.....	..	..	.....
217	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
218	F	2	4619.74	F	4	4619.36	F	4	4619.48	..	..	.....	..	..	.....	..	..	.....
219	..	..	.....	F	3	4623.43	..	..	.....	Dif.	2	4624.29	..	..	.....	..	..	.....
220	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
221	W	3	4626.03	F	2	4625.99	P	3	4626.20	..	..	.....	..	..	.....	..	..	.....
222	..	..	.....			4628.87	F	3	4628.95	..	..	.....	..	..	.....	..	..	.....
223	..	..	.....	F	4	4629.26	..	..	.....	..	..	.....	..	..	.....	..	..	.....
224	W	4	4630.10	F	7	4631.26	..	..	.....	..	..	.....	..	..	.....	..	..	.....
225	..	..	.....	F	6	4633.46	..	..	.....	..	..	.....	..	..	.....	..	..	.....

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667. CHARACTER, INTENSITY, AND WAVE-LENGTH.	−3° 1685. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+57° 702. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+14° 2048. CHARACTER, INTENSITY, AND WAVE-LENGTH.	MEAN AND IDENTIFICATION. ANGSTROMS.	ELEMENT.
181	.. ..	N 4 4541.98	F 3 4542.08	F 3 4542.15	.....	
182	.. ..	.. ..	.. ..	.. ..	.....	
183	F 3 4545.66	F 3 4545.74	F 5 4545.30	F 3 4546.07	4545.5	V.
184	Br. 2 4547.80	Br. 2 4547.72	.. ..	.. ..	.....	
185	.. ..	.. ..	.. ..	.. ..	.....	
186	.. ..	.. ..	4548.89	.. ..	.....	
187*	F 3 4549.74	F 3 4549.50	.. ..	F 4 4550.13	4549.8	V. Ti.
188	4552.03	.. ..	.. ..	.. ..	.....	
189	F 10 4552.96	Str. 4552.91	W 7 7452.25	Str. 4552.98	4553.4	Cy.
190	4554.58	4555.00	4553.76	4554.01	4555.	
191	.. ..	4559.51	.. ..	.. ..	.....	
192	G 6 4560.42	W 8 4561.60	W 6 4561.43	F 6 4561.11	4560.7	V.
193	.. ..	4563.91	4563.40	.. ..	.....	
194	.. ..	.. ..	.. ..	.. ..	.....	Ti.
195*	Dif. 4 4564.51	.. ..	.. ..	W 3 4565.18	4564.8	Fe.
196	.. ..	F 2 4566.12	.. ..	.. ..	.....	
197	Br. 3 4568.00	.. ..	Br. 1 4567.35	.. ..	4567.7	
198	.. ..	.. ..	.. ..	.. ..	.....	
199	F 3 4571.87	F 6 4572.00	W 5 4571.95	W 4 4572.00	4571.9	Ti.
200	F 5 4576.40	F 5 4576.45	W 6 4576.43	F 4 4576.75	4576.7	
201	.. ..	.. ..	.. ..	.. ..	4580.4	Cr.
202	F 6 4586.73	F 3 4587.46	W 4 4586.36	W 4 4587.59	.....	
203	4587.56	.. ..	4588.52	.. ..	.....	
204	.. ..	.. ..	Br. 2 4591.01	.. ..	4589.9	
205	.. ..	.. ..	.. ..	.. ..	.....	Cr.
206	F 2 4593.62	F 3 4593.30	F 3 4592.97	P 2 4593.63	4593.8	V.
207	.. ..	.. ..	.. ..	.. ..	.....	
208	.. ..	.. ..	.. ..	F 2 4597.54	4597.3	
209	.. ..	.. ..	.. ..	.. ..	.....	
210	4601.51	P 4 4601.12	.. ..	F 3 4601.41	4601.4	Cr. ?
211	.. ..	.. ..	.. ..	W 8 4603.03	4603.2	Fe.
212	.. ..	.. ..	W 6 4605.28	.. ..	4605.6	Cy.
213	4607.25	.. ..	.. ..	4606.28	.....	
214	.. ..	.. ..	.. ..	.. ..	.....	
215	F 2 4612.98	.. ..	.. ..	.. ..	4613.2	Fe.
216	.. ..	W 5 4614.57	W 6 4614.55	F 3 4614.68	4614.7	
217	Br. 2 4617.50	.. ..	.. ..	.. ..	.....	Ti.
218	.. ..	P 3 4620.45	.. ..	.. ..	4619.7	V.
219	.. ..	.. ..	W 4 4623.19	.. ..	.....	
220	.. ..	.. ..	.. ..	F 3 4625.39	.....	
221	.. ..	.. ..	.. ..	.. ..	4626.1	Cr.
222	.. ..	.. ..	.. ..	.. ..	4628.9	Cr.
223	.. ..	.. ..	.. ..	.. ..	.....	
224	.. ..	.. ..	.. ..	.. ..	.....	
225	Dif. 7 4633.56	.. ..	W 5 4632.99	F 4 4633.16	4633.3	Fe. Cr.

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 5057. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 5° 5223. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 20° 5071. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+ 34° 1929. CHARACTER, INTENSITY, AND WAVE-LENGTH.	— 10° 513. CHARACTER, INTENSITY, AND WAVE-LENGTH.
226	.. ..	.. ..	.. ..	.. ..	.. ..	F 3 4637.14
227	.. ..	F 3 4640.58	F 2 4640.63	W 3 4641.63	.. ..	F 2 4640.18
228	.. ..	Br. 1 4642.75	.. ..	.. ..	.. ..	.. ..
229	P 5 4648.01	F 6 4647.13	W 5 4646.50	.. ..	.. ..	.. ..
230	W 5 4656.55	F 5 4656.52	F 4 4656.54	.. ..	.. ..	.. ..
231	.. ..	.. ..	.. ..	F 2 4658.38	.. ..	.. ..
232	Br. 4660.01	.. ..	.. ..	.. ..	.. ..	.. ..
233	.. ..	.. ..	F 2 4663.23	W 4 4664.73	.. ..	.. ..
234	W 4 4668.10	.. ..	F 4 4668.33	.. ..	.. ..	.. ..
235	Dif. 3 4680.85	.. ..	.. ..	.. ..	.. ..	.. ..
236	.. ..	.. ..	P 3 4684.22	.. ..	.. ..	.. ..
237	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
238	W 3 4696.90	F 4 4696.73	F 4 4698.14	.. ..	P 6 4697.34	.. ..
239	.. ..	.. ..	.. ..	P 4 4699.05	.. ..	.. ..
240	.. ..	.. ..	.. ..	G 6 4709.04	.. ..	.. ..
241	.. ..	.. ..	.. ..	.. ..	.. ..	4713.45
242*	F 7 4715.04	F 10 4714.07	G 9 4714.83	.. ..	F 6 4715.01	F 6 4714.48
243	.. ..	.. ..	.. ..	F 5 4716.87	.. ..	.. ..
244	F 3 4722.68	.. ..	F 4 4722.61	.. ..	F 3 4722.37	F 5 4721.61
245	.. ..	.. ..	.. ..	F 2 4724.29	.. ..	.. ..
246	F 4 4728.83	F 3 4728.77	F 3 4728.79	.. ..	F 3 4728.13	F 3 4727.65
247	.. ..	.. ..	.. ..	G 5 4730.93	.. ..	.. ..
248*	G 4736.36	G 4736.02	G 4636.37	.. ..	G 4736.42	G 10 4736.11
249	.. ..	.. ..	.. ..	G 4738.81	.. ..	.. ..
250	Head 4737.55	Head 4737.47	.. ..	.. ..	Head 4737.27	Head 4737.07
251	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
252	W 7 4743.80	.. ..	F 6 4743.74	.. ..	G 7 4743.65	G 8 4743.66
253	Edge 4745.59	F 2 4745.86	Edge 4744.40	.. ..	Edge 4744.66	Edge 4744.80
254	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
255	F 1 4748.78	.. ..	.. ..	.. ..	.. ..	.. ..
256	.. ..	.. ..	.. ..	.. ..	F 3 4750.62	F 4 4750.99
257	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
258	F 2 4755.50	F 2 4756.30	.. ..	.. ..	F 3 4754.94	.. ..
259	.. ..	.. ..	.. ..	.. ..	F 2 4757.17	.. ..
260	.. ..	.. ..	.. ..	.. ..	.. ..	4761.79
261	F 1 4762.25	S 2 4762.50	.. ..	.. ..	.. ..	.. ..
262	.. ..	F 4 4763.78	.. ..	.. ..	4762.97	.. ..
263	W 5 4765.13	F 3 4765.46	W 4 4765.96	.. ..	W 3 4765.95	W 3 4764.06
264	.. ..	.. ..	.. ..	.. ..	4767.66	.. ..
265	.. ..	G 3 4771.37	.. ..	.. ..	.. ..	.. ..
266	Dif. 2 4772.42	.. ..	P 2 4772.18	.. ..	.. ..	4772.33
267	.. ..	F 2 4775.88	F 3 4775.24	.. ..	.. ..	.. ..
268	.. ..	.. ..	4780.25	.. ..	.. ..	.. ..
269	4783.57	P 3 4783.45	.. ..	F 2 4783.19	F 3 4784.24	.. ..
270	W 5 4787.63	F 3 4786.30	.. ..	.. ..	.. ..	.. ..

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667. CHARACTER, INTENSITY, AND WAVE-LENGTH.	−3° 1685. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+57° 702. CHARACTER, INTENSITY, AND WAVE-LENGTH.	+14° 2048. CHARACTER, INTENSITY, AND WAVE-LENGTH.	MEAN AND IDENTIFICATION. ANGSTROMS.	ELEMENT.
226	F 3 4637.16	P 3 4636.68	.. ..	.. ..	4637.0	Ca.
227	F 3 4640.64	.. ..	.. ..	.. ..	4640.7	
228	Br. 2 4643.08	.. ..	.. ..	.. ..	4642.9	
229	.. ..	.. ..	.. ..	F 3 4645.96	.....	
230	.. ..	F 3 4656.88	.. ..	.. ..	4656.6	Ti. Cr.
231	.. ..	.. ..	.. ..	.. ..	.....	
232	.. ..	.. ..	.. ..	.. ..	.....	
233	.. ..	.. ..	.. ..	.. ..	.....	
234	.. ..	.. ..	.. ..	.. ..	4668.2	Fe.
235	.. ..	.. ..	.. ..	.. ..	.....	
236	.. ..	.. ..	.. ..	.. ..	.....	
237	F 4 4694.24	.. ..	.. ..	.. ..	.....	
238	.. ..	.. ..	.. ..	.. ..	4697.2	C.
239	.. ..	.. ..	.. ..	.. ..	.....	
240	.. ..	.. ..	.. ..	.. ..	.....	
141	.. ..	.. ..	.. ..	.. ..	.....	
242*	F 6 4715.18	.. ..	.. ..	.. ..	4714.8	C. Ni.
243	.. ..	.. ..	.. ..	.. ..	.....	
244	F 6 4722.43	.. ..	.. ..	.. ..	4722.3	Zn.
245	.. ..	.. ..	.. ..	.. ..	.....	
246	.. ..	P 3 4729.17	.. ..	.. ..	4728.6	Fe.
247	.. ..	.. ..	.. ..	.. ..	.....	
248*	F 6 4736.62	F 6 4736.59	F 6 4736.56	W 6 4736.90	4736.4	C.
249	.. ..	.. ..	.. ..	.. ..	.....	
250	Head 4737.70	Head 4737.67	.. ..	Head 4737.92	4737.6	Cr.
251	Br. 3 4739.71	.. ..	.. ..	Br. 2 4739.61	4739.7	
252	G 10 4743.60	G 10 4743.17	G 10 4743.20	G 10 4743.60	4743.6	
253	Edge 4745.40	Edge 4745.11	Edge 4745.14	4745.14	4745.1	
254	Br. 2 4747.63	Br. 3 4747.46	.. ..	Br. 3 4747.78	4747.6	
255	.. ..	.. ..	.. ..	.. ..	.....	
256	F 6 4751.51	G 10 4751.02	G 10 4751.05	G 9 4751.51	4751.1	
257	Edge 4753.31	Edge 4752.45	Edge 4752.91	Edge 4752.51	4752.8	
258	.. ..	Br. 3 4755.32	.. ..	.. ..	.....	
259	.. ..	F 3 4758.38	F 2 4757.83	F 2 4758.58	4758.0	
260	.. ..	.. ..	.. ..	.. ..	.....	
261	.. ..	.. ..	.. ..	.. ..	4762.4	
262	.. ..	.. ..	.. ..	.. ..	.....	
263	.. ..	.. ..	.. ..	.. ..	4765.3	
264	Neb. 4766.51	F 3 4765.98	W 3 4764.79	P 2 4765.92	.....	
265	.. ..	.. ..	.. ..	.. ..	.....	
266	F 3 4772.30	F 2 4772.30	F 2 4772.39	P 3 4772.91	4772.4	
267	Br. 2 4777.85	.. ..	Br. 2 4775.26	.. ..	.....	
268	4780.15	.. ..	.. ..	.. ..	.. ...	
269	.. ..	.. ..	F 3 4784.77	.. ..	.....	
270	Neb. 4 4787.23	W 5 4787.02	.. ..	F 4786.65	.....	

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811.			— 10° 5057.			+ 5° 5223.			+ 20° 5071.			+ 34° 1929.			— 10° 513.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
271			4793.17	P	3	4792.40			4792.04	..	..	.....	..	..	.....	F	3	4793.59
272	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
273	F	3	4799.52	F	3	4799.09	F	4	4799.40	..	..	.....	F	3	4799.42	..	..	.....
274	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	W	3	4801.28
275	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
276	..	..	.....	P	3	4804.80	..	..	.....	..	..	.....	..	..	.....	..	..	.....
277	Dif.	3	4807.86	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
278			4809.90	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
279	..	..	.....	P	2	4811.78	..	..	.....	..	..	.....	..	..	.....	..	..	.....
280	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
281	..	..	.....	P	3	4817.03	..	..	.....	..	..	.....	..	..	.....	..	..	.....
282	..	..	.....	..	..	.....	..	..	.....	P	2	4818.90	..	..	.....	..	..	.....
283	..	..	.....	..	..	.....	F	3	4823.74	..	..	.....	..	..	.....	F	2	4822.73
284	W	3	4824.05	P	3	4824.72	..	..	.....	..	..	.....	..	..	.....	..	..	.....
285	..	..	.....	P	3	4826.66	..	..	.....	..	..	.....	..	..	.....	..	..	.....
286	..	..	.....	P	2	4828.89	..	..	.....	..	..	.....	..	..	.....	..	..	.....
287	P	3	4833.00	..	..	.....	F	3	4833.08	..	..	.....	..	..	.....	..	..	.....
288	..	..	.....	F	4	4836.83	F	3	4838.60	..	..	.....	F	3	4838.18	F	3	4839.31
289	Dif.	3	4841.41	..	..	.....	..	..	.....	..	..	.....	F	3	4841.94	P	4	4841.77
290	..	..	.....	P	2	4843.75	..	..	.....	..	..	.....	..	..	.....	F	1	4844.14
291	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
292	F	2	4848.64	F	3	4848.76	F	3	4848.86	..	..	.....	..	..	.....	..	..	.....
293	F	4	4855.85	F	4	4855.05	..	..	.....	G	5	4856.32	P	2	4855.54	F	2	4855.86
294*	G	5	4861.39	..	..	.....	F	4	4860.60	F	1	4861.34	F	2	4861.26	F	5	4861.42
295	..	..	.....	P	2	4865.81	..	..	.....	F	2	4866.64	Br.	1	4866.67	..	..	.....
296	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	F	2	4868.40
297			4870.29	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
298*	..	..	.....	F	4	4871.41	F	4	4871.88	F	2	4871.67	Dif.	3	4872.31	F	2	4872.08
299	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
300	..	..	.....	P	3	4875.54	..	..	.....	..	..	.....	..	..	.....	..	..	.....
301			4877.14	..	..	.....	..	..	.....	..	..	.....	..	..	.....	F	3	4877.95
302	..	..	.....	F	3	4881.29	..	..	.....	..	..	.....	..	..	.....	..	..	.....
303			4884.60	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
304*	W	3	4886.07	F	4	4886.23	F	3	4886.10	W	4	4886.12	F	5	4885.95	F	4	4886.49
305	..	..	.....	P	2	4889.28	..	..	.....	..	..	.....	..	..	.....	..	..	.....
306			4892.80	..	..	.....	F	3	4892.05	..	..	.....	Dif.	4	4892.60	F	3	4892.05
307	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
308	..	..	.....	..	..	.....	..	..	.....	F	3	4896.82	..	..	.....	..	..	.....
309	..	..	.....	F	3	4900.79	..	..	.....	..	..	.....	G	4	4900.14	..	..	.....
310	..	..	.....	..	..	.....	F	2	4903.43	..	..	.....	..	..	.....	..	..	.....
311	..	..	.....	..	..	.....	..	..	.....	F	3	4905.54	..	..	.....	..	..	.....
312	F	3	4911.07	F	4	4910.69	F	3	4911.12	..	..	.....	..	..	.....	..	..	.....
313	..	..	.....	..	..	.....	..	..	.....	P	4	4915.81	..	..	.....	..	..	.....
314			4917.85	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
315	Str.		4920.98	G	7	4922.32	G	4	4920.58	..	..	.....	F	4	4921.70	F	4	4920.67

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667.			−3° 1685.			+57° 702.			+14° 2048.			MEAN AND IDENTIFICATION.	
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			ANGSTROMS.	ELEMENT.
271			4792.20	F	2	4793.53	..	..	.....	..	..	.....	.....	
272	Br.	2	4795.21	..	..	.....	..	..	.....	..	..	.....	.....	
273	W	4	4799.34	F	4	4798.99	F	3	4798.84	W	3	4799.64	4799.3	
274	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
275	Br.	2	4803.00	..	..	.....	..	..	.....	..	..	.....	.....	
276	..	..	.....	W	2	4805.81	..	..	.....	..	..	.....	.....	
277	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
278	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
279	F	3	4811.45	Br.	1	4811.23	..	..	.....	..	..	.....	4811.5	
280	..	..	.....	F	2	4816.46	..	..	.....	F	3	4816.50	4816.5	
281	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
282	Br.	2	4818.15	Br.	2	4819.32	Br.	2	4818.72	Br.	3	4820.22	.....	
283	F	4	4823.28	F	4	4822.57	F	2	4822.70	F	4	4822.52	4822.9	
284	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
285	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
286	..	..	.....	F	2	4827.16	F	2	4828.32	F	3	4828.58	4828.2	V.
287	F	4	4832.57	F	2	4832.19	F	2	4833.24	F	2	4833.00	4832.8	Fe.
288	..	..	.....	G	5	4839.40	..	..	.....	..	..	.....	.....	
289	F	3	4842.89	..	..	.....	..	..	.....	..	..	.....	.....	
290	..	..	.....	G	6	4843.10	F	3	4843.69	F	3	4843.38	4843.6	
291	..	..	.....	W	4	4845.58	..	..	.....	..	..	.....	.....	
292	F	2	4848.78	..	..	.....	..	..	.....	S	2	4847.72	4848.6	
293	F	3	4855.64	F	4	4854.36	F	3	4854.74	P	2	4854.95	4855.4	
294*	W	3	4861.14	W	5	4860.91	F	3	4861.38	W	4	4862.35	4861.3	Hβ.
295	S	1	4865.68	..	..	.....	P	2	4866.58	..	..	.....	.....	Ni. ?
296	..	..	.....	..	..	.....	Br.	2	4868.62	..	..	.....	.....	
297	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
298*	..	..	.....	P	2	4871.77	F	1	4872.15	..	..	.....	4871.9	V. Fe.
299	F	3	4875.15	..	..	.....	..	..	.....	..	..	.....	.....	
300	..	..	.....	F	3	4875.25	F	4	4875.90	F	3	4875.77	4875.7	V.
301	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
302	..	..	.....	F	3	4881.34	F	3	4881.09	F	2	4881.25	4881.2	V. ?
303	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
304*	W	4	4886.29	F	3	4885.23	F	4	4886.20	G	3	4886.57	4886.2	Fe. V.
305	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
306	F	5	4891.51	F	4	4891.01	F	2	4890.85	..	..	.....	.....	V.
307	Edge		4892.13	..	..	.....	..	..	.....	..	..	.....	.....	
308	Br.	2	4896.52	..	..	.....	F	3	4896.78	..	..	.....	.....	
309	..	..	.....	Br.	2	4898.29	..	..	.....	Br.	2	4899.51	.....	
310	F	5	4902.89	F	3	4901.69	F	3	4901.08	..	..	.....	.....	
311	..	..	.....	F	3	4905.74	F	2	4906.64	..	..	.....	4906.0	V.
312	..	..	.....	F	4	4910.97	W	4	4911.33	F	4	4910.91	4911.0	Fe.
313	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
314	..	..	.....	..	..	.....	..	..	.....	..	..	.....	.....	
315	G	5	4920.12	G	7	4921.31	W	5	4919.72	F	4	4921.64	4921.0	Fe.

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+ 42° 2811.			— 10° 5057.			+ 5° 5223.			+ 20° 5071.			+ 34° 1929.			— 10° 513.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
316			4926.11 j	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
317	..	..	.....	F	3	4933.77	..	..	.....	F	6	4931.06	..	..	.....	..	..	.....
318	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
319	W	4	4939.01	P	4	3937.84	P	2	3940.21	..	..	.....	..	..	.....	F	4	3938.19
320	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
321	F	4	4957.86	F	3	4957.77	F	4	4958.20	..	..	.....	..	..	.....	..	..	.....
322	..	..	.....	..	..	.....	F	3	4965.73	..	..	.....	..	..	.....	..	..	.....
323	Dif.	2	4969.74	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
324	..	..	.....	..	..	.....	F	2	4978.07	..	..	.....	..	..	.....	..	..	.....
325	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
326	W	5	4984.09	G	4	4983.19	F	4	4984.06	F	5	4985.68	..	..	.....	P	3	4984.96
327	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
328	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....	..	..	.....
329	F	4	5006.89	..	..	.....	F	3	5006.70	..	..	.....	..	..	.....	..	..	.....

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TABLE III. MEAN WAVE-LENGTHS. CLASS R—CONTINUED.

NO.	+61° 667.	−3° 1685.	+57° 702.	+14° 2048.	MEAN AND IDENTIFICATION.	
	CHARACTER, INTENSITY, AND WAVE-LENGTH.	CHARACTER, INTENSITY, AND WAVE-LENGTH.	CHARACTER, INTENSITY, AND WAVE-LENGTH.	CHARACTER, INTENSITY, AND WAVE-LENGTH.	ANGSTROMS.	ELEMENT.
316	.. ..	.. ..	.. ..	.. ..	.....	
317	.. ..	F 2 4929.76	.. ..	.. ..	.....	
318	.. ..	F 2 4932.25	.. ..	.. ..	.....	
319	.. ..	F 6 3936.63	W 4 3937.69	W 4 3935.74	.....	
320	.. ..	3954.39	.. ..	.. ..	.....	
321	.. ..	.. ..	.. ..	W 3 4958.94	4958.2	Fe.
322	.. ..	.. ..	.. ..	.. ..	.....	
323	.. ..	4968.77	.. ..	F 3 4968.38	.....	
324	.. ..	W 6 4977.62	F 4 4977.85	.. ..	4977.8	Ti. Fe.
325	.. ..	Rev. 1 4982.08	Br. 1 4982.22	.. ..	4982.1	Ti.
326	.. ..	F 6 4985.74	F 6 4986.63	W 5 4987.49	.....	
327	.. ..	4990.65	.. ..	.. ..	.....	
328	.. ..	F 3 5000.61	F 5 5001.20	.. ..	.....	
329	.. ..	.. ..	.. ..	.. ..	.....	

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TABLE IV. WAVE-LENGTHS. CLASS N.

NO.	19 PISCIUM.			+ 76° 734.			+ 34° 4500.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
1	End		4242.98	..	..	.....	..	..	.....
2	P	5	4247.68	..	..	.....	..	..	.....
3	F	1	4251.08	..	..	.....	..	..	.....
4	P	5	4254.12	..	..	.....	..	..	.....
5	P	6	4258.86	..	..	.....	..	..	.....
6	P	2	4260.69	..	..	.....	..	..	.....
7	F	5	4262.94	..	..	.....	..	..	.....
8	F	4	4271.91	..	..	.....	..	..	.....
9	F	6	4275.41	..	..	.....	..	..	.....
10	F	4	4277.70	..	..	.....	..	..	.....
11	Br.	2	4278.74	..	..	.....	..	..	.....
12	F	3	4280.81	..	..	.....	..	..	.....
13	F	3	4283.10	..	..	.....	..	..	.....
14			4288.52	..	..	.....	..	..	.....
15	F	6	4289.37	..	..	.....	..	..	.....
16	F	3	4291.90	..	..	.....	..	..	.....
17			4292.50	..	..	.....	..	..	.....
18	F	3	4294.93	..	..	.....	..	..	.....
19	G	6	4307.00	F	6	4306.70	P	5	4307.39
20	F	3	4319.04	F	4	4318.82	..	..	.....
21	P	3	4321.51	..	..	.....	..	..	.....
22	G	10	4325.47	..	..	.....	F	7	4325.59
23	F	3	4330.13	..	..	.....	..	..	.....
24	P	3	4333.85	..	..	.....	P	4	4334.05
25	F	2	4337.76	P	4	4338.28	F	5	4339.22
26	Br. ?		4339.62	..	..	.....	..	..	.....
27	F	2	4341.70	..	..	.....	..	..	.....
28	..	..		..	..	.....	P	3	4346.22
29	G	3	4348.06	F	4	4347.53	F	4	4347.98
30	G	8	4351.90	..	..	.....	F	6	4351.96
31	F	3	4355.97	..	..	.....	..	..	.....
32	F	9	4360.46	..	..	.....	W	5	4360.38
33			4365.16	..	..	.....	..	..	.....
34	F	6	4368.48	..	..	.....	W	5	4368.37
35	F	1	4373.13	..	..	.....	..	..	.....
36	F	3	4376.09	F	4	4375.78	F	4	4375.94
37	Br.	1	4377.98	..	..	.....	..	..	.....
38	F	5	4380.64	..	..	.....	..	..	.....
39	G	7	4384.50	F	8	4384.21	..	..	.....
40	..	..				4385.27	Head		4385.42
41	..	..	.....	G	3	4387.44	..	..	.....
42	F	4	4390.27	F	5	4390.04	F	4	4390.44
43	G	8	4395.01	F	7	4394.99	F	7	4394.86
44	..	..	.....	..	..	.....	Br.		4402.62
45	F	5	4400.56	F	6	4400.30	F	8	4400.29

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TABLE IV. WAVE-LENGTHS. CLASS N—CONTINUED.

NO.	19 PISCIUM.			+ 76° 734.			+ 34° 4500.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
46	G	6	4404.92	G	8	4404.86	F	7	4405.19
47	F	5	4408.62	F	6	4408.47	F	5	4408.64
48	P	2	4412.61	F	3	4412.01	..	..	.....
49	F	4	4415.31	..	..	.....	F	4	4415.53
50	..	..	.....	P	6	4416.37	..	..	.....
51	F	4	4417.34	..	..	.....	..	..	.....
52	F	3	4420.95	..	..	.....	..	..	.....
53	..	..	.....	F	6	4422.26	F	5	4422.16
54	P	2	4423.21	..	..	.....	..	..	.....
55	F	1	4426.10	..	..	.....	..	..	.....
56	F	3	4428.00	..	..	.....	G	4	4427.65
57	F	3	4430.05	F	4	4429.74	F	3	4430.33
58	Br.	1	4432.34	..	..	.....	Br.		4432.71
59	G	7	4435.46	G	10	4435.48	G	8	4435.72
60	..	..	.....	..	..	.....	F	2	4438.59
61	..	..	.....	..	..	.....	Br.		4439.56
62	F	4	4443.43	W	7	4444.03	W	6	4443.08
63	..	..	.....	..	..	.....	F	2	4447.44
64	F	4	4450.36	F	6	4450.25	G	5	4450.40
65	F	4	4455.80	P	6	4455.20	P	4	4455.98
66	F	2	4459.72	..	..	.....	..	..	.....
67	F	4	4462.04	F	7	4462.11	F	6	4462.11
68	Br.		4463.95	..	..	.....	Br.		4464.33
69	F	2	4465.70	F	5	4465.86			4465.61
70	P	3	4468.64	F	4	4468.84	..	..	.....
71	P	3	4471.50	..	..	.....	..	..	.....
72	..	..	.....	..	..	.....			4472.21
73	..	..	.....	..	..	.....	Br.		4473.04
74	P	3	4476.39	F	4	4476.18	..	..	.....
75	P	4	4480.23	F	7	4480.53	F	7	4480.39
76	..	..	.....	..	..	.....	F	3	4487.47
77	F	4	4489.60	P	6	4489.18	F	4	4489.58
78	F	1	4493.80	F	2	4494.50	..	..	.....
79	G	6	4496.97	G	6	4496.83	G	7	4496.92
80	F	3	4501.51	F	4	4501.65	F	5	4501.44
81	Br.		4501.81	..	..	.....	Edge		4501.90
82	G	4	4506.88	F	5	4506.81	F	5	4507.00
83	P	4	4513.04	F	6	4512.83	W	6	4513.70
84	F	3	4518.47	P	5	4518.63	F	3	4518.63
85	..	..	.....	F	2	4520.10	..	..	.....
86	F	3	4523.30	G	5	4522.97	G	4	4523.30
87	..	..	.....	..	..	.....			4526.06
88	F	4	4527.87	F	6	4527.31	P	6	4528.17
89	P	2	4531.45	..	..	.....	..	..	.....
90	P	5	4535.47	G	8	4535.28	..	..	.....

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TABLE IV. WAVE-LENGTHS. CLASS N—CONTINUED.

NO.	19 PISCIUM. CHARACTER, INTENSITY, AND WAVE-LENGTH.			+ 76° 734. CHARACTER, INTENSITY, AND WAVE-LENGTH.			+ 34° 4500. CHARACTER, INTENSITY, AND WAVE-LENGTH.		
91	..	..	.....	..	..	.....	Edge		4536.34
92	Br.	2	4538.14	..	..	.....	..	..	.....
93	G	4	4540.71	F	5	4540.95	F	5	4541.27
94	F	3	4545.81	F	4	4545.74	G	5	4545.93
95	..	..	.....	G	3	4549.11	..	..	.....
96	F	3	4549.72	..	..	.....	F	6	4549.93
97	G	10	4553.85	F	9	4553.34	G	10	4553.61
98	P	3	4560.04	F	4	4560.26	F	3	4560.16
99	P	3	4563.44	..	..	.....	F	4	4563.39
100	W	4	4564.68	P	4	4564.06	..	..	.....
101	F	1	4565.10	..	..	.....	..	..	.....
102	..	..	.....			4570.34	..	..	.....
103	..	..	.....	..	..	.....			4571.12
104			4571.92	..	..	.....	..	..	.....
105			4576.78			4577.20			4578.26
106	..	..	.....	F	4	4590.86	..	..	.....
107	..	..	.....	F	2	4593.94	F	4	4593.74
108	..	..	.....	F	1	4597.37	F	3	4598.10
109	..	..	.....	P	2	4600.58	..	..	.....
110	..	..	.....	F	1	4603.18	..	..	.....
111	G	10	4606.72	G	10	4606.60	G Max.		4606.25
112	..	..	.....	F	1	4610.45	..	..	.....
113	..	..	.....	F	2	4613.45	F	4	4613.75
114	F	2	4616.48	F	2	4616.22	F	4	4616.84
115	F	3	4619.96	G	4	4619.52	G	6	4620.06
116	..	..	.....	..	..	.....	F	3	4623.12
117	F	3	4629.38	W	4	4629.12	F	4	4629.56
118			4632.61	..	..	.....			4632.14
119	..	..	.....	W	6	4635.30	..	..	.....
120			4637.35	..	..	.....			4637.59
121	G	5	4640.47	G	5	4640.29	G	6	4640.65
122	Br.	2	4642.22	..	..	.....	Br.		4642.67
123	..	..	.....			4644.03			4644.11
124	W	7	4646.55	F	8	4646.28	..	..	.....
125	F	6	4656.94	F	6	4657.18	F	5	4657.15
126	W	5	4663.34	..	..	.....	F	3	4663.55
127	W	7	4668.85	F	5	4668.04	F	6	4668.77
128	..	..	.....	F	4	4682.27	..	..	.....
129	..	..	.....	..	..	.....	W	8	4695.78
130	..	..	.....			4698.21	..	..	.....
131	F	3	4704.04	F	3	4704.40	..	..	.....
132	P	4	4707.42	..	..	.....	..	..	.....
133	G	9	4714.62	F	8	4714.43	G	10	4714.14
134	F	2	4722.97	F	6	4722.83	F	3	4722.88
135	F	3	4729.43	..	..	.....	..	..	.....

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TABLE IV. WAVE-LENGTHS. CLASS N—CONTINUED.

NO.	19 PISCIUM.			+ 76° 734.			+ 34° 4500.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
136	G	10	4735.96	G	10	4735.87	G	10	4735.96
137	Head		4736.82	Head		4737.11	..	..	.....
138	F	6	4743.79	G	9	4743.52	G	10	4743.76
139	..	..	.....	F	4	4745.67	..	..	.....
140	..	..	.....	F	2	4758.67	..	..	.....
141	..	..	.....	F	1	4761.49	F	2	4763.52
142	..	..	.....	F	4	4772.43	P	2	4773.29
143	..	..	.....	F	3	4779.34	..	..	.....
144	..	..	.....			4782.33	..	..	.....
145	Neb.		4784.59	W	5	4784.75	..	..	.....
146	..	..	.....			4789.54	P	2	4789.45
147	..	..	.....	F	3	4791.18	..	..	.....
148	..	..	.....	Dif.	4	4799.69	..	..	.....
149	..	..	.....	P	3	4805.75	W	4	4806.84
150	..	..	.....	..	..	.....	F	2	4812.13
151	F	2	4816.03	G	4	4815.82	F	3	4816.24
152	P	2	4823.50	F	5	4822.80	F	4	4823.44
153	..	..	.....	F	4	4827.82	F	3	4828.36
154	P	2	4832.72	F	5	4832.54	G	3	4833.00
155	..	..	.....	F	3	4839.82	..	..	.....
156	..	..	.....	P	2	4848.93	..	..	.....
157	..	..	.....	F	3	4852.18	..	..	.....
158	..	..	.....	F	4	4854.78	P	4	4854.28
159	Head		4855.37	..	..	.....	..	..	.....
160	..	..	.....	W	4	4865.66	F	3	4865.47
161	..	..	.....	..	..	.....	F	2	4868.48
162	F	2	4871.74	..	..	.....	F	3	4872.02
163	P	2	4874.92	F	4	4875.54	F	3	4876.03
164	F	2	4881.46	G	4	4881.74	F	3	4882.13
165	F	1	4885.40	..	..	.....	..	..	.....
166	..	..	.....	P	4	4891.02	W	4	4891.43
167	..	..	.....	F	3	4900.72	..	..	.....
168	..	..	.....	..	..	.....	F	4	4902.77
169	..	..	.....	P	3	4905.33	..	..	.....
170	..	..	.....	F	2	4910.28	..	..	.....
171	..	..	.....	P	4	4914.40	..	..	.....
172	Red Edge		4914.50	..	..	.....	..	..	.....
173	F	3	4921.44	G	4	4920.73	G	6	4921.12
174	F	4	4934.35	F	4	4934.22	F	6	4934.64
175	P	3	4940.12	..	..	.....	..	..	.....
176	..	..	.....	..	..	.....	F	3	4950.18
177	..	..	.....	P	4	4957.16	F	3	4958.57
178	..	..	.....	F	4	4966.86	..	..	.....
179	P	4	4981.39	F	5	4980.33	P	2	4982.35
180	..	..	.....	F	4	4986.42	..	..	.....

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TABLE IV. WAVE-LENGTHS. CLASS N—CONTINUED.

NO.	19 PISCIUM.			+ 76° 734.			+ 34° 4500.		
	CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.			CHARACTER, INTENSITY, AND WAVE-LENGTH.		
181	W	5	4988.20	..	..	.....	..	..	.....
182	..	..	.....	F	4	4990.08	..	..	.....
183	W	5	5006.56	F	4	4999.63	F	4	5000.70
184	F	4	5014.76	F	5	5014.44	F	3	5014.62
185	..	..	.....	F	6	5038.96	..	..	.....
186	Red Edge		5041.30	..	..	.....	..	..	.....
187	..	..	.....	F	7	5095.29	..	..	.....
188	..	..	.....	Red Edge		5167.95	..	..	.....
189	..	..	.....	F	6	5182.71	..	..	.....
190	..	..	.....	F	3	5192.91	..	..	.....
191	..	..	.....	W	8	5205.82	..	..	.....

RADIAL VELOCITIES

The accompanying table (Table V) of radial velocities contains the results obtained for 10 stars belonging to Class R. Although the list is not large enough to give more than an indication of the average radial velocity of the class, it furnishes the only data available at the present time.

Column 1 gives the star's designation; column 2, the plate numbers. The third column gives the quality of the stellar spectrum, S, and of the comparison spectrum, C, of the plate; G, good; F, fair; P, poor; W, wide; and Dif., diffuse. Column 4 contains the number of the lines used in each plate. The radial velocities determined from the individual plates and reduced to the sun are recorded in column 5, also the probable error based upon the internal agreement of the velocities given by the lines used in the reduction of the plate. Column 6 gives the adopted radial velocity of the star reduced to the sun and its probable error. The last column tabulates the residual radial velocity after deducting the component due to the motion of the sun through space, or the radial velocity of the star reduced to the sidereal system.

The number of lines for radial velocity determination of star — 10° 5057 recorded in the table is smaller than the number available for use on the individual plates, as only the lines giving the best results on all the plates were included. Making use of 27 lines of plate 2882 A, 30 lines of plate 2997 A, 31 lines of plate 2914 A, and 32 lines of plate 2915 A, the result gives practical agreement with the value given in the table. The star + 20° 5071 presented some difficulty on account of its broad absorption lines and the presence of bright lines near some of the lines of the standard table. The probable error of the velocity of this star indicates that the close agreement of the separate velocities derived from the three plates is in a measure fortuitous. The relative velocity of this star with reference to — 10° 5057 was determined by comparing the displacements of 20 lines in common and gave good agreement with the tabulated values.

The radial velocity for each star was obtained by weighting the plate velocities according to probable error and the quality of the plate. On account of the small number of available plates for each star, the probable error accompanying the adopted value of the radial velocity of the star was based upon the internal agreement of the lines of the individual plates instead of following the ordinary method of forming residuals from the mean velocity. For the determination of the residual radial velocities the apex of the sun's way was taken at the point, $\alpha = 270°$, $\delta = +30°$, and the velocity of the sun with reference to the sidereal system was taken to be 20 km. per sec. This is the position of the apex of the sun's way and value of the velocity of the sun through space favored by Boss for general use

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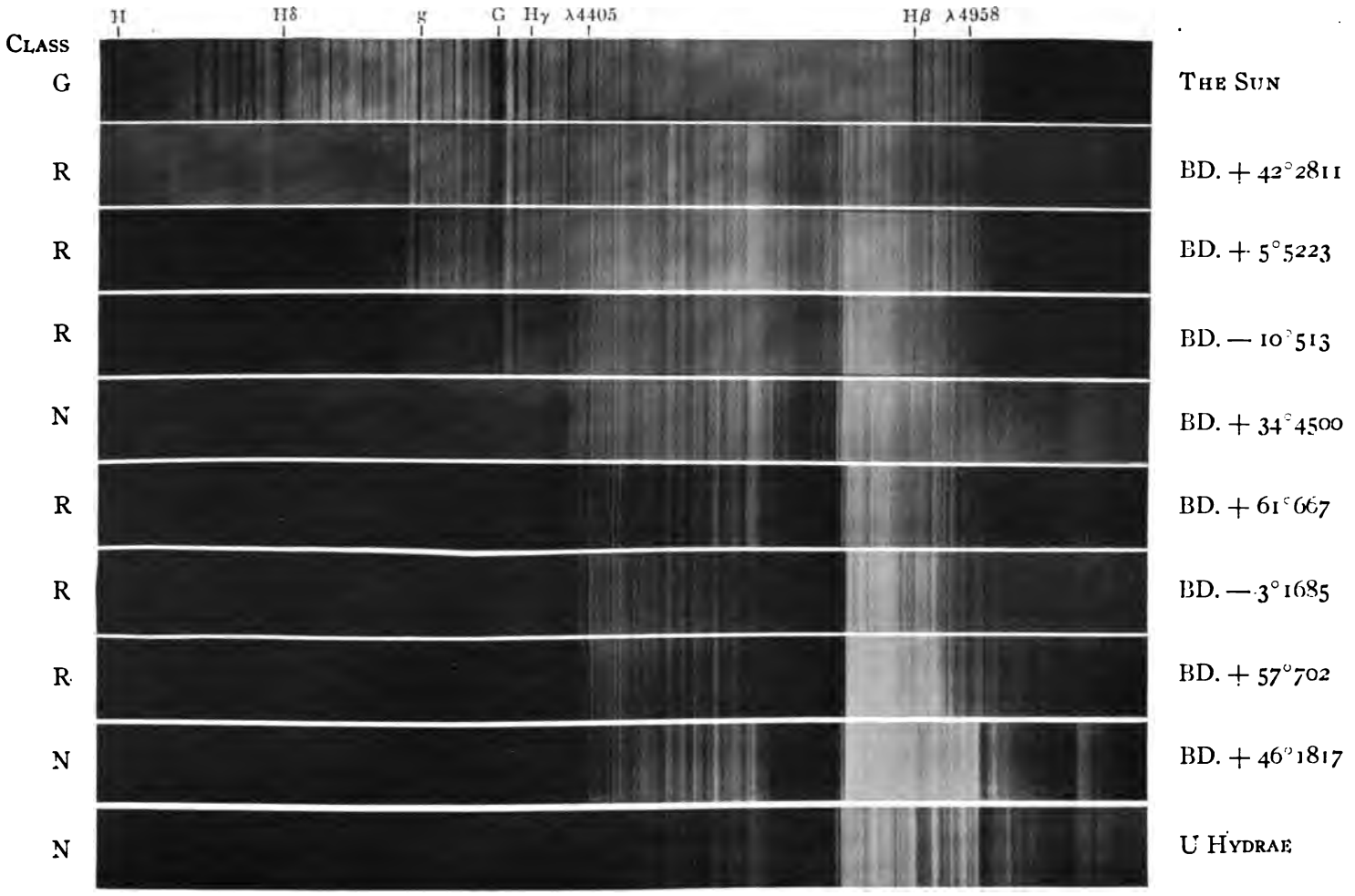

Plate I. Stellar Spectra Illustrating Steps in the Transition from the Sun to Standard Class N

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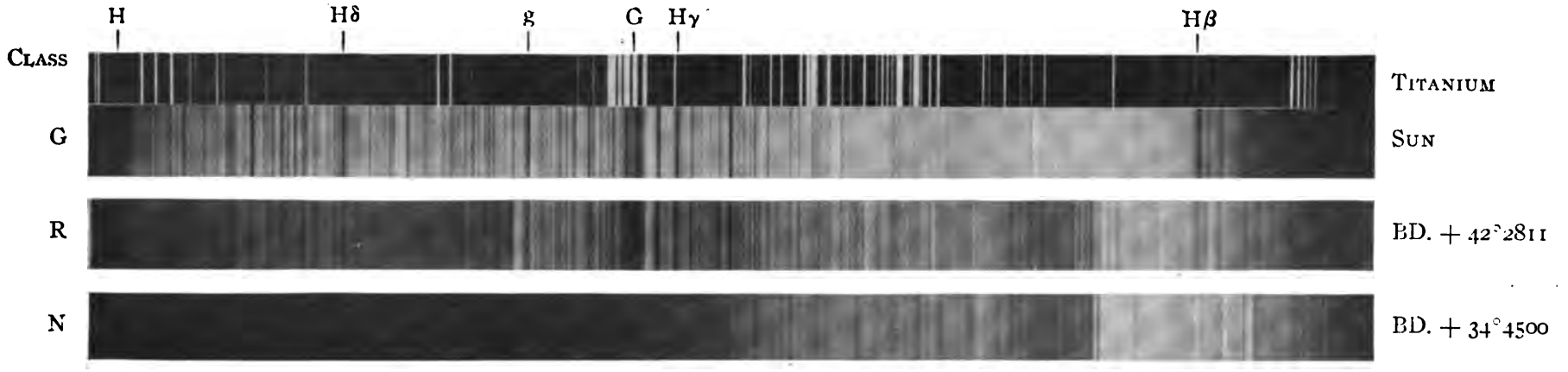

Plate J. Class G, Early Class R, and Early Class N Spectra, with Titanium Spark Comparison

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TABLE V. RADIAL VELOCITIES, CLASS R.

STAR.	PLATE.	QUALITY.		NO. OF LINES.	PLATE VELOCITY TO SUN.	STAR VELOCITY TO SUN.	RESIDUAL RADIAL VELOCITY.
		S.	C.				
DM. + 42°2811	3128 B	F	F	32	−24.5 ± 1.3		
	3135 B	G	F	33	−26.0 ± 1.0		
	3144 B	G	G	37	−25.2 ± 1.2	−25.2 ± 0.8	− 6.
− 10 5057	2882 A	G	F	17	−43.2 ± 1.7		
	2887 A	G	G	18	−42.5 ± 1.3		
	2914 A	G	G	20	−42.6 ± 1.3		
	2915 A	G	G	21	−45.0 ± 0.6		
	2923 A	P	F	14	−43.6 ± 4.3	−43.8 ± 0.5	−30.
+ 5 5223	2988 A	F	G	26	−26.0 ± 1.5		
	2989 B	F	F	26	−26.4 ± 2.2	−26. ± 1.1	−24.
+ 20 5071	2966 C	F	G	19	−49.8 ± 1.6		
	Remeasured.				−48.4 ± 2.2		
	2967 A	D	F	12	−48.8 ± 5.2		
	2968 A	P	F	14	−50.1 ± 4.1	−49. ± 2.	−38.
+ 34 1929	3138 B	F	P	33	+27.8 ± 1.6		
	3143 A	W	G	26	+22.6 ± 1.5	+25. ± 1.	+20.
− 10 513	3055 A	P	F	12	+21.9 ± 3.9		
	3063 A	F	G	24	+19.8 ± 1.2	+21. ± 1.	+ 9.
+ 61 667	3054 C	F	G	16	− 6.9 ± 1.4		
	3088 A	W	F	7	− 4.1 ± 3.5		
	3089 A	F	F	21	− 8.8 ± 2.1	− 7. ± 1.	− 6.
− 3 1685	3094 D	G	F	25	+19.0 ± 1.8		
	3097 A	F	G	27	+17.6 ± 2.0	+18. ± 1.	+ 1.
+ 57 702	3044 A	W	G	16	−13.6 ± 2.6		
	3079 A	G	F	29	−10.6 ± 1.5	−11. ± 1.	−10.
+ 14 2048	3126 A	F	G	21	+ 3.5 ± 1.3		
	3127 A	G	G	16	+ 5.9 ± 2.1		
	3134 A	G	G	19	+ 2.6 ± 1.8	+ 4. ± 1.	− 5.
Average velocities considered positive						23. ± 1.	14.9

and represents a compromise between the results obtained by proper motion and radial velocity determination. (*Astr. Jour. 28*, 167, 1914.) This direction was used by Campbell in his solution for solar motion by spectral types (*Lick Observatory Bulletin*, No. 196, 127, 1911.)

At the foot of the columns are given the averages of the radial velocities of Class R stars, 23 km. per second, and of the residual radial velocities, 15 km. per second. In this connection it is interesting to compare the average radial velocity of 8 stars of Class N determined by Hale, Ellerman, and Parkhurst and exhibited in Table VI. The average radial velocity of 8 Class N stars is 11 km. per second. The residual radial velocities were not given in their paper, but were determined by the method given above for the sake of comparison with the results for Class R stars. The average *residual radial velocity* of 8 stars with spectra of Class N is 13 km. per second. On account of the small number of stars of Classes R and N for which the radial velocities have been

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determined, their averages may only approximately represent the classes.

Average residual radial velocity of 10 Class R stars, 15 km. per sec.

Average residual radial velocity of 8 Class N stars, 13 km. per sec.

TABLE VI. RADIAL VELOCITIES, CLASS N.

STAR.	RADIAL VELOCITY	RESIDUAL RADIAL VELOCITY.
74 Schj.	+ 5 km.	— 9 km.
78 Schj.	— 1	— 8
115 Schj.	— 13	— 22
132 Schj.	— 28	— 38
318 Birm.	— 10	— 3
152 Schj.	+ 1	+ 10
19 Piscium	— 2	0
280 Schj.	— 25	— 16
Average (positive)	11	13.2

For the sake of comparison with other types the foliowing table is inserted. Part of the data is taken from *Lick Observatory Bulletin*, No. 196, 126, 1911. The values for Classes R and N were added by the writer.

TABLE VII. RESIDUAL RADIAL VELOCITIES ACCORDING TO SPECTRAL TYPE.

SPECTRAL TYPES.	NO. OF STARS.	AVERAGE RESIDUAL RADIAL VELOCITY.
O and B	141	8.99
A	133	9.94
F	159	13.90
G and K	529	15.15
M.	72	16.55
R	10	14.9
N	8	13.2

QUALITATIVE RESULTS

General Characteristics. The general features of the spectrum of Class R stars have been given in the section on spectrograms; here we shall discuss the details at greater length.

The most conspicuous feature of the region of the spectrum under consideration (violet end to λ 5000), as previously pointed out, is the broad absorption band with head at λ 4737, usually attributed to some form of carbon compound. The progression of intensity of this band in the spectrum of the different stars, suggesting an evolutionary series, has been used to establish the sequence, + 42° 2811, — 10° 5057, + 5° 5223 + 20° 5071, + 34° 1929, — 10° 513, + 61° 667, — 3° 1685, + 57° 702, + 14° 2048.

In general the spectrum having the strongest carbon absorption band shows the least light action in the violet. It is interesting at this point to inquire if other changes can be found accompanying the change in this underlying characteristic.

Naturally we expect the color index to indicate this transition. Data are lacking excepting for two stars of the list.

STAR.	COLOR INDEX.
— 10° 5057	1.09
+ 20° 5071	1.82

These two follow the proposed order. The visual colors of the 10 stars as recorded in our observational notes appear to be consistent with the sequence. The first one closely resembles the solar type.

Carbon. Accompanying the increase of intensity of the λ 4700 band, a peculiar transformation takes place in the bright region of the spectrum adjoining the head of the band. The absorption line at λ 4743.6 grows much stronger until at the middle of the series it resembles a new head of the band at λ 4745.1. Another line then appears at λ 4751.1, which follows a similar course and at the end of the series appears like a head of the band at λ 4752.8. No explanation is here offered.

In addition to λ 4737 other members of Group IV of the Swan spectrum, λλ 4715, 4697, 4684, are found in the spectra of some of the stars; in other cases, especially late in the sequence, they are not distinguishable on account of the strong absorption in this region. Group V, λλ 4381, 4371, 4365, is weak, if present; lines corresponding only to the second were measured on any of the plates, and they may be identifiable as a chromium line that is widened in sun spots.

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Cyanogen Group II, λλ 4606, 4578, 4553, 4532, 4515, 4502, is represented in all of the stars by some of its lines. The intensity of the first of the group varies greatly, but not progressively; λ 4553 appears to grow stronger in the order of the sequence. The first of Cyanogen Group III, 4216, 4197, 4181, is strong; the second is present. In the case of the later stars, the spectrum is weak in this region and consequently the intensity of the lines cannot be compared from star to star. The "chief carbon line q"[17] λ 4268 does not change appreciably in intensity with the sequence, but was not measured in all of the stars.

Chromium. A large number of strong chromium lines appear in the spectra of Class R stars. Many of them are identical with the strong lines of Classes I and II of the electric furnace emission lines investigated by King.[18] We note especially λλ 4254.5, 4275.0, 4289.9, 4337.7, (4339.6-40.3?), 4352.0, 4497.0, 4530.9, 4580.3. No systematic change in intensity can be detected. Some of the strong furnace and arc emission lines, however, are not represented by absorption lines in the stellar spectra.

Vanadium. This element is also represented by numerous lines, the greater number of which are of King's classes I and II. The table of wave-lengths indicates the lines ascribed to Vanadium. No progressive change is noticeable.

Iron. Numerous iron lines are present, chiefly of King's Classes II and III.[19] The most prominent are λλ 4187.9, 4250.6 (blend), 4260.6, 4271.9, 4325.9, 4404.9, 4415.3, 4494.7, 4871.9 (blend), 4920.7. No systematic change is noticeable.

Titanium. The presence of this element is well attested. Several of the lines show a weakening or broadening in the spectra of the later stars. Lines of Classes I, II, and III are most numerous.

Calcium. The line at λ 4227 is strong and broadens, passing along the sequence. The H and K lines appear only on plates of the first star of the sequence.

Nickel. This element is probably present giving λ 4470.7 in some of the stars. It may contribute to λ 4714.6, which does not give accordant values of the wave-length in the series. λ 4866 is very uncertain.

Magnesium. λ 4352.1 may blend with chromium, λ 4351.9, to produce the strong line at λ 4352, which decreases in intensity along the series. λ 4481 is strong, apparently widening in the latter part of the series.

Hydrogen. Hydrogen lines appear on many of the plates, but they are not strong. Hγ grows poorer in the later stars, where it can scarcely be separated from a strong line at λ 4337.8 attributed to chromium. Hβ appears as a good line with intensity 5 in the spectrum of the first star of the series. It was not measured in the second; it is weak in others, and widens in the later stars of the list.

The displacement of some lines or shift with spectral type determined by Albrecht[20] is indicated in the series of Class R stars. Fifteen of the lines traced through Albrecht's *"Table of Wave-lengths Varying Progressively with Spectral Type"* show a change in the same direction passing along the series of Class R stars to Class N. Since the change in wave-length with spectral type is small, seldom exceeding 0.2 Å from F to K, our test establishes only the direction of the variation, on account of the probable error of ± 0.12 Å in wave-length determination for a single line on a plate.

Plate VIII shows graphically the change in wave-length of λ 4435 with spectral class.

The progressive change in intensity of various lines as we pass along the series has been noted for the different elements. Comparison of these lines and their changes in intensity with a list of lines, whose intensity changes with spectral type[21] from Miss Maury's Class VI to Class XVIII, indicates that, in general, the progressive change in intensity in our series corresponds with the direction given in the Harvard table.

Comparison with solar spectrum. The general resemblance of the spectra of Class R stars to

[17] Crew and Baker; *Ap. J. 16*, 67, 1902.

[18] Lines of Class I are relatively strong at a low temperature (1900°C) and strengthen slowly at higher temperature. Class II lines appear at the low temperature and strengthen rapidly with increase of temperature. *Ap. J. 41*, 1915.

[19] Class III lines are absent or faint at low temperature (1900°C), appear at medium (2200°C) and strengthen rapidly at higher temperature.

[20] *Cordoba Boletin*, No. 1, 1911.

[21] *Harvard Annals*, Vol. 28, p. 60.

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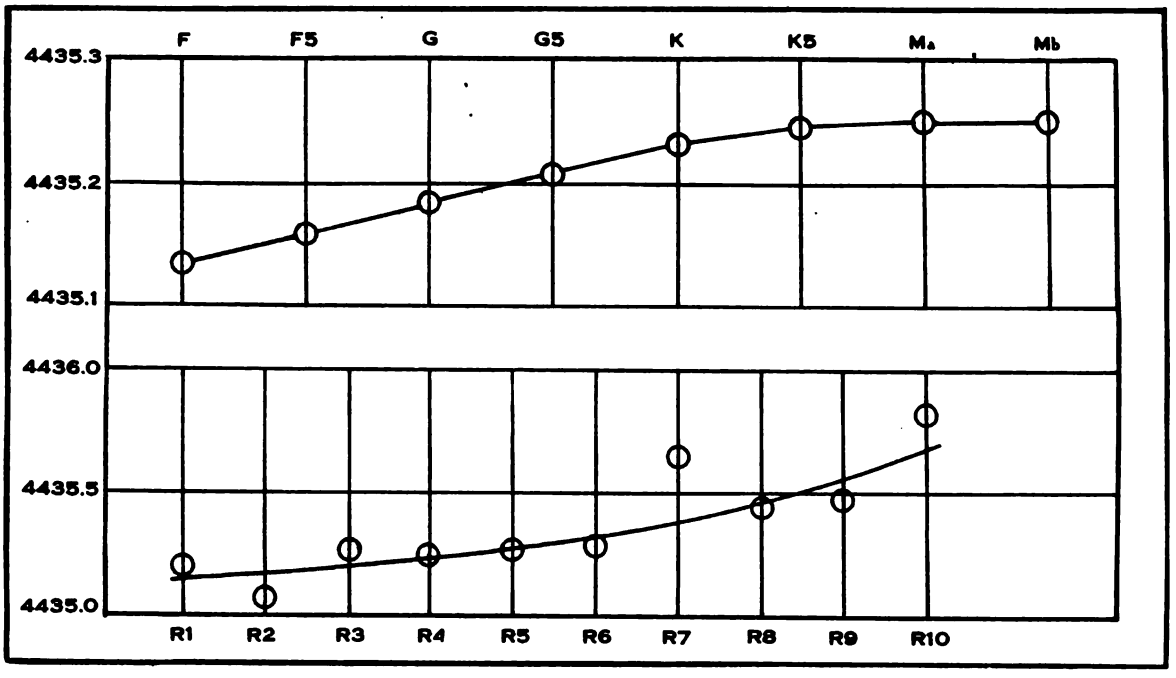

PLATE VIII. CHANGE IN WAVE-LENGTH OF λ4435 WITH SPECTRAL CLASS
OBSERVATIONS OF UPPER CURVE BY ALBRECHT
OBSERVATIONS OF LOWER CURVE BY RUFUS

the solar spectrum with reference to metallic lines harmonizes with the similarity of Types II, III, and IV, pointed out by other writers. Referring to star — 10° 5057, Parkhurst says:[22] "The spectrum resembles the solar type, with the addition of the dark λ 4700 band." He apparently considers that its spectrum differs greatly from the spectra of other stars of Classes N and R, which constituted his list for the determination of color indices, for he places it in a class by itself. We find, however, a very close resemblance between its spectrum and the spectrum of + 42° 2811, one of the recently discovered Class R stars which precedes it in our series. Its spectrum seems to fit well into the sequence adopted on page 136 above. We wish here simply to call attention to the close solar relationship of early Class R stars.

Hale, Ellerman, and Parkhurst have compared fourth type spectra with sun spot spectra and found that a large number of the lines widened in sun spots are strengthened in the spectra of fourth type stars. This result was based upon the region λ 5190 to the red end of the spectrum. From the work of Adams[23] we are able to make a similar comparison for Class R stars from λ 5000 to the violet end.

We have limited our count to the strongest lines of Adams' list, intensity at least 2, because with our slit width and dispersion, many of the faint lines do not appear. About two-thirds of the lines widened in the sun spots appear in Class R stars. Some others may be present, but concealed by the carbon and cyanogen absorption bands. We also note the omission of lines on the less refrangible side of the heads of these bands; e. g. opposite cyanogen λ 4197, the iron lines, λ 4198.2, slightly strengthened in spots; λ 4198.4-.5, a strong blend slightly weakened; λ 4198.8, slightly strengthened and λ 4199.3, strengthened in spots, do not appear on our plates. Also below λ 4447.3 the strengthened lines λ 4447.9 Fe. and λ 4449.3 Ti. are not distinguishable. Among the lines weakened in sun spots, we find many weak or absent in Class R stars. Al-

[22] *Ap. J. 35*, 131, 1912.

[23] *Ap. J. 27*, 45, 1908.

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though there is some conflicting evidence the resemblance of the spectra of Class R stars and sun spot spectra seems to be well established.

Comparison with Class O. In the region of the spectrum under consideration, only the following coincidences occur in the bright lines of Classes O and R. The second column gives the number of Class R stars in which the bright line was measured.

CLASS R.	NO.	CLASS O.
λ 4464	4	4465-67
		4473-74
4507	1	4504-10
		4515-18
4538	3	4534-44
		4592-98
4617	1	4614-16
		4650-54
		4862

These coincidences are too few in number to establish any physical relationship. Both classes have bright lines, but this characteristic is shared with other classes.

In addition to the absence of some of the bright lines of Class O stars in the spectra of Class R, many bright lines found in the spectra of Class R stars are lacking in Class O.

There seems to be a shift in wave-length toward the red in the case of λλ 4589-91, 4818-20 and 4896-99, within the series of Class R stars.

The bright lines have not been identified. An explanation for the presence of some, at least, may be sought in the reversal of lines of known elements under certain conditions, rather than by the postulation of unknown elements.

Comparison with Class N. The chief difference between the spectra of stars of Class R and stars of Class N, from the violet end of the spectrum to λ 5000, is in the relative intensity of the spectrum on the two sides of the dark band with head at λ 4737. In general the spectra of stars of Class R extend farther into the violet than the spectra of Class N stars under similar conditions of exposure on stars of comparatively equal photographic magnitude. Ellerman's long exposure on 19 Piscium[24] shows the presence of violet light in this Class N star, giving a spectrum beyond the Fraunhofer H and K lines; and Parkhurst expresses the opinion concerning the spectra of certain stars of Class N, "It is probably a mere question of exposure to extend the other spectra to this region, except perhaps that of 152 Schjellerup." The difference, then, appears to be due to a difference in the quantity rather than in the quality of the violet light emitted by stars of the two classes.

The presence of Hβ as an absorption line in the spectra of many of the Class R stars, and its absence in Class N stars, or presence as a bright line, marks another difference. Hβ has been observed, however, as a bright line in the Class R variables.[25]

The cyanogen bands appear to be somewhat stronger in the spectra of Class N stars, especially λ 4553.6 and λ 4606. An unknown line λ 4443 (Ti spark λ 4444?) appears to be stronger in Class R than in Class N. On some of our plates it is very prominent, having a maximum line intensity of 10, while Hale does not mention it among prominent lines of Class N.

The progressive change in the wave-length of certain lines with spectral type passing from Class R to Class N has already been mentioned as taking place in the direction indicated by Albrecht.

The change in intensity of certain lines with spectral type has also been pointed out. These two features, however, could be better tested by the selection of some later Class N stars,[26] also by the observation of a larger number for wave-length determination.

Another noticeable difference is in the number and intensity of bright lines. The emission lines found in the spectra of Class R stars are in general very weak. In the case of the earlier stars, only a few faint ones were found, any one of which might be considered merely a slight increase in intensity of the continuous spectrum. Later in the series they become more numerous

[24] The Spectra of Stars of Secchi's Fourth Type, *Pub. of Yerkes Ob.*, Vol. II, page 260.

[25] *Harvard Circular*, 76.

[26] We have measured the plates of 19 Piscium, +76° 734, and +34° 4500, all of which are comparatively early Class N stars.

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and are stronger, but are not so prominent on the plates of Class R stars as they are on Class N, especially on the plates of U Hydræ, the standard star of Class N.

Concerning the relative strength of carbon absorption in the two classes, we are able to compare the spectra of our series of Class R stars with the series of Class N stars of Hale, Ellerman, and Parkhurst.[27] Since the strength of the λ 4700 band was a determining factor in the arrangement of both sequences the series are practically duplicates. In both Classes R and N, we find stars in which the carbon absorption is relatively weak and others in which parallel degrees of intensity are evident down to the strong bands of $+57^{\circ}702$ (R) and 152 Schjellerup (N). Hale, Ellerman, and Parkhurst placed $+57^{\circ}702$ (R) at the end of their list of stars of type IV. It is evident, therefore, that the strength of carbon absorption is not a distinguishing feature between the two classes, but a factor which enters alike into the spectra of both types. We are thrown back, then, chiefly upon the criterion of general absorption in the violet to differentiate between the spectra of the two classes. This feature appears to be fundamental in the sequence B A F G K M. Although its application is difficult and fails to establish a definite line of demarkation between Classes R and N, the same difficulty is met in the case of other classes. In any natural system of classification, intermediate varieties frequently merge or overlap.

SUMMARY

Relation between Classes R and N. We shall now apply the accumulated data to the first part of our problem, viz., to establish a relationship between Classes N and R. Although the data are meager on some points and may be changed by later investigation, we shall use all in our possession. In Table VIII we tabulate a few results, using the solar type G as a basis of reference.

On all the points it appears that Class R precedes Class N, when the solar type is used as the point of reference.

The close resemblance of the spectra of Classes R and N, and the gradual transition from one to the other with indications of overlapping in the case of the data of individual stars, prove that the two classes are closely related.

These facts indicate clearly that stars belonging to Classes R and N form a continuous sequence with R preceding N.

The place of Classes R and N in the Evolutionary Sequence. We shall now apply our results to the second part of our problem, viz., the relationship between Classes R and N and the sequence B A F G K M. In approaching this problem, two alternatives immediately present themselves: Do the Classes R and N follow M and form one continuous evolutionary sequence, or do they constitute a separate branch of the sequence coordinate with K and M? A more detailed study of the spectra of Classes K and M would be valuable in this connection, but is outside the scope of the present investigation.

Let us first consider some general questions relating to the assumption that spectral type is a linear function, with time as the independent variable. Evidently we are not at liberty to introduce another independent variable, so we must postulate an equal mass of primordial matter for each star, possessing exactly the same properties, and *surrounded by the same conditions.* Then we might expect all the stars to pass through the same evolutionary stages. However, the unequal distribution of stars in space, the presence of nebulous matter in some regions, and the unequal masses of determined systems, indicate that the conditions imposed are imaginary rather than real.

An application of the theory of evolution in other realms of science affords room for the genus to develop into species along diverging branches, possessing common genetic characteristics and different specific characteristics. This type of development may also obtain in stellar evolution.

Hale[28] has summarized the common characteristics of stars of Secchi's types III and IV.[29] 1. Red color. 2. Tendency to variability. 3. Resemblance of dark lines. 4. Presence of lines widened in sun spots. 5. Similar physical con-

[27] *Pub. of Yerkes Obs.*, Vol. II, Plate VIII of *The Spectra of Stars of Secchi's Fourth Type.*

[28] *Pub. of Yerkes Obs.*, Vol. II, 385.

[29] Type III approximately includes Classes K and M; type IV includes R and N.

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ditions revealed by spectra. 6. Presence of bright lines. 7. Dark flutings; cyanogen flutings in common. 8. Connection of both types of spectra with spectra of solar stars. We may consider these to be the genetic characteristics.

The chief difference between the spectra of types III and IV is in the special absorption. Type III contains wide absorption bands sharp toward the violet and degraded toward the red, usually attributed to titanium oxide; type IV has similar bands, but reversed in appearance, being sharp toward the red, attributed to some form of carbon compound.[30] This constitutes the specific difference. We have, then, two types of spectra closely resembling each other, with respect to the line spectrum, with a different kind of absorption band superposed. These characteristics meet the conditions for coordinate branches of the evolutionary sequence.

The serious objection against this system of coordination raised by Pechule is that no examples of an intermediate type between the solar spectrum and type IV have been found. The spectra of the earliest stars of Class R seem to refute that objection. The carbon absorption of the first star of our series is comparatively weak, the spectrum extends as far as H and K, and the metallic lines give fair agreement with late solar type. The second star of the series according to Parkhurst has a spectrum resembling the solar type with the bands superposed. In this case the bands are stronger than in the preceding. The faintness of Class R stars and their scarcity lessen the probability of finding better intermediaries. Also, if the difference from the solar type were less, the probability of this difference being detected on a weak plate would be small.

The balanced condition of elements necessary to produce the Swan spectrum (distinguishing feature of Classes R and N) may lessen the probability that many stars exist with carbon bands. Baly[31] calls attention to the fact that the presence of a slight trace of oxygen in the source of light destroys the Swan spectrum due to carbon monoxide and produces the second band spectrum due to carbon dioxide.

However, in spite of these difficulties, as larger instruments are brought into use and the fainter red stars are more thoroughly studied, it is quite probable that examples will be found with weaker carbon bands than the first of the series photographed.

In addition to the qualitative similarity between types III and IV and the close correspondence between the earliest Class R stars and the solar type, we can make a comparison between the Classes R and K. These two classes should correspond quite closely, if the branching takes place following Class G. Perhaps R and K5 are more

TABLE VIII. COMPARISON OF DATA.

	G.	R.	N.
Color Index	1.02	1.7	2.5
In galactic region	58 per cent.	63 per cent.	87 per cent.
Number of variables		8 per cent.	20 per cent.
Residual radial velocity	15.15 km.	14.9 km.	13.2 km. per sec.
Shift in wave-length with type		Slight.	Increased.
Change in intensity of lines		Noted.	Same sense.
Bright lines		Few and weak.	More numerous and stronger.
Hydrogen, Hγ		Weak.	Weaker.
Hydrogen, Hβ		Present, and bright in variables.	Absent or bright.
Calcium, λ 4227	Present.	Widened.	Widened (Hale).
Cyanogen flutings	Absent.	Weak.	Stronger.

[30] Baly thinks the evidence is conclusive in favor of carbon monoxide. *Spectroscopy*, p. 444, 1905.

[31] *Spectroscopy*, p. 444, 1905.

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nearly parallel. We shall use data for K5 when available. A comparison follows showing a close correspondence.

	K OR K5	R.
Color index	1.64	1.7
Percentage in galactic region	56	63
Residual radial velocity, G and K	15.15	14.9 km. per sec.
Percentage of variables	Small	8

The correspondence is maintained also by the close similarity of the absorption lines, the shift in wave-length of certain lines with spectral type, the change in intensity of certain lines, absence or weakness of bright lines, weakness of cyanogen flutings, widening of λ 4227, and presence of hydrogen lines.

The close resemblance of the spectra of early Class R stars to the solar type, the parallelism between Classes R and K, together with the fulfillment of the conditions for coordinate branches by the two great classes of red stars (Secchi's types III and IV) lead to the conclusion of the second part of our problem, that the evolutionary sequence is divided, Classes K and M forming one branch and Classes R and N forming the other.

PHYSICAL CONDITIONS

The question arises: What physical conditions differentiate between stars of Classes N and R? Hale has interpreted the physical conditions of fourth type stars. Carbon vapor lies close to the photosphere above which are gases producing the bright lines of the spectra. The thick layer of dense atmosphere produces a strong general absorption. On the basis of this interpretation, the atmosphere of Class R stars is not so thick nor so dense as that of Class N stars and produces less general absorption, permitting the escape of a larger proportion of violet rays, which suffer most from general absorption.

Kapteyn[32] has accumulated much evidence bearing upon the question of the change of spectrum due to the absorption of light in space. Parallax data are not available to determine whether this effect is evident as a differentiating factor between the stars of Classes R and N.

[32] *Ap. J.* 40, 187, 1914.

Another question may arise: How can a solar type evolve two different types of stars? This question suggests another, the burden of proof of which rests with the advocate of the linear evolutionary sequence with R and N following M. How can a star, in which titanium oxide gives the characteristic feature of the spectrum, change into a star in which carbon monoxide predominates? An attempt to answer the first of these two questions seems more probable of success.

Given two stars of solar type under the same conditions, with equal masses, equal volumes, the same constituent elements, and surrounded by the same conditions in space, and the evolution of two different types would appear to be improbable. However, two stars of solar type drifting in different directions, as proper motion indicates, through time and space unlimited, may eventually encounter dissimilar conditions and differentiation may take place.

Moreover, it is quite probable that a differentiation begins before the solar type is reached. Nascent stars will be changed according to the properties of their own constituent elements and the characteristic elements of the matter pervading the surrounding region of space. Both of these factors may vary in different stars. If one region of space or a certain set of conditions is more favorable to the production of carbon and another to the existence of titanium, we may have in different stars of the solar type the right relationship between the various elements for the evolution of two different types of stars, one giving a spectrum in which the carbon bands predominate, the other producing a spectrum characterized by the bands of titanium oxide.

It is a curious fact that we find both elements, carbon and titanium, present in both classes of stars, type III with titanium bands and type IV with carbon bands. The type of spectrum apparently is due to the adjustment between the various elements. Possibly the predominance of carbon vapor in the form of carbon monoxide in the star's atmosphere suppresses the absorbing power of titanium oxide or *vice versa*. The discovery of a stellar spectrum showing the presence of both kinds of bands, carbon and titanium, would be both interesting and instructive. On

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account of the delicate balance of elements that would perhaps be necessary to produce such a hybrid, its existence is quite improbable. The delicate adjustment between carbon and oxygen suitable for the production of the Swan spectrum, which has previously been mentioned, may account for the comparatively small number of stars belonging to Classes R and N.

CONCLUSIONS

The radial velocities of ten Class R stars have been determined, with an average probable error ± 1 km. These radial velocities range from — 49 km. to + 25 km. per second, and yield an average residual radial velocity of 14.9 km. per second.

Wave-lengths of heads of bands, absorption lines and emission lines of Class R stars have been determined, (probable error of the mean in ten plates ± 0.05 Å) for the region λ 4185 to λ 5000.

The following substances were identified: Carbon (carbon monoxide and cyanogen), hydrogen, chromium, vanadium, titanium, iron, sodium, calcium, magnesium, nickel, and manganese.

Spectrograms of five Class O stars were made for comparison with Class R. No close resemblance was found.

Spectrograms of five Class N stars were obtained, part of which were measured and reduced, for qualitative comparison with Class R stars.

The chief difference between the spectra of stars of Class R and stars of Class N is due to general absorption.

Sixty-three per cent. of Class R stars are in the galactic region (+ 30° to — 30° galactic latitude).

The average color index of five Class R stars (Parkhurst) is 1.7.

The number marked variable in the Harvard tables is 8 per cent.

There is a shift in wave-length and change in intensity of certain lines with spectral type through the R N sequence in the same order as G to M.

Bright lines are not so numerous nor so strong in Class R as in Class N.

Class R precedes Class N in the evolutionary sequence taken in the usual order B A F G K M.

There is a close resemblance between the solar spectrum and the spectra of the first stars of the Class R series.

A marked correspondence exists between Classes R and K in color index, distribution with reference to galaxy, residual radial velocity, tendency to variability, similarity of absorption lines, bright lines, cyanogen flutings, λ 4227, and hydrogen lines.

The evolutionary sequence divides at the solar type, Classes K and M forming one branch, and Classes R and N constituting the other.

Stars of Class R form the connecting links between Class N and the solar type.

The writer wishes to express his grateful acknowledgement to Professor R. H. Curtiss for supervision and direction throughout this investigation, to various members of the Observatory staff for assistance and encouragement, and to those who so kindly arranged the programs of work with the 37½-inch Reflector to make possible the observations on which this study is based.

Ann Arbor, Michigan, June, 1915.

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A STUDY OF BETA CEPHEI[1]

By CLIFFORD C. CRUMP

INTRODUCTION

The study of a spectroscopic binary of very short period is of especial interest because of the material it affords for the furtherance of our knowledge of stellar evolution. Through the observation of such a system we are able to derive data with reference to the development of binaries in their early stages and to investigate the laws which govern their evolution. If a star in addition to being a short period binary be a variable of the δ Cephei type the interest is enhanced, for while there are considerable data and many theories concerning Cepheid stars, astronomers agree that a satisfactory solution of the problem of their variation has not been reached and all contributions to the facts already known are particularly valuable.

It is the purpose of this paper to give the results of the study of β Cephei, which is both a short period binary and a variable of the δ Cephei type. The period of β Cephei is the shortest that has thus far been determined for a spectroscopic binary, namely, 4 hours 34 minutes and 17.4 seconds, enabling one to follow it for more than a revolution in a single night. It is remarkable that plates taken with the time of mid-exposure only six minutes apart show a measurable change in the radial velocity of this star. The shortness of the period and the brightness of the star, 3.3 magnitude, make it possible to determine in a comparatively short time the definitive elements of the orbit with the observations grouped around a well defined epoch. It is also possible to compare readily complete cycles of the curve. The recent investigations of the light variation of β Cephei by Guthnick of the Berlin-Babelsberg Observatory have added to the importance of a complete spectroscopic study of this star.

It is proposed to determine definitively the orbital elements of β Cephei and to bring together the existing data concerning it.

[1] A dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of Michigan.

TOTAL LIGHT

As a result of the photoelectric measures of the light variation of β Cephei, Guthnick has announced the star to be a variable. He reaches this conclusion from a series of observations which extend over six months and comprise one hundred and seventy-seven determinations of the light intensity. The light curve plotted from the normal places of his observations indicates an amplitude of 0.05 magnitude as measured with the sodium cell and a range in time from maximum to minimum light of 0.09 days, a little more than one-half the spectroscopic period. The light curve determined by Guthnick compared with the radial velocity curve as observed by Frost showed, on account of the position of the points of maximum and minimum light, that the star was not an Algol nor a β Lyræ variable. The general form of the light curve and the degree of agreement of the point of maximum light with the point of maximum velocity of approach led to the assumption that this variable was of the δ Cephei type. Plate IX, Figure 2, shows the light curve plotted from Guthnick's observations, which may easily be compared with Figure 1 of the same plate, which shows the velocity curve determined at the Detroit Observatory.

According to the general characteristics of Cepheid variables the maximum of light should coincide approximately with the maximum velocity of approach. In most cases this agreement lies within one-fifteenth of the period. Here we observe the difference in time to be 0.029 days, or about one-seventh of the period, a quantity approximately equal to that observed in the case of the Cepheid variable, Y Sagittarii, but with light maximum occurring on the descending branch instead of the ascending branch of the velocity curve. In β Cephei the variation from the general rule of close coincidence between light maximum and maximum velocity of approach may be connected with the closeness of the components of the system. The correspondence between the minimum of light and the mini-

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mum velocity of approach in the cases so far observed has not been so close on the average as that between the maximum of light and the maximum velocity of approach. The discrepancy between light minimum and the maximum velocity of recession in the case of β Cephei, though it amounts to about one-fifth of the period, should not exclude it from the δ Cepheid class. This disagreement may be due here and in other cases to variations caused by the ellipsoidal form of the members of the system.

HISTORICAL

From December 18, 1901, to April 16, 1902, six spectrograms of β Cephei ($\alpha = 21^h\ 27^m$, $\delta = +70^\circ\ 7'$) were obtained by Mr. W. S. Adams with the Bruce spectrograph of the Yerkes Observatory. The measures of the third plate of this series suggested a variable radial velocity for this star. This suggestion was confirmed by subsequent investigations of Professor Edwin B. Frost and the binary character of the star was established. An early set of radial velocity determinations showed a range of from — 20.3 km. to + 11.3 km. The period was at first thought to be rather long, but Professor Frost, from two spectrograms taken on the night of May 14, 1902, during an interval of five and one-half hours, observed a change in velocity of 14 km. and inferred that the period was short. Owing to the insufficient data at hand Professor Frost at that time made no attempt to draw further conclusions.

In the spring of 1906 the problem was again taken up by Professor Frost with the idea of determining the period. On the nights of May 18, 19, 20, and 21, eight plates were secured. The approximate measures of these plates indicated that the period was about four hours and thirty minutes. The star was continuously observed through several nights, and from May 28 to August 27 more than sixty plates were made. From the observations of July 6, the following elements were derived:

ELEMENTS OF JULY 6, 1906.

$P = 0.190479$ days,
$e = 0.03 +$,
$\omega = 165^\circ\ 13'$,
$T = 20^h\ 37^m$ G. M. T., or J. D.2417398.859,
$K = 16.9$ km.,
$\gamma = -4.9$ km.

At the meeting of the National Academy of Sciences in December, 1914, Professor Frost gave a brief discussion of this system. In a letter which I have recently received he gives the value of γ on the night of April 29, 1912, as — 10.2 km. The discrepancy between the velocity of the center of mass as given for July 6, 1906, and that determined for this later date is of particular significance and will be discussed in connection with the elements derived by the writer.

OBSERVATIONS

Observations of β Cephei were begun at the Detroit Observatory on August 8, 1912, and were continued until November 21, of that year. During this interval 175 spectrograms were made with the one-prism spectrograph attached to the 37½-inch Reflector. Of the total number of spectrograms obtained 163 were used in the determination of the radial velocities. For the most part Seed 23 plates were used. On the night of October 15 Seed 30 plates were used, with the exception of plates 1247 B, 1249 C, 1250 D, and 1269 B, which were made on Seed 23 plates. The time of exposure varied from five to fifteen minutes. On November 7, conditions being very favorable, thirty-six plates were secured with an average exposure of six minutes. The last series of observations, November 21, was taken with a narrow slit for the purpose of searching the spectrum for double lines. It was the aim from the beginning to follow the star continuously through the period assumed by Frost and on four different nights we were successful in this endeavor.

SPECTRUM

The spectrum of β Cephei is of the Orion type. It is designated in the *Harvard Annals,* Vol. 50, by B1, and in Miss Maury's classification in the *Harvard Annals,* Vol. 28, by IIIa. The chief lines of the spectrum are due to helium, hydrogen, oxygen, and silicon. For a spectrum of this class the lines are well defined and on the whole easy to measure. The series of plates taken with a narrow slit for the purpose of detecting a second component's spectrum gave no evidence that such a spectrum existed. The region used in this investigation extended from λ 3933 to λ 4922. The comparison spectrum was that of the titanium spark.

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MEASUREMENT

A part of the spectrograms, namely those of the night of November 21, were measured with Measuring Engine No. 1 of this Observatory, and the remainder of the plates were measured at the Observatorio Nacional de La Plata, Argentina, with a Gærtner engine belonging to the Cordoba Observatory. The screw error of both these engines was so small that correction of the micrometer readings was unnecessary. Twenty-two comparison lines were used on each plate. Three settings were made on the upper comparison line and three on the lower, and the average of these readings was taken as the reading for that line. The average of three settings on the star line was taken as the reading for this line. The correction for curvature of the spectral lines, although small, was taken into account in the reductions. All of the plates were measured direct and reversed in order to eliminate personal equation as far as possible.

WAVE-LENGTHS

Twenty-two star lines were selected for use in the determination of the radial velocities. The wave-lengths were taken from published tables containing such quantities and are incorporated in Table I. Wherever a line was used which could not be identified in some published table the wave-length was computed from the well known Hartmann Formula,

$$\lambda = \lambda_0 + \frac{c}{R_0 - R}.$$

The constants of this formula for the spectrograph used had previously been determined by Mr. Mellor, whose results are published on Volume 1, page 140, of the *Publications of the Detroit Observatory*. The standard of dispersion used by Mr. Mellor having been adopted, the constants determined by him have been used in these reductions.

RADIAL VELOCITIES

The velocities were determined from the measures in the usual manner. Corrections were applied for the orbital velocity of the earth, for the earth's rotation, for the curvature of the lines, and for the small increments which were applied to the wave-lengths of the several star lines to make them homogeneous.

TABLE 1. WAVE-LENGTHS OF LINES MEASURED.

ASSUMED WAVE LENGTH.	WAVE-LENGTH IN β CEPHEI.	AUTHORITY.	ELEMENT.
Å	Å		
3933.83	3933.78	Rowland.	K, Calcium.
3964.88	3964.84	Runge and Paschen.	Helium.
3968.62	3968.65	Rowland.	He, Hydrogen.
3973.48	3973.40	Eisig.	Oxygen.
4009.12	4009.16	Runge and Paschen.	Helium.
4026.37	4026.37	Runge and Paschen.	Helium.
4070.05	4069.94	Crump.	
4072.16	4072.06	Crump.	
4076.08	4076.01	Exner and Haschek.	Oxygen. (?)
4089.00	4089.16	Lunt.	Silicon.
4101.92	4101.92	Curtiss.	Hδ, Hydrogen.
4121.02	4120.92	Runge and Paschen.	Helium.
4143.92	4143.93	Runge and Paschen.	Helium.
4253.81	4254.01	Crump.	
4267.15	4267.22	Eder and Valenta.	Carbon.
4340.63	4340.64	Rowland.	Hγ, Hydrogen.
4388.10	4388.06	Runge and Paschen.	Helium.
4471.68	4471.65	Runge and Paschen.	Helium.
4552.76	4552.82	Albrecht.	Silicon.
4713.31	4713.53	Runge and Paschen.	Helium.
4861.53	4861.56	Rowland.	Hβ, Hydrogen.
4922.10	4922.10	Runge and Paschen.	Helium.

The correction for curvature of the spectral lines, which was obtained on the assumption that the lines were parabolic in form, was made from measures of well defined lines. This correction although small was applied to each determination.

The correction for making the lines homogeneous was made in the following manner. Residuals were found from a comparison of the velocity as given by the line and the velocity as given by the plate. A correction was then applied to each line which would reduce to zero the algebraic sum of the residuals of that line for all of the plates on which it was measured.

The method used in determining the final weights for the lines has been described by Professor Curtiss in his article on δ Orionis, in the *Publications of the Detroit Observatory*, Volume 1, page 123.

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Table II contains the observed velocity data. Column 1 gives the number of the plate; column 2, the time of mid-exposure reckoned in days from January 0.0, 1912; column 3, the phase computed with the period determined below, 0.1904795 days, and referred to the epoch of the first observation, 1912, August 23,7576 G.M.T.; column 4, the number of lines measured; column 5, the weight assigned to the plate; column 6, the radial velocity; column 7, the residuals from the final elliptic elements; and column 8, general remarks.

TABLE II. ANN ARBOR OBSERVATIONS.

PLATE NO.	G. M. T. 1912.	PHASE.	NO. OF LINES.	WT.	VELOCITY.	O–C	REMARKS.
					km.	km.	
1019 D	236.7576	0.0000	15	13	—21.07	—3.6	
1023 B	239.7847	0.1699	15	11.5	—19.65	+6.2	
1027 B	239.8321	0.0268	17	12.5	— 0.33	+3.7	
1028 C	239.8448	0.0395	17	14	— 2.50	—2.3	
1029 A	239.8601	0.0548	11	6.5	+ 2.84	+0.6	
1035 A	240.6979	0.1307	19	15	—27.86	—0.8	
1038 C	242.6437	0.1717	19	22	—23.58	+1.7	
1039 D	242.6528	0.1808	21	31	—27.72	—6.1	
1040 A	242.6618	0.1898	19	25.5	—22.05	—4.6	
1041 B	242.6799	0.0174	18	18.5	—11.55	—3.2	
1042 C	242.6896	0.0271	17	10.5	— 5.61	—1.9	
1043 D	242.6999	0.0374	17	19	— 1.38	—1.3	
1044 A	242.7125	0.0500	17	16.5	— 0.52	—2.8	
1045 B	242.7243	0.0618	19	18.5	— 2.11	—3.6	
1116 B	275.5646	0.1397	15	26.5	—35.32	—7.5	
1117 C	275.5736	0.1487	17	21.5	—34.73	—5.4	
1118 D	275.5833	0.1584	17	41	—34.23	—5.7	
1119 A	275.5924	0.1675	17	36.5	—29.94	—3.3	
1120 B	275.6000	0.1751	16	29	—25.86	—1.9	
1121 C	275.6090	0.1841	15	23	—16.51	+3.6	Poor.
1122 D	275.6159	0.0005	13	18.5	—16.54	+0.6	
1123 A	275.6250	0.0096	17	24.5	— 9.43	+3.0	
1124 B	275.6317	0.0163	12	16.5	—10.77	—1.9	
1125 C	275.6410	0.0256	14	34.5	— 3.39	+1.0	
1126 D	275.6486	0.0332	16	31	+ 3.08	+4.4	
1127 A	275.6562	0.0408	16	23.5	+ 5.56	+4.7	
1128 B	275.6639	0.0485	20	24	+ 9.51	+7.4	
1129 C	275.6722	0.0568	18	31.5	+ 9.90	+7.7	
1130 D	275.6771	0.0617	18	31	+10.13	+8.6	
1131 A	275.6875	0.0721	16	38	— 0.43	+0.8	
1132 B	275.6969	0.0815	18	38.5	— 8.67	—3.6	
1133 C	275.7049	0.0895	18	27.5	— 7.15	+1.8	
1134 D	275.7130	0.0976	15	25	—16.10	—3.0	
1135 A	275.7211	0.1057	17	29.5	—11.69	+5.4	
1136 B	275.7285	0.1131	19	22	—16.35	+4.4	
1137 C	275.7361	0.1207	16	26	—21.34	+2.6	
1138 D	275.7437	0.1283	15	18.5	—27.31	—0.8	
1139 A	275.7521	0.1367	18	25	—32.75	—4.4	
1140 B	275.7604	0.1450	18	26	—28.63	+0.5	
1141 C	275.7705	0.1551	19	23	—23.40	+5.6	

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TABLE II. ANN ARBOR OBSERVATIONS—Continued

PLATE NO.	G. M. T. 1912.	PHASE.	NO. OF LINES.	WT.	VELOCITY.	O–C.	REMARKS.
1142 D	275.7802	0.1648	18	27.5	−23.56	+3.7	
1247 B	289.5579	0.0375	15	38	+ 5.10	+5.1	
1248 A	289.5642	0.0438	15	22.5	+ 0.55	−0.9	
1249 C	289.5712	0.0508	18	27	− 0.48	−2.7	
1250 D	289.5802	0.0598	18	35	+ 5.52	+3.8	
1251 A	289.5885	0.0681	14	19	− 2.33	−2.3	
1252 B	289.5962	0.0758	13	14.5	− 2.19	+0.4	Poor.
1253 C	289.6017	0.0813	15	33	− 0.08	+4.9	
1254 D	289.6066	0.0862	14	26.5	− 5.41	+2.0	
1255 A	289.6121	0.0917	16	22.5	− 8.82	+1.3	
1256 B	289.6194	0.0990	17	36.5	−15.41	−1.4	
1257 C	289.6302	0.1098	14	24	−22.06	−2.8	
1258 D	289.6351	0.1147	16	29	−17.73	+3.7	
1259 A	289.6392	0.1188	14	14.5	−23.99	−0.7	
1260 B	289.6434	0.1230	15	22.5	−32.51	−7.7	
1261 C	289.6479	0.1275	11	15.5	−33.48	−7.3	Poor.
1262 D	289.6521	0.1317	15	16.5	−32.22	−4.8	
1263 A	289.6562	0.1358	11	14	−29.19	−0.9	Poor.
1264 B	289.6604	0.1400	11	23.5	−27.23	+1.6	
1265 C	289.6646	0.1442	13	15	−23.30	+6.0	
1266 D	289.6694	0.1490	15	18	−33.49	−4.2	
1267	289.6748	0.1544	12	18	−25.20	+3.8	
1268 A	289.6794	0.1590	13	24	−36.76	−8.4	
1269 B	289.6864	0.1660	19	48	−20.25	+6.8	
1288 A	291.6958	0.0801	15	22.5	− 8.56	−4.0	
1289 B	291.7062	0.0905	19	34.5	− 3.78	+5.8	
1290 C	291.7152	0.0995	15	25	−13.11	+1.0	Poor.
1291 D	291.7243	0.1086	15	12	−11.59	+7.0	Poor.
1292 A	291.7333	0.1174	15	20	−18.28	+4.3	
1293 B	291.7424	0.1267	20	15.5	−28.21	−2.2	Poor.
1294 C	291.7514	0.1357	18	18	−28.73	−0.5	
1295 D	291.7611	0.1454	18	22	−28.66	+0.6	
1296 A	291.7701	0.1544	20	17.5	−20.71	+8.3	
1297 B	291.7792	0.1635	18	18	−29.41	−1.9	
1298 C	291.7882	0.1725	16	18.5	−21.64	+3.5	
1299 D	291.7972	0.1815	16	18.5	−14.42	+7.0	
1304 D	298.6111	0.1381	17	31.5	−29.19	+0.5	
1305 A	298.6229	0.1499	17	24.5	−29.29	+0.0	
1306 B	298.6340	0.1610	17	26	−26.54	+1.5	
1307 C	298.6414	0.1684	18	27.5	−24.41	+1.9	
1308 D	298.6479	0.1749	16	26.5	−21.23	+3.0	
1309 A	298.6555	0.1825	18	25	−23.94	−3.0	
1310 B	298.6590	0.1860	16	15	−16.32	+3.0	Poor.
1311 C	298.6653	0.0019	16	23.5	−10.85	+6.1	
1312 D	298.6705	0.0071	17	29.5	−11.21	+2.5	

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TABLE II. ANN ARBOR OBSERVATIONS—Continued.

PLATE NO.	G. M. T. 1912.	PHASE.	NO. OF LINES.	WT.	VELOCITY.	O–C.	REMARKS.
					km.	km.	
1313 A	298.6746	0.0112	20	27.5	— 10.13	+ 1.4	
1314 B	298.6790	0.0156	19	25.5	— 2.97	+ 6.3	
1315 C	298.6830	0.0196	17	26.5	— 10.78	— 3.5	
1316 D	298.6871	0.0237	17	26.5	— 5.29	+ 0.1	
1317 B	298.6969	0.0335	18	40	+ 3.34	+ 4.6	
1318 C	298.7010	0.0376	14	17	— 4.78	— 4.8	
1319 D	298.7052	0.0418	14	21	+ 2.42	+ 1.4	
1320 A	298.7101	0.0467	16	21	+ 0.69	— 1.2	Poor.
1321 B	298.7163	0.0529	18	24.5	— 0.77	— 3.0	
1322 C	298.7212	0.0578	18	27	+ 1.74	— 0.2	
1323 D	298.7260	0.0626	19	27.5	+ 4.14	+ 2.8	
1324 A	298.7302	0.0668	17	35	— 7.15	— 7.4	
1325 B	298.7344	0.0710	19	44.5	— 3.13	+ 2.2	
1326 C	298.7385	0.0751	17	29.5	— 7.19	— 4.8	
1327 D	298.7434	0.0800	19	40	— 4.48	+ 0.0	
1328 A	298.7483	0.0849	19	31.5	— 11.03	— 4.3	Poor.
1330 C	298.7573	0.0939	18	37	— 7.99	+ 3.3	
1331 D	298.7621	0.0987	19	31.5	— 19.19	— 5.6	
1332 A	298.7670	0.1036	17	27	— 8.69	+ 7.6	
1333 B	298.7746	0.1112	20	39	— 22.06	— 2.0	
1334 C	298.7805	0.1171	20	44.5	— 29.30	— 6.8	
1335 D	298.7941	0.1307	19	34.5	— 30.57	— 3.5	
1336 A	298.7983	0.1359	8	17	— 24.14	+ 4.1	Poor.
1337 B	298.8031	0.1397	20	37.5	— 27.54	+ 1.3	
1338 C	298.8073	0.1439	19	42.5	— 27.60	+ 1.6	
1410 C	312.5437	0.1657	15	27	— 28.53	— 1.5	Poor.
1411 B	312.5528	0.1748	18	27	— 24.70	— 0.4	
1412 D	312.5576	0.1796	19	34.5	— 20.77	+ 1.5	
1413 A	312.5628	0.1848	20	27.5	— 24.82	— 4.9	
1414 B	312.5677	0.1897	17	24.5	— 19.90	— 2.2	
1415 C	312.5727	0.0043	17	28.5	— 18.13	— 0.9	
1416 D	312.5788	0.0104	20	31	— 15.81	— 3.8	
1417 A	312.5837	0.0153	17	21.5	— 5.34	+ 4.1	
1418 B	312.5896	0.0212	16	30	— 11.42	— 4.9	
1419 D	312.5944	0.0260	16	25	— 1.02	+ 3.2	
1420 D	312.5996	0.0312	17	31	— 4.22	— 2.2	
1421 A	312.6045	0.0361	16	23.5	— 2.27	— 1.8	Poor.
1422 B	312.6097	0.0413	18	32	+ 4.99	+ 4.1	
1423 C	312.6146	0.0462	14	18.5	— 1.73	— 3.6	
1425 A	312.6246	0.0562	18	34.5	+ 0.69	— 1.3	
1426 B	312.6299	0.0615	17	27.5	— 3.63	— 5.1	
1427 C	312.6399	0.0715	16	25.5	— 5.58	— 4.6	
1428 D	312.6448	0.0764	14	25	— 5.14	— 2.3	
1429 A	312.6500	0.0816	16	26	— 10.69	— 5.6	
1430 B	312.6548	0.0864	13	17.5	— 14.22	— 6.7	Poor.

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TABLE II. ANN ARBOR OBSERVATIONS—Continued.

PLATE NO.	G. M. T. 1912.	PHASE.	NO. OF LINES.	WT.	VELOCITY.	O–C.	REMARKS.
					km.	km.	
1431 C	312.6601	0.0917	15	22	— 17.22	— 7.1	
1432 D	312.6649	0.0965	13	21.5	— 14.04	— 1.4	
1433 A	312.6701	0.1017	17	25.5	— 20.95	— 5.8	
1434 B	312.6750	0.1066	15	24.5	— 23.18	— 5.7	
1435 C	312.6861	0.1177	15	20.5	— 24.66	— 2.0	
1436 D	312.6920	0.1236	18	25	— 28.50	— 3.5	
1437 A	312.6976	0.1292	18	21	— 25.72	+ 1.0	
1438 B	312.7028	0.1344	19	25.5	— 27.85	— 0.2	
1439 C	312.7076	0.1392	19	24	— 35.23	— 6.5	
1440 D	312.7128	0.1444	16	20.5	— 32.45	— 3.2	
1441 A	312.7177	0.1493	18	26.5	— 31.65	— 2.3	
1442 C	312.7229	0.1545	19	34	— 27.08	+ 1.9	
1443 D	312.7277	0.1593	18	26	— 22.90	+ 5.4	
1444 D	312.7337	0.1653	14	19.5	— 28.58	— 1.5	
1445 B	312.7384	0.1700	7	8.5	— 21.07	+ 5.9	Poor.
1446 C	312.7451	0.1767	11	8	— 21.57	+ 2.0	Poor.
1481 B	326.6246	0.1512	22	34.5	— 32.48	— 3.2	
1482 C	326.6358	0.1624	21	32.5	— 25.87	+ 1.9	
1483 D	326.6469	0.1735	20	20	— 22.54	+ 2.2	
1484 A	326.6580	0.1846	21	22	— 17.48	+ 2.5	
1485 B	326.6691	0.0052	21	21.5	— 18.06	— 3.3	
1486 C	326.6802	0.0163	21	21.5	— 16.72	— 7.6	Poor.
1487 D	326.6913	0.0274	20	38.5	— 7.17	— 3.6	
1489 A	326.7024	0.0385	19	32.5	— 0.05	+ 0.1	
1490	326.7135	0.0496	17	26.5	— 1.84	— 4.0	
1491 C	326.7246	0.0607	18	28	+ 0.09	— 1.6	
1492 D	326.7358	0.0719	18	27.5	— 0.69	+ 0.4	
1493 A	326.7469	0.0830	17	25.5	— 9.59	— 3.8	
1494 B	326.7580	0.0951	18	27	— 5.20	+ 6.8	
1495 C	326.7691	0.1052	17	23.5	— 20.69	— 3.6	
1496 D	326.7913	0.1274	17	27	— 18.14	+ 8.0	
1497 A	326.8024	0.1385	14	19.5	— 25.34	+ 3.3	
1498 B	326.8184	0.1545	17	28.5	— 25.63	+ 3.4	

PERIOD

The period as determined by Frost was used as the basis for obtaining the period which has been used in this discussion. In obtaining the new period determinations were made of the epochs of the cross-points with the center of mass axis of the velocity curves obtained on the different nights at this Observatory. These values were compared with the epochs for the points as given by the curve drawn by Frost for July 6, 1906. The differences between the corresponding times of these determinations divided by the assumed period gave the number of cycles which had elapsed between these two dates and thereby a means was afforded for the correction of the

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assumed period. The following values of the period were obtained by this method.

The mean period of 0.1904795 days was adopted and used throughout this investigation.

PERIOD DETERMINATIONS.

DATE.	PERIOD.
August 29, 1912	0.1904804 days.
October 1, 1912	0.1904792 days.
October 1, 1912	0.1904796 days.
October 24, 1912	0.1904794 days.
October 24, 1912	0.1904792 days.
November 7, 1912	0.1904784 days.
November 7, 1912	0.1904796 days.
November 21, 1912	0.1904799 days.
November 21, 1912	0.1904799 days.
Mean	0.1904795 days.

NORMAL PLACES

The observed radial velocities given in column 6 of Table II were combined into normal places with phase as the basis for grouping. One hundredth of a day was the phase interval over which each normal place extended and accordingly nineteen normal places were formed. The weight for each place was determined on the basis of the sum of the weights of the plates combined to make the normal place. The largest of these sums was taken as unity and the rest reduced to this basis. The phases were reduced to the sun.

The normal places were plotted and a smooth curve was drawn through them. Correction to the normal place velocity for curvature of the curve was then investigated at the maximum and minimum points where such a correction would make itself most manifest. This correction proved to be very small and was not needed outside these regions. Within the limits where it was appreciable it was applied.

The following table gives the data for the normal places.

TABLE III. NORMAL PLACES.

NO.	PHASE.	LIMITS OF PHASE.	VELOCITY.	O–C.	WT.	NO. OF PLATES.
1	0.0056	0.00 to 0.01	−14.48	+0.11	0.522	7
2	0.0166	0.01 to 0.02	−10.07	−1.19	0.616	8
3	0.0269	0.02 to 0.03	− 3.88	+0.08	0.583	7
4	0.0367	0.03 to 0.04	+ 0.38	+0.67	0.808	9
5	0.0464	0.04 to 0.05	+ 2.84	+0.98	0.621	8
6	0.0572	0.05 to 0.06	+ 2.83	+0.71	0.665	8
7	0.0648	0.06 to 0.07	+ 0.01	−0.91	0.612	7
8	0.0748	0.07 to 0.08	− 3.37	−1.17	0.672	7
9	0.0850	0.08 to 0.09	− 7.50	−0.77	0.947	10
10	0.0973	0.09 to 0.10	−11.83	+1.24	0.928	10
11	0.1074	0.10 to 0.11	−17.09	+1.14	0.545	7
12	0.1178	0.11 to 0.12	−22.57	+0.33	0.622	7
13	0.1273	0.12 to 0.13	−26.26	−0.06	0.561	8
14	0.1380	0.13 to 0.14	−30.04	−1.48	1.000	13
15	0.1472	0.14 to 0.15	−29.65	−0.36	0.788	10
16	0.1575	0.15 to 0.16	−28.43	+0.21	0.809	9
17	0.1672	0.16 to 0.17	−25.11	+1.55	0.928	11
18	0.1768	0.17 to 0.18	−22.49	+1.02	0.609	8
19	0.1863	0.18 to 0.19	−21.07	−1.62	0.696	9

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LEAST SQUARE SOLUTIONS

A definitive discussion of the orbit requires that the most probable value of ω and of T be found. Both of these values can not be conveniently determined from the least square solution on account of the low eccentricity. From an intensive study of the observations in which different values of ω were assumed and the corresponding values of T computed by the method of least squares the preliminary elements which are given below were adopted for the final least square solution.

PRELIMINARY ELEMENTS.

$P = 0.1904795$ days,
$T = 0^d.0543$ Phase, or
1912, August 23.8119 G. M. T.,
$e = 0.044$,
$\omega = +5^\circ .175$,
$K = 16.125$ km.,
$\gamma' = -13.38$ km.

The method used in the least square solution is essentially that outlined by Dr. Schlesinger in Volume I of the *Allegheny Publications*. Assuming the constancy of the period and the epoch of periastron and employing at Professor Curtiss's suggestion $d\gamma'$, the correction to the mean velocity, instead of Γ as an unknown, the normal equations were formed from which the corrections to the elements were found. The normal equations are as follows:

NORMAL EQUATIONS.

$$+13.532d\gamma' - 1.083\kappa + 0.207\pi + 5.732\epsilon + 0.177 = 0,$$
$$+7.137\kappa + 0.316\pi + 0.122\epsilon + 1.965 = 0,$$
$$+6.395\pi + 0.318\epsilon - 4.499 = 0,$$
$$+3.808\epsilon - 0.121 = 0.$$

From these equations the values of the unknown quantities were found to be

$$\epsilon = +0.161,$$
$$\pi = +0.7157,$$
$$\kappa = -0.3277,$$
$$d\gamma' = -0.1185.$$

The corrections to the elements were now obtained and applied in the manner indicated. The corrections to the elements are as follows:

$$\Delta e = -0.0045,$$
$$\Delta\omega = -2^\circ .543,$$
$$\Delta K = -0.327 \text{ km.},$$
$$\Delta\gamma' = -0.1185 \text{ km.}$$

By adding these corrections to the *Preliminary Elements* we obtain the following

FINAL ELEMENTS.

$P = 0.1904795$ days.
$T = 0^d.0543$ Phase, or
1912, August 23.8119 G. M. T.,
$e = 0.040 \pm 0.015$,
$\omega = +2^\circ .632 \pm 0^\circ .917$,
$K = 15.798 \pm 0.24$ km.,
$\gamma' = -13.4985 \pm 0.21$ km.,
$a \sin i = 40{,}541$ km.,
$\gamma = -14.13$ km.

A second ephemeris was now computed on the basis of the corrected elements and the residuals formed in the usual sense. These residuals were compared with the corrected residuals of the first ephemeris. The resulting differences, the largest of which was $+0.04$ km., showed that a second solution was unnecessary. The sum of the weighted squares of the residuals was reduced from 17.10 to 13.39. The change in this quantity is small, which shows that the observed velocities were well represented by the assumed elements. The probable error of a normal place of weight unity was determined as ± 0.637 km. The probable error of a single plate was ± 2.95 km.

The residuals of the normal place velocities computed from the final elements are found in column 5 of Table III. A glance at these residuals reveals at once an alternation of positive and negative values, suggestive of a secondary oscillation. In Fig. 1 of Plate IX the normal place observed velocities are plotted as small circles, and the velocity curve corresponding to the final elements is drawn as a continuous line. The regular oscillation of the observed velocities above and below the curve corresponding to the final elliptic elements seems well indicated. In this figure the horizontal heavy line represents the mean velocity of the system. The line of long and short dashes oscillating about this horizontal line represents the secondary variation, which when superposed upon the elliptic velocity curve yields the curve of short dashes. This composite curve evidently satisfies the plotted velocities better than the elliptic curve.

At this point the advisability is suggested of a further least square solution containing terms

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determinative of the elements of the secondary oscillation. It is hoped that this solution may be made in connection with further studies of this star. For the present in drawing the curves of Fig. 1, Plate IX, after several trials in which the period of the secondary oscillation was taken to be one-third the major period, the semi-amplitude was assumed to be 1.25 km. For the present these values of the elements of the secondary oscillation may be taken as close approximations to definitive values.

OBSERVATIONS OF OCTOBER 1, 1912.

The observations made on the night of October 1, 1912, show a shift of both the points of maximum and minimum from those which were generally observed. The spectrograms secured on this night have been investigated for possible blend effects with negative results. These spectrograms seemed not to be different from those taken on other nights. The lines of the spectrum of the star seemed to be of the same average intensity and the average weight of a plate of this set was in close agreement with the average weight of a plate of a set taken on the night of October 24, which showed no variation from the generally observed results. The Observations of October 1, when plotted, (see Plate X,) seemed to be well satisfied by two straight lines. However, this may have been due to accident and motion in a conic section has been assumed in the reductions.

In order to investigate more thoroughly whether the divergence of the velocities of October 1 from the mean curve was real or due to error in measurement, or to other sources, the data were plotted in Plate X, a set of preliminary elements was computed, and a least square solution was undertaken. The assumed elements, determined by the 45° chordal point method, (see page 178 of this volume,) were as follows:

PRELIMINARY ELEMENTS.

$P = 0.1904795$ days,
$\mu = 1890° .0$,
$T = 0^d.0628$ Phase,
$e = 0.045$,
$\omega = +13°.53$,
$K = 20.31$ km.,
$\gamma' = -11.69$.

The usual method of procedure was followed, the residuals formed, and the normal equations written as indicated.

NORMAL EQUATIONS.

$$\begin{aligned} 11.194 d\gamma' - 1.286_K - 0.075\pi + 4.200e + 1.687 &= 0, \\ + 6.323_K + 0.380\pi + 0.014e + 7.076 &= 0, \\ + 4.871\pi + 0.165e + 5.620 &= 0, \\ + 2.549e + 1.679 &= 0. \end{aligned}$$

From these equations the values of the unknowns were determined as follows:

$$\begin{aligned} e &= -0.308, \\ \pi &= -1.062, \\ K &= -1.088, \\ d\gamma' &= -0.167. \end{aligned}$$

and the corresponding corrections to the elements are

$$\begin{aligned} \Delta e &= +0.007, \\ \Delta\omega &= +2° .996, \\ \Delta K &= -1.088 \text{ km.}, \\ \Delta\gamma' &= -0.167 \text{ km.} \end{aligned}$$

Adding these corrections to the preliminary elements given above we get the

FINAL ELEMENTS FOR OCTOBER 1, 1912.

$P = 0.1904795$ days,
$T = 0^d.0628$ Phase, or
1912, August 23.8204 G. M. T.,
$e = 0.052 \pm 0.058$,
$\omega = +16° .526 \pm 3° .438$,
$K = 19.222 \pm 0.741$ km.,
$\gamma' = -11.857 \pm 0.744$ km.,
$\gamma = -12.88$ km.

An ephemeris was computed from these elements and the residuals formed in the usual manner. These quantities compared with the corrected residuals formed from the ephemeris based on the assumed elements showed that a second solution was unnecessary. The sum of the weighted squares of the residuals was reduced from 95.300 to 85.710. The velocity curve corresponding to these final elements is compared with the observations and the *mean curve* in Plate X.

The difference between the value of K from the mean curve and that for October 1, 1912, is considerably greater than the sum of the probable errors of the determinations. The indication is that the difference is not due to accidental error and that the change in amplitude on Octo-

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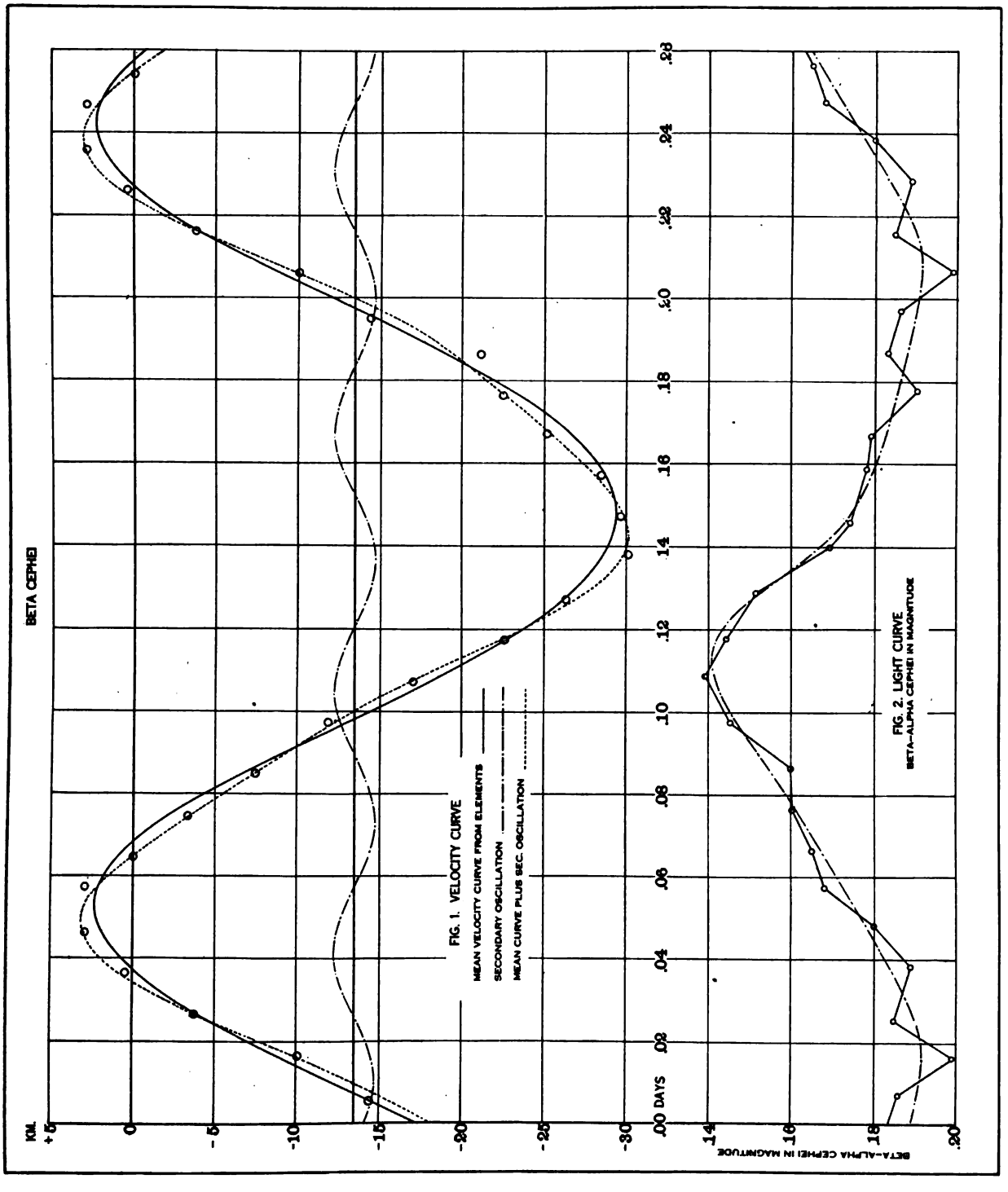

PLATE IX. VELOCITY AND LIGHT CURVES OF β CEPHEI

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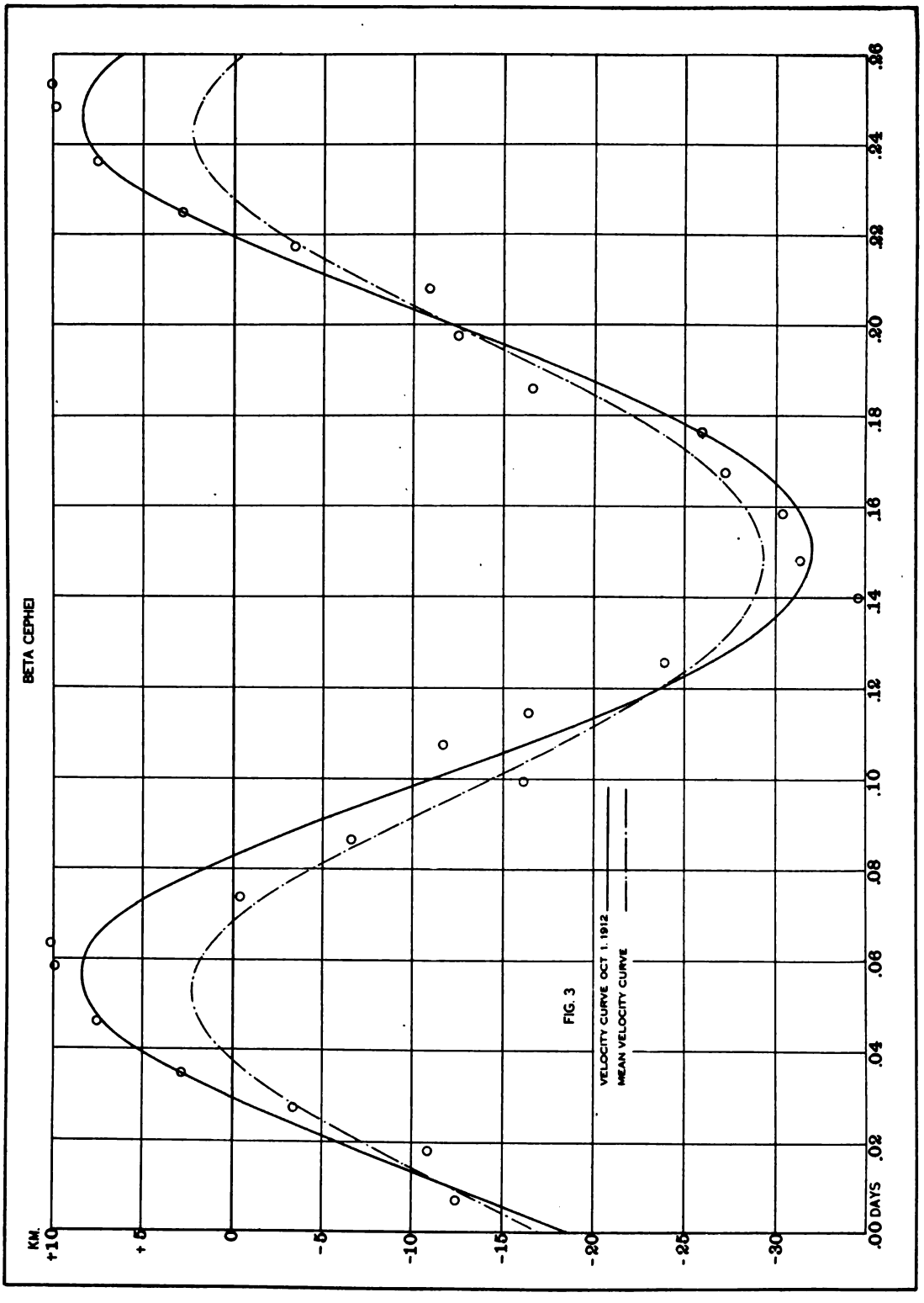

PLATE X. VELOCITY CURVE OF β CEPHEI FOR OCTOBER 1, 1912, COMPARED WITH MEAN CURVE

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ber 1 was due to an unknown cause. Further investigation of the nature of this change would undoubtedly be of value.

Variations of the maxima and minima of velocity curves have been observed in the cases of β Orionis by Plaskett and β Coronæ Borealis by Cannon. In the case of β Orionis, Plaskett attributes the change in amplitude to blends or to the perturbations caused by a third body. Cannon found that the shift of the maximum of β Coronæ Borealis was periodic, repeating itself in about twelve periods. In this case he attributes the effect to "a three body system, the body which gives the spectrum measured revolving about another in 40.9 days, and the two, with a third, forming a period of perturbation of 490.8 days."

DISCUSSION OF THE ELEMENTS

Period. The period of β Cephei may be compared with those of the short period variables W Ursæ Majoris, $4^h\ 0^m$, and 14.1904 Cygni, $3^h\ 14^m$, the shortest periods known to the writer. The agreement between the length of the period observed by Frost in the spring of 1906 and that determined at this Observatory in the fall of 1912 is interesting. We can assume the value of $4^h\ 34^m\ 17^s.4$ to be, on the average, the period for the last six years.

Eccentricity. If we apply the probable error in the most favorable way to the value of the eccentricity as given by the definitive elements, we find an agreement of this element with that given by Frost for July 6, 1906.

FROST, 1906.	CRUMP, 1912.
$e = 0.03 +$	0.04 ± 0.015.

These values indicate that the orbit is nearly circular and the agreement shown above would indicate constancy of form.

The Angular Distance from Periastron. A considerable degree of uncertainty pertains to the value of ω. Possibly the large difference between Frost's value of 165° 13′ and that of 2°.63 determined here is due to the uncertainties connected with the small eccentricity.

The Time of Passage through Ascending Node. The value of T depends upon ω and is therefore difficult to fix in a nearly circular orbit. In this discussion it has been found to be 1912 August 5.8119 G.M.T.

Velocity of the System. The two determinations of the element γ by Frost and that which I made are as follows:

July 6, 1906	— 4. 9 km.	Frost.
April 29, 1912	— 10. 2 km.	Frost.
August 5, 1912	— 14.13 km.	Crump.

The values indicate a change in the center of mass velocity. A variation of this element has been established in the case of Polaris and suspected by Belopolsky in the case of δ Cephei. In no other cases of Cepheid variables are observations concerning this variation available for study. The change in the velocity of the center of mass may possibly be attributed to the action of a third body.

The Semi-Amplitude. That the half amplitude of the velocity curve K is subject to variation was shown by the variant curve observed on October 1, 1912. During the series of observations this was the only instance of large variation observed. If we compare the value as determined in 1906, of 16.9 km., with the value as obtained at this Observatory in 1912, of 15.8 km., we observe a fair agreement. Though this element may vary from cycle to cycle we do not know whether the average value of this quantity is changing.

SUMMARY AND CONCLUSION

1. As a result of this investigation, definitive elements of the orbit of β Cephei have been determined from 163 velocity observations, whose average probable error determined from an elliptic curve was found to be $\pm$ 2.95 km.

The period of $4^h\ 34^m\ 17^s.4$ is the shortest period which has thus far been observed for a spectroscopic binary. The agreement between the value determined by Frost in 1906, and that determined by the writer in 1912, to the sixth decimal place in days, indicates that on the average the period remained constant during the interval of six years.

The eccentricity of 0.040 shows that the orbit is almost circular. This element is in agreement with the general observation of astrophysicists

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that spectroscopic binaries of early type have small eccentricities. It is the smallest eccentricity that has been determined for a variable of the δ Cephei type.

In nearly circular orbits the value of the angular distance of periastron is more or less uncertain. In this case the eccentricity is so small that both ω and T can not be easily determined from a least square solution, and consequently, different values of ω were chosen and the corresponding times of periastron passage computed. The value of this element which seemed the best determined was used with the corresponding value of T in the preliminary elements. Assuming this value of T as fixed, a final least square solution gave the corrected value of ω as $+ 2°.63$. This determination seems as good as could be expected. The large difference between this value and that of Frost in 1906 may be due to uncertainties affecting its determination.

2. The observations indicate a secondary oscillation whose period appears to be about one-third of the primary. Such an oscillation has been observed by Professor Curtiss in W Sagittarii, and a similar one, though reversed, has been found by Campbell in the case of ζ Geminorum. At present the cause of such oscillations is not known, although Professor Curtiss has suggested a possible explanation in these *Publications*, Vol. I, p. 109. The presence of a secondary oscillation may prove to be a further indication of variation of the δ Cephei type.

3. The velocity curve determined by the writer together with the light curve observed at the Berlin-Babelsberg Observatory confirms Guthnick's conclusion that β Cephei is a variable star of the δ Cephei type. The disagreement observed of about one-seventh of the period between the points of light maximum and maximum velocity of approach is the same as that observed in Y Sagittarii, but occurs on the descending instead of the ascending branch of the velocity curve. This variation may possibly be connected with the closeness of the components of the system. The time interval between the minimum of light and the minimum velocity of approach is observed to be about one-fifth of the period. Although this disagreement is large it would not necessarily exclude it from the δ Cepheid class of variables. In the cases so far observed the agreement between the points of minimum light and minimum velocity of approach has not been so close, on the average, as that between the maximum of light and the maximum velocity of approach. The disagreement may be due to variations caused by the ellipsoidal form of the members of the system, or by other conditions related to the extremely short period.

4. The observations made at this Observatory indicate, as does the unpublished observation of April 29, 1912, kindly communicated to me by Professor Frost, a shift of the center of mass velocity from that determined on July 6, 1906. Only in three cases of Cepheid variables, β Cephei, Polaris, and δ Cephei, are the data available for the determination of the existence of a variation of the velocity of the center of mass. In the first two cases named the variation is present and in the third its presence is suspected. Further observations would be of interest to determine whether this is a characteristic of variables of the δ Cephei type.

5. A variation in the amplitude of the velocity curve was detected and studied. While the amplitude has varied in one case at least we do not know that the average value of this quantity is changing.

It is with pleasure that I acknowledge my indebtedness to Professor R. H. Curtiss for help and supervision throughout this investigation, to Professor G. W. Hess for aid in securing the spectrograms, and to Doctor Laurence Hadley for valuable assistance.

Detroit Observatory, Ann Arbor, Michigan,
August, 1915.

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THE SPECTRUM AND RADIAL VELOCITY OF RHO LEONIS

By BERNHARD H. DAWSON

Rho Leonis, $\alpha = 10^h\ 27^m.5$, $\delta = +9^\circ\ 49'$, was announced as a spectroscopic binary by Campbell in *Lick Observatory Bulletin, 5,* 174 on the basis of six plates by the Lick observers. Although it was later withdrawn from their list, there had been obtained by Harper in the meantime sixty-five spectrograms which had a range of thirty kilometers and thus seemed to confirm the variability of the radial velocity, but without indicating any period.

This star was placed on the observing list at Ann Arbor and forty-three spectrograms were obtained by Curtiss and Mellor on twenty-one nights in 1913 and 1914. These have all been measured by the writer.

THE TOTAL LIGHT

The visual magnitude, as given in the *Harvard Photometry,* is 3.8; and since this star is of early type, the photographic magnitude is probably about 0.2 magnitude brighter. The writer is not aware of its ever having been suspected of variability.

THE SPECTRUM

The spectrum is classed in the *Harvard Annals* as Bop, it being peculiar in having more lines than are usually found in B type spectra. As many as 71 lines have been found measurable on a single plate, and a total of 93 lines have been measured on two or more plates and can thus be considered as certainly in the star. The wave lengths, intensities and a partial identification of these lines, together with other data explained below, as given in Table I. The probable errors in column 5 of this table are based on the agreement of the wave-lengths deduced from the several plates, and do not include the systematic uncertainties (of the order of 0.02 Å) of the corrected interpolation curve.

As noted by Curtiss (these *Publications,* Vol. 1, p. 131) the carbon (?) group at λ 4649.6 appears as a single, rather broad line. The remaining lines of the spectrum are generally sharply defined and easily measurable with this dispersion. On a few plates, notably Nos. 1721, 1804 and 2683, the star lines in the violet, excepting the K line, are considerably broader than in the majority, while the K line and the lines of the comparison spectra remain sharp. This would perhaps indicate that the spectrum was that of a binary with both components bright. But the remarkably sharp definition of the lines in the green and blue of plate No. 1804, and the fact that only in a few sporadic cases has anything like doubling been actually observed would indicate that this was not the true cause, or at least cast considerable doubt on the validity of such a conclusion. In the few cases that a line has appeared double or blended, as often as not values markedly inconsistent with the means of the other lines (which would of course include both components) were obtained by setting on the mean of the pair. Again, if this broadening of the lines be due to relative velocity of two components, they must revolve in a very short period, for against plate No. 1721 we have plate No. 1720, taken less than an hour earlier, in which the lines are sharp, and against plate No. 2683 we have plates both before and after in which the effect is much less marked.

The K line, contrary to Mr. Harper's experience at Ottawa, seems to be at all times well defined on these plates. In those plates in which it was not measured the cause was not poor definition, but the fact that the exposure was insufficient to make anything measurable in that part of the spectrum, for which the (old) prism is rather strongly absorptive. As the measures were made and the reductions begun with the idea that the K line velocity would not agree with that from the other lines, the measures of this line and the few measures of the H line that were made were kept separate throughout the discussion.

None of the plates were sensitized for the visual region.

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TABLE I. WAVE-LENGTHS OF LINES IN THE SPECTRUM OF RHO LEONIS.

WAVE-LENGTH.	INT.	NO. OF MEAS.	CLASS.	P. E.	SOURCE.	ASSUMED WAVE-LENGTH.	AUTHORITY.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Å				Å		Å	
3889.1	15	3	c	.....	Hζ	.....	
3912.3	1	2	c	.....	..	.....	
3919.1	1½	2	c	.....	..	.....	
3926.8	1½	2	c	.....	..	.....	
.....	4½	23	.	.....	Ca	3933.825	K, Rowland.
3945.2	1	2	c	.....	..	.....	
3954.5	1	3	c	.....	..	.....	
3955.8	1½	2	c	.....	..	.....	
3964.84	3½	20	b	± 0.022	..	.....	
.....	2	6	.	.....	Ca	3968.625	H, Rowland.
3970.25	15	7	c	0.009	Hε	.....	
3973.51	2½	9	c	0.027	..	.....	
3995.13	6	35	b	0.010	..	.....	
4009.41	5	25	b	0.020	..	.....	
4026.33	12	42	a	0.010	He	4026.37	Runge and Paschen.
4035.23	1	4	c	.....	..	.....	
4041.57	1	5	c	0.016	..	.....	
4044.3	1	3	c	.....	..	.....	
4069.88	2	26	b	0.021	..	.....	
4072.37	2	17	b	0.029	..	.....	
4076.03	4	33	b	0.018	O	.....	
4079.1	1	3	c	.....	..	.....	
4081.2	1	2	c	.....	..	.....	
4084.6	1	2	c	.....	..	.....	
4085.2	1	3	c	.....	..	.....	
4086.8	1	2	c	.....	..	.....	
4089.21	4½	29	a	0.019	Si	4089.00	Lunt.
4097.22	1	7	c	0.039	Si or N	.....	
4101.85	23	42	a	0.011	Hδ	4101.92	
4116.34	1	11	c	0.036	Si	.....	
4119.6	½	2	c	.....	..	.....	
4120.88	7	16	c	0.028	..	.....	
4138.9	1	2	c	.....	..	.....	
4143.90	6½	39	b	0.013	..	.....	
4153.59	1	7	c	0.041	..	.....	
4185.6	1½	3	c	.....	..	.....	
4190.02	1½	5	c	0.052	..	.....	
4227.90	1	4	c	.....	..	.....	
4237.21	1½	7	c	0.044	..	.....	
4240.27	2½	19	b	0.021	..	.....	
4253.87	2	39	a	0.018	S	4253.77	
4267.34	2½	32	a	0.017	C	4267.15	Eder and Valenta.
4285.16	1½	4	c	.....	..	.....	
4304.2	1	2	c	.....	..	.....	
4317.37	2	14	b	± 0.040	O	.....	

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TABLE I. WAVE-LENGTHS OF LINES IN THE SPECTRUM OF RHO LEONIS—CONTINUED.

LENGTH. WAVE-	INT.	NO. OF MEAS.	CLASS.	P. E.	SOURCE.	ASSUMED WAVE-LENGTH.	AUTHORITY.
(1) Å	(2)	(3)	(4)	(5) Å	(6)	(7) Å	(8)
4319.89	2	15	b	± 0.036	O	.....	
4332.7	½	3	c	.....	..	.....	
4337.19	1	4	c	.....	..	.....	
4340.62	28	43	a	0.009	Hγ	4340.634	Rowland.
4345.61	2	17	b	0.039	O	.....	
4349.61	3	26	b	0.014	O	.....	
4351.6	1	2	c	.....	..	.....	
4366.95	2½	21	b	0.029	..	.....	
4372.1	½	2	c	.....	..	.....	
4388.05	8	43	a	0.011	He	4388.10	Runge and Paschen.
4414.91	2½	18	b	0.026	O	.....	
4417.20	1½	7	c	0.032	O	.....	
4425.2	½	2	c	.....	..	.....	
4427.7	½	2	c	.....	..	.....	
4437.3	1	2	c	.....	..	.....	
4447.27	1½	8	c	0.061	..	.....	
4452.6	1	2	c	.....	..	.....	
4471.65	13	43	a	0.012	He	4471.68	Runge and Paschen.
4481.23	2	19	b	0.052	Mg	.....	
4530.4	1	3	c	.....	..	.....	
4552.81	5	43	a	0.010	Si	4552.762	Albrecht.
4567.97	3	41	a	0.011	Si	4567.967	Albrecht.
4574.92	2	37	a	0.023	Si	4574.918	Albrecht.
4587.4	½	2	c	.....	..	.....	
4591.21	1½	15	c	0.041	..	.....	
4596.62	1¼	10	c	0.055	O	.....	
4601.78	1¼	10	c	0.038	..	.....	
4607.23	1	7	c	0.053	O	.....	
4614.05	1	6	c	0.079	..	.....	
4621.48	1	8	c	0.139	..	.....	
4626.9	¾	2	c	.....	..	.....	
4630.66	2½	34	b	0.019	..	.....	
4634.2	½	2	c	.....	..	.....	
4638.78	1	8	c	0.043	..	.....	
4642.36	2	36	b	0.023	..	.....	
4649.64	6	32	a	0.018	C ?	4649.68	Hartmann.
4661.87	2	18	b	0.034	..	.....	
4676.22	½	4	c	.....	..	.....	
4705.46	½	4	c	.....	..	.....	
4713.37	2	34	a	0.018	He	4713.31	Runge and Paschen
4803.7	½	2	c	.....	..	.....	
4819.4	½	2	c	.....	..	.....	
4838.0	½	2	c	.....	..	.....	
4861.47	10	39	a	0.013	Hβ	4861.527	Rowland.
4906.08	½	4	c	.....	..	.....	
4911.8	1	3	c	.....	..	.....	
4922.11	7½	36	a	± 0.024	He	4922.096	Runge and Paschen.
4946.7	½	2	c	.....	..	.....	

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FORMER RADIAL VELOCITIES

Six spectrograms of Rho Leonis were obtained by the Lick observers in 1902-1908, and from the discordance of the fifth Campbell announced the variability of the radial velocity in *Lick Observatory Bulletin*, Vol. 5, p. 174. But in a list of errata, (*ib.*, Vol. 6, p. iv.) the values of the velocity from this plate are corrected and the star removed from the list of binaries. As thus corrected, the velocity from this plate is + 41.3 km., using the mean of two measures, while the other plates range from that to + 35.3 km. The mean of all, giving half weight to the fourth, which is published as being somewhat uncertain, is

$$V = + 38.8 \pm 0.74 \text{ km.},$$

where the probable error of ± 0.74 km. does not contain the effects of elements systematic to the series.

TABLE Ia. LINES SUSPECTED IN THE SPECTRUM.

WAVE-LENGTH.	PLATE NUMBER.	WAVE-LENGTH.	PLATE NUMBER.
3962.1	1878	4419.6†	1903
3983.5	1878	4422.3†	1903
4032.9	1707	4442.4	2704
4093.1	2681	4581.4†	2704
4111.0	2682	4636.6	2704
4169.4	1707	4665.8	1804
4174.0	1805	4669.8	1804
4176.2	1805	4698.8	1656
4200.2*	1707	4709.6	1804
4233.6	2660	4741.9	1744
4249.6	2660	4774.0	1804
4262.4	1878	4800.3	1804
4295.8	2702	4807.1	1804
4327.4	2695	4813.8	1804
4383.1	1707	4970.0	2687
4410.0	1879	4993.8	2687

* H8'. † Perhaps parts of bands.

Harper at Ottawa obtained 65 plates in 1910-1912, and using only eight lines in the determination, deduced radial velocities ranging from + 30 km. to + 52 km., excluding two doubtful plates. After an unsuccessful attempt to find a period to fit the variation indicated he expressed it as his belief that the variation was real, though not definitely established. The measures are published in some detail in *Publications of the Dominion Observatory*, Vol 1, p. 337, where, on the supposition of constant velocity, he derives from them, (page 351), the value:

$$V = + 43.2 \pm 3.4 \text{ km.}$$

Using the data of these plates, Schlesinger (*Astrophysical Journal*, Vol. 41, p. 166) deduced a period of 12.28 days, saying that the orbit was probably highly eccentric with periastron near the descending node and an amplitude of the velocity curve of about 20 km. On the basis of this period all the extremely low velocities fall within a day of the phase 7.3 days, the date of Harper's first plate being the zero. The possible range of period for which they all fall within 1.5 days of a single phase is from 12.19 to 12.38 days.

THE ANN ARBOR RADIAL VELOCITIES

The Ann Arbor radial velocities were determined from 43 plates taken with the 37½" Reflector and single prism spectrograph. All of the plates except No. 1708 are on Seed 23 emulsion with titanium spark comparison. In No. 1708 a Red Label Lantern Slide plate was used by mistake, and the spectrum is consequently considerably underexposed. These plates were all measured by the writer with the engine mentioned on page 80 of Vol. 1 of these *Publications*, according to the customary method here, assigning weights to the various lines on the return measures. All the lines that could be well seen, except the Hε, were generally set on, although only a part of these measures were intended for velocity determination. The K line, and the H line where measured, were kept separate in the reductions. After reduction to mean dispersion fifteen lines were selected, each of which had been measured on nearly all the plates. and for which wavelengths were at hand. With these lines a preliminary reduction was made, and upon them alone the final mean velocity depends. They are designated by a in column 4 of Table I., and have the originally assumed wave-length and authority given in columns 7 and 8 of the same table. As the star was supposed to be a spectroscopic binary, it was desirable to have better values of the relative velocities from the different

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plates, and these lines were accordingly reduced to homogeneity by applying corrections to the wave-lengths such that the weighted mean of the residuals of each individual line was reduced to zero. At the same time these preliminary velocities were used to determine the wave-lengths of eighteen other lines and to make them homogeneous with the first group. These eighteen lines are designated by b in column 4 of Table I. With these 33 homogeneous standards a final reduction was made and velocities obtained which are exhibited in Table II, together with the number of lines used in obtaining each and the probable error deduced from their agreement. These velocities were then used to determine approximate wave-lengths of the remaining lines. The number of lines so determined on each plate is given in column 9 of Table II, and the lines are designated by c in Table I.

In view of the fact that a large number of lines was used and that their character in the spectrum varies considerably from plate to plate it was not thought advisable to go through the process of reweighting them, and the weights assigned to the individual lines at the time of measurement were retained unaltered in the reduction of each plate. But as the measures extended over several months it is not only likely from *a priori* considerations, but also evident from the work itself, that the weight assigned to a line of given value was different in different parts of the series, while still sensibly constant for any one plate. Since the sum of the weights of the individual lines could not be used as the weight of the plate in comparing the various plates, the weighting of the plates became a problem. Assigning values proportional to the inverse squares of the probable errors was seen to give fictitiously high weights to several of the plates with few lines, and to be no more reliable than the sums of the original line weights. It was found that by dividing the number of lines used in velocity determination by the probable error of the plate, quantities were obtained which express very nearly the writer's estimate of the values of the plates from a rapid review of them under the microscope, and this method of weighting was accordingly adopted. The weights thus assigned are given in column 7 of Table II.

The velocities obtained were first plotted on the basis of Schlesinger's period, but showed no sign of variation in any such period. The weighted mean of the plates was accordingly taken, giving:

$$V = +41.1 \pm 0.23 \text{ km.}$$

At the same time the probable error of an average plate was found to be

$$\epsilon = \pm 1.50 \text{ km.},$$

while the probable error deduced from the internal agreement of the lines is

$$\epsilon_1 = \pm 1.23 \text{ km.}$$

Introducing the quantity ϵ_2 used by Curtiss (these *Publications,* Vol. 1, p. 134) and defined by the relation

$$\epsilon^2 = \epsilon_1^2 + \epsilon_2^2,$$

we derive the result,

$$\epsilon_2 = \pm 0.86 \text{ km.}$$

Since this value of ϵ_2 is no greater than experience with the spectrograph employed here would lead us to expect for this star in case its velocity were constant the indication is that the Ann Arbor radial velocities contain no evidence of variability.

The fact that the Ottawa observations of this star satisfy a period makes it necessary to limit as above the statement relative to the velocity variability, especially as none of the Ann Arbor observations are at phases exactly corresponding to Schlesinger's minimum, but since there should be some variation indicated in the remainder of the period as well as at minimum, and the Ann Arbor velocities show none, it can at least be said that while the Ann Arbor observations do not supply sufficient evidence for stating definitely that the radial velocity is constant, they nevertheless materially strengthen the evidence in favor of such a conclusion and make the agreement of Harper's low velocities with Schlesinger's period seem more probably fortuitous than the result of any real variation.

THE K LINE

The K line was measured on twenty-three of the forty-three plates, and as before stated, these measures were kept separate. Taking the weighted mean of all, we deduce:

Velocity from the K line $= +1.7 \pm 1.21$ km.

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TABLE II. RADIAL VELOCITIES OF RHO LEONIS.

NO. OF PLATE.	OBSERVER.	DATE, G. M. T.	GENERAL LINES.						K LINE.	
			VEL.	NO.	P. E.	WT.	RESID.	NO.	VEL.	WT.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1913 d.	km.		km.		km.		km.	
1651	Curtiss	April 5.742	+40.8	16	±1.3	12	−0.3	1	+26.1	1
1655	Curtiss	6.636	+40.4	23	1.1	20	−0.7	3	− 3.2	2
1656	Curtiss	6.650	+40.3	23	1.3	18	−0.8	2	+ 9.0	2
1679	Curtiss	13.625	+39.8	24	1.1	23	−1.3	1	0.0	1
1680	Curtiss	13.648	+42.1	25	1.4	19	+1.0	0	− 6.7	1
1707	Mellor	16.647	+40.8	33	0.9	36	−0.3	29	− 3.0	3
1708	Mellor	16.669	+36.2	10	3.9	3	−4.9	0	....	.
1720	Curtiss	19.588	+42.3	27	1.1	24	+1.2	1	− 5.7	2
1721	Curtiss	19.620	+39.7	26	1.0	26	−1.4	0	− 5.8	1
1731	Curtiss	20.613	+37.7	27	0.9	32	−3.4	3	+ 5.2	2
1732	Curtiss	20.631	+42.2	20	0.6	32	+1.1	0	....	.
1743	Mellor	23.622	+36.8	25	1.3	19	−4.3	1	+ 5.3	½
1744	Mellor	23.644	+40.6	24	1.2	20	−0.5	1	....	.
1745	Mellor	23.653	+42.2	30	1.0	32	+1.1	0	+ 5.3	½
1804	Curtiss	May 1.664	+41.9	27	1.3	21	+0.8	14	+ 8.0	½
1805	Curtiss	1.680	+43.1	25	1.2	20	+2.0	8	+10.8	½
1818	Curtiss	3.608	+39.2	28	1.3	22	−1.9	10	....	.
1819	Curtiss	3.626	+40.2	27	1.6	18	−0.9	6	....	.
1839	Mellor	7.611	+44.3	28	1.4	20	+3.2	8	....	.
1840	Mellor	7.630	+40.7	24	1.1	21	−0.4	3	....	.
1878	Curtiss	18.598	+38.1	33	0.8	43	−3.0	30	− 7.5	3
1879	Curtiss	18.616	+38.7	30	1.2	25	−2.4	9	+ 3.8	½
1903	Curtiss	31.628	+43.5	23	1.2	20	+2.4	6	....	.
1904	Curtiss	31.648	+43.2	28	1.3	22	+2.1	11	....	.
1960	Curtiss	June 8.599	+36.2	18	1.4	13	−4.9	1	....	.
1961	Curtiss	8.623	+41.2	8	2.0	4	+0.1	0	....	.
		1914								
2650	Curtiss	March 20.671	+41.2	19	1.0	19	+0.1	18	+14.6	1
2653	Curtiss	24.696	+44.8	7	1.9	7	+3.7	0	....	.
2669	Curtiss	April 5.787	+44.9	21	2.1	10	+3.8	3	....	.
2672	Curtiss	8.615	+44.3	26	1.3	20	+3.2	3	+19.7	¼
2673	Curtiss	8.645	+42.0	31	1.0	30	+0.9	16	+ 1.7	1
2681	Curtiss	12.615	+43.3	30	1.1	28	+2.2	18	+ 3.2	½
2682	Curtiss	12.656	+43.2	23	1.5	15	+2.1	2	....	.
2683	Curtiss	12.680	+44.2	15	0.9	17	+3.1	2	+12.4	½
2684	Curtiss	12.722	+43.8	27	1.4	20	+2.7	3	− 4.6	½
2685	Curtiss	12.744	+45.0	18	1.3	14	+3.9	1	....	.
2686	Curtiss	12.789	+43.1	26	1.7	15	+2.0	4	....	.
2687	Curtiss	12.827	+43.1	21	1.7	13	+2.0	2	....	.
2695	Mellor	13.683	+41.1	23	1.6	15	0.0	5	+ 8.3	¼
2696	Mellor	13.712	+38.7	25	1.2	21	−2.4	5	+ 9.2	½
2702	Mellor	17.634	+42.2	16	1.9	8	+1.1	2	....	.
2703	Mellor	17.665	+36.3	18	2.3	8	−4.8	2	....	.
2704	Mellor	22.661	+39.2	29	±1.2	24	−1.9	22	....	.
Means.			+41.1 ± 0.23 km.						+1.7 ± 1.21	

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This result is markedly less positive than that for the other lines and confirms Harper's conclusion with respect to this star. It also agrees in this respect with the results of other observers in cases of other stars of this spectral type. Since in the present case we have the difference occurring at the same time with an apparently constant velocity in the case of each, Hartmann's hypothesis, that the K line is due to a calcium cloud separate from the star and not sharing its velocity, is given added strength.

The width of the Hϵ line was so great that it in most cases obscured the H, which was measurable on only six plates, and which is not discussed. It may be said that the scanty evidence at hand indicates that what is true of the K line applies to the H line as well.

ADDITIONAL LINES SUSPECTED IN THE SPECTRUM

In addition to the 93 lines measured in more than one plate there are several that, although measured in only one, were so definitely indicated on the one where measured that their real existence in the star's spectrum seems probable. Their approximate wave-lengths and the numbers of the plates in which they were measured are given in Table Ia. None of these lines are counted in column 9 of Table II.

CONCLUSION

The results of this investigation may be summed up as follows:

1. The wave-lengths of 91 lines have been determined from measures on two or more plates, and approximate wave-lengths of 32 additional lines probably present in the spectrum have been deduced from single measures.

2. The 43 Ann Arbor radial velocities contain no evidence of variation, and consequently the reality of both the variation deduced from the Ottawa observations by Harper and the period derived by Schlesinger are rendered doubtful.

3. The variations in character of the lines from plate to plate may be taken as evidence of complexity, but the general indications are against such a conclusion.

4. The mean radial velocities deduced are $+1.7 \pm 1.21$ km. for the K line and $+41.1 \pm 0.23$ km. for the others.

5. The K line velocity differs so radically from that of the other lines as to make a common source highly improbable.

I wish to express my thanks to Dr. Curtiss for advice and direction throughout the work.

Ann Arbor, Michigan, June, 1916.

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THE REGISTRATION OF EARTHQUAKES AT THE DETROIT OBSERVATORY DURING THE YEARS 1914 AND 1915

By PAUL W. MERRILL

The record is in the same form as for preceding periods, as given in Volume I of these *Publications,* pp. 54-72 and 191-199, except that, commencing with January 1, 1914, all times are given in Greenwich Civil Time, counted from midnight to midnight. The recorded amplitude is the semi-amplitude of the oscillation of the pen. The character * denotes well defined; † gradual.

On December 31, 1914, the magnification of all three components of the Wiechert instruments was changed from 40 to 80. The weight of the horizontal Wiechert was taken apart and readjusted on February 2, 1915. The magnifications and periods of the instruments are now as follows:

	MAGNIFICATION.	PERIOD.
B—EW	40	11.5 sec.
B—NS	50	12.0
W—EW	80	4.9
W—NS	80	6.0
W—V	80	4.2

There is no applied damping.

The distance between the recording pen and the time indicator on each of the Bosch machines is found as follows: Every morning before removing the sheets the pen is swung by a touch of the finger at the exact instant of the minute signal; the distance between this sudden departure from the record line and the preceding minute mark is the "parallax" to be applied to the times read from the sheet.

REMARKS.

110. Another maximum occurred at $4^h 26^m$ with amplitude 0.2 mm. Period at first maximum (M) 30 ± sec., at second maximum 18 sec. Some additional very weak waves a few minutes after F. W—NS shows a trace of S but L is not visible. Bosch instruments were not running at this time.

111. Minutes assumed on W—horizontal records, since hour signals are missing. The main portion consists of very short period vibrations superposed on longer waves. This earthquake was reported by Ottawa observers as "local."

112. A very peculiar disturbance, possibly of artificial origin, (blasting?). Preliminaries very feeble and doubtful.

113. Distance probably accurate. W—NS shows one group of waves $5^h 15^m.0$ to $16^m.3$, amplitude 0.4 mm; another group $5^h 17^m.4$ to $18^m.3$, amplitude 1.0 mm.

114. There are two distinct parts to the main portion: M in the table, records the first maximum; the second maximum occurs about 3.5 min. later.

115. A slow flat shock or groups of slow regular sinusoidal microseisms. B—EW waves from $17^h 37^m.2$† to 43^m† having a maximum from 40-42^m, amplitude 0.35 mm., period 18 sec., with other smaller groups a few minutes later. B—NS waves from $17^h 32^m.2$† to $39^m.2$†, having a maximum at $32^m.9$; amplitude 0.3 mm., period 19 sec.

W—EW waves from $17^h 32$-33^m; from $37^m.4$† to $42^m.7$†; from 47^m to 49^m. Maximum amplitude 0.15 mm., period 18 sec.

116. Shock apparently in progress when record began. Long waves end at $13^h 56^m$. F is indeterminate, the tail running into irregular microseisms. On W—EW small vibrations began at $13^h 39^m$ or before, stronger motion at $45^m.6$, amplitude 0.5 mm. The W—NS record is weak; the stronger motion began at $44^m.4$.

117. S uncertain; no well marked phases.

118. A very long flat shock. A slight trace on W—V. S possibly misidentified.

119. The Panama earthquake.

120. A long weak shock about 18 hrs.

June 2 Very strong irregular disturbances about 13 hrs. Perhaps artificial.

122. A feeble indefinite shock, being somewhat better marked on EW instruments.

123. Horizontal W not running.

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NO.	DATE.	INST. COMP.	P	S	L	M	F	A	Δ
	1914		h m	h m	h m	h m	h m	mm.	mgm.
110	Jan. 30	W—EW	.. ....	3 57.9*	4 3†	4 14	4 59	0.1	...
111	Feb. 10	B—EW	.. ....	18 33.9	18 34.6	18 35.1	18 40	1.2	...
		B—NS	.. ....	33.9	34.4	35.1	39	1.3	...
		W—EW	.. ....	33.8	34.5	35.2	40	1.5	Small
		W—NS	.. ....	33.8	34.5	35.1	39	1.0	...
		W—V	.. ....	33.8	34.5	35.1	36.3	0.2	...
112	Feb. 26	B—EW	5 8.5	.. ....	5 16.8	.. ....	.. ....	0.5	...
		B—NS	9.4	.. ....	17.0	.. ....	.. ....	2.9	...
		W—EW	8.3	.. ....	16.8	.. ....	.. ....	1.0	...
		W—NS	8.5	.. ....	16.8	.. ....	.. ....	0.5	...
113	Feb. 28	B—EW	5 8.4	5 12.4	5 15.0	5 16.2	5 29	3.2	...
		B—NS	8.3	12.5	17.4	17.9	34	14.0	2.4
		W—EW	8.3	12.3	15.0	15.7	34	3.9	...
		W—NS	.. ....	Remark					
114	Mar. 30	B—EW	0 47.4	0 52.3	0 58.7?	1 0.4	1 58	6.2	...
		B—NS	0 47.3	52.2	1 1.0	2.4	59	13.3	3.7
		W—EW	.. ....	52.3	0 58.7†	0.1	56	0.7	...
		W—NS	.. ....	52.2	59.7†	2.2	20	1.5	...
115	April 11	B—EW	.. ....	.. ....	17 37.2†	17 41	.. ....	0.35	...
		B—NS	.. ....	.. ....	32.2†	32.9	.. ....	0.2	...
116	April 20	B—EW	.. ....	.. ....	.. ....	13 49.5	.. ....	2.6	...
		B—NS	.. ....	.. ....	.. ....	50.6	.. ....	2.2	...
117	April 24	B—EW	.. ....	8 48.3?	8 51.1	8 51.6	9 1	0.25	...
		B—NS	.. ....	47.5?	48.5	48.7	8 57	0.3	...
		W—EW	.. ....	48.5?	51.3	51.7	58	0.2	...
118	May 26	B—EW	.. ....	14 44.3†	15 28.7	15 32.8	16 43	2.6	...
		B—NS	.. ....	44	29.3	32.5	44	2.0	...
		W—EW	.. ....	44.7†	28.9	32.7	45	0.4	...
		W—NS	.. ....	.. ....	.. ....	32.3	.. ....	0.3	...
119	May 28	B—EW	.. ....	3.35.8*	3 43.4	3 44.5	3 55	0.4	...
		B—NS	3.30.4	.. ....	42.9	44.4	57	0.6	4.
		W—EW	31.6†	35.9*	43.6	44.6	55	0.2	...
120	May 28	B—NS	.. ....	.. ....	.. ....	18 0±	.. ....	...	...
121	May 29	B—EW	.. ....	.. ....	.. ....	6 13	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....	6 13	.. ....	...	...
122	June 20	B—EW	7 { 40.1? 45.6?	7 49.1?	8 14.7	8 19.9	8 51	0.15	...
		B—NS	.. ....	.. ....	.. ....	29.1	.. ....	0.1	...
		W—EW	{ 40.4? 45.7?	49.4	13.1	19.6	.. ....	0.1	...
123	June 25	B—EW	19 26.8	19 48.1	20 11.5	20 19.0	21 11	1.0	...
		B—NS	26.2	.. ....	15.0	28.5	5	1.5	...
124	June 26	B—EW	.. ....	.. ....	.. ....	5 49	.. ....	...	...

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NO.	DATE.	INST. COMP.	P	S	L	M	F	A	Δ
	1914		h m	h m	h m	h m	h m	mm.	mgm.
125	July 3	B—EW	.. ....	.. ....					
		B—NS	.. ....	.. ....		21 0±	.. ....	...	...
		W—NS	.. ....	.. ....					
126	July 5	B—EW	.. ....	.. ....	3 15.7	.. ....	.. ....	0.15	...
		B—NS	.. ....	.. ....	15.4	3 15.7	.. ....	0.2	...
		W—NS	.. ....	.. ....	15.6	.. ....	.. ....	0.05	...
127	July 5	B—EW	.. ....	.. ....	4 9.7	.. ....	.. ....	0.1	...
		B—NS	.. ....	.. ....	9.8	.. ....	.. ....	0.15	...
		W—NS	.. ....	.. ....	9.9	.. ....	.. ....	0.05	...
128	July 17	B—EW	.. ....	.. ....	7 35.6	7 43.4	.. ....	0.05	...
		B—NS	7 20.8	.. ....	34.8	44	.. ....	0.05	...
129	July 21	B—EW	22 36.8	22 48.4	22 51.1	22 51.5	23 20	2.0	...
		B—NS	35.9	48.2	50.8	51.2	22	1.0	...
		W—EW	.. ....	48.7	51.4	51.8	22 56	0.2	...
		W—NS	.. ....	48.4	51.3	51.4	23 16	0.4	...
130	Aug. 3	B—EW	11 32.0	11 37.7?	11 41.4	11 41.6	11 54	0.3	...
		B—NS	30.9	?	39.4	41.7	.. ....	0.2	3½
		W—EW	.. ....	.. ....	41.5	41.6	43	0.05	...
		W—NS	31.1*	.. ....	40.9	41.8	51	0.1	...
131	Aug. 4	B—EW	23 4.9?	.. ....	23 24†	23 29.7	0 21	1.5	...
		B—NS	5.6*	.. ....	29†	38.7	30	1.5	7?
		W—EW	.. ....	.. ....	28.0	45.7	23 49	0.25	...
		W—NS	5.6	.. ....	29	35.9 / 43.6	0 29	0.3	...
		W—V	.. ....	.. ....	.. ....	44.7	.. ....	0.1	...
132	Aug. 8	B—EW	19 16.0	.. ....	19 25.1	19 28.4	20 23	2.0	...
		B—NS	.. ....	.. ....	25.2	27.3	7	5.6	...
		W—EW	.. ....	.. ....	25.4	26.0	19 39	0.5	...
		W—NS	.. ....	.. ....	25.0	27.3	20 8	0.7	...
133	Aug. 22	B—EW	5 34.8*	5 45.7*	5 48.0	5 48.8	6 32	9.2	...
		B—NS	40.2	45.6†	48.1	48.5	28	8.8	...
		W—EW	37.7	45.7	47.7	48.6	12	0.9	...
		W—V	.. ....	.. ....	48.0	48.6	5 57	0.4	...
134	Aug. 28	B—NS	.. ....	.. ....	.. ....	9 6.0	.. ....	...	...
135	Aug. 28	B—NS	.. ....	.. ....	.. ....	16 4.5	.. ....	0.1	...
136	Aug. 28	B—NS	.. ....	.. ....	17 34	.. ....	.. ....	...	...
137	Sept. 25	B—NS	.. ....	.. ....	.. ....	10 53	.. ....	0.1	...
138	Oct. 3	B—EW	17 30.6*	17 35.9	17 40.9†	17 42.9	18 41	3.9	...
		B—NS	30.7	36.1	40.4?	41	37	3.0	...
		W—EW	30.7	36.0	41.0†	42.6	46	0.7	3.5
		W—NS	30.9	36.0	40.5	41	17 58	0.5	...
		W—V	.. ....	.. ....	40.7	.. ....	.. ....	0.1	...
139	Oct. 22	B—EW	6 57.7	.. ....	7 0.7	7 2.9	7 10	0.2	...
		B—NS	57.7	.. ....	1.3	2.6	6	0.15	...

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NO.	DATE.	INST. COMP.	P	S	L	M	F	A	Δ
	1914		h m	h m	h m	h m	h m	mm.	mgm.
140	Oct. 23	B—EW	.. ....	.. ....	.. ....	7 24	.. ....	0.2	...
		B—NS	.. ....	.. ....	.. ....	24	.. ....	0.2	...
141	Nov. 10	B—EW	.. ....	11 22.7	11 25.8?	.. ....	.. ....	0.6	...
		B—NS	11 19.0	23.0	25.8	11 26.1	.. ....	0.5	2.4
		W—EW	.. ....	23.0	?	.. ....	.. ....	0.8	...
142	Nov. 18	B—EW	9 52.1	micros.	?	9 58.9	? micros.	0.8	...
		W—EW	52.2	9 55.9	9 57.3	58.8	10 8	0.4	2.
143	Nov. 24	B—EW	.. ....	12 17.4	.. ....	.. ....	13.21	(2.5)	...
		B—NS	.. ....	17.2	.. ....	.. ....	24	(4.5)	...
		W—EW	.. ....	17.2	.. ....	.. ....	12 55	(1.5)	...
		W—NS	.. ....	17.2	.. ....	.. ....	13 15	(1.1)	...
144	Dec. 4	B—EW	.. ....	22 43.9	22 46.6†	22 46.8	? micros.	0.15	...
		B—NS	.. ....	44.3	46.4	47.2	.. ....	0.1	...
145	Dec. 20	B—EW	.. ....	.. ....	.. ....				
		B—NS	.. ....	.. ....	.. ....	15 0±	.. ....	...	...
		W—NS	.. ....	.. ....	.. ....				
146	Dec. 25	B—EW	.. ....	3 51.4	3 53.8	3 54.3	.. ....	1.3	...
		B—NS	.. ....	54.2	56.7	57.4	.. ....	0.3	...
		W—EW	.. ....	.. ....	.. ....	54.9	.. ....	0.6	...
		W—NS	.. ....	.. ....	.. ....	55.2	.. ....	0.2	...
	1915								
147	Jan. 13	B—EW	.. ....	.. ....	7 28	7 39.6	7 46	0.2	...
		B—NS	.. ....	.. ....	7 24	7 30.3	>8 0	0.5	...
		W—EW	.. ....	.. ....	7 25	7 33	7 44	...	...
148	Mar. 5	B—EW	.. ....	.. ....	4 32.8	.. ....	.. ....	0.7	...
		B—NS	.. ....	.. ....	4 32.9	.. ....	.. ....	0.6	...
		W—EW	.. ....	.. ....	4 32.9	.. ....	.. ....	0.4	...
		W—NS	.. ....	.. ....	4 33.0	.. ....	.. ....	0.4	...
149	April 23	B—EW	15 37.5*	15 43.9*	.. ....	.. ....	16 3+	...	...
		B—NS	15 37.4*	15 44.0*	.. ....	.. ....	16 0+	...	4.8
		W—EW	15 37.5*	15 44.0*	.. ....	.. ....	16 2	...	...
		W—NS	15 37.5*	15 44.0	.. ....	.. ....	? micros.	...	...
150	May 1	B—EW	5 12.0*	5 21.6*	5 43.3	5 43.7	8 6	9.5	...
		B—NS	5 12.1*	5 22.1	5 42.4	5 46.3	7 24	13.6	7.8
		W—EW	5 12.0*	5 21.6	5 43.2	5 43.6	8 6	1.6	...
		W—NS	5 12.0*	5 21.7	5 42†	5 46.3	8 6	3.5	...
151	May 3	B—EW	.. ....	.. ....	.. ....	5 11†	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....	5 11	.. ....	...	...
		W—EW	.. ....	.. ....	.. ....	5 12	.. ....	...	...
		W—NS	.. ....	.. ....	.. ....	5 11+	.. ....	...	...
152	May 6	B—EW	.. ....	12 15.6?	.. ....	12 29.1	13 0±	0.9	...
		B—NS	.. ....	.. ....	12 25.8	12 28.0	13 0±	1.2	7.1?
		W—EW	.. ....	12 15.5?	12 28	12 29.0	13 0+	0.5	...
		W—NS	.. ....	12 15.5?	12 27.7	12 30.5	13 0+	1.8	...

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NO.	DATE.	INST. COMP.	P	S	L	M	F	A	Δ
	1915		h m	h m	h m	h m	h m	mm.	mgm.
153	June 1	B—EW	14 59.8?	15 3.3	15 9.7	15 11.0	15 40+	5.4	...
		B—NS	14 53.6?	15 3.3	15 10.3	15 13.5	15 42	3.3	4.9
		W—EW	.. ....	15 2.9	15 9†	15 11.3	15 45	1.4	...
		W—NS	.. ....	.. ....	15 9.9	15 13.5	15 47	1.8	...
154	June 6	B—EW	21 39.7	21 47.8*	.. ....	21 54	22 45+	1.8	...
		B—NS	21 39.6*	21 47.9*	.. ....	22 10	22 35	0.9	6.7
		W—EW	21 39.8	21 48.0*	.. ....	.. ....	23 0±	0.7	...
		W—NS	21 39.8	21.47.9	.. ....	22 10	22 50	0.7	...
155	June 23	B—EW	.. ....†	.. ....	4 14.7	4 15.6	.. ....	1.1	...
		B—NS	.. ....	.. ....	4 12.9	13.7	.. ....	3.0	3.
		W—EW	.. ....	4 12.5?	4 14.7	15.6	.. ....	0.7	...
		W—NS	.. ....	.. ....	4 13.0	14.7	.. ....	2.0	...
156	June 23	B—EW	.. ....†	.. ....	5 11.6	5 12.0	.. ....	1.9	...
		B—NS	.. ....	.. ....	5 9.7	5 11.0	.. ....	2.8	3.
		W—EW	.. ....	5 9. ?	5 11.4	5 11.9	.. ....	0.8	...
		W—NS	.. ....	5 6.1?	5 9.4*	5 10.9	.. ....	2.6	...
157	July 31	B—EW	1 42.6	1 51.8	2 6.6	2 11	3 7	1.6	...
		B—NS	1 42.6	1 51.8	2 6.2	2 13.4	3 8	1.5	8.0
		W—EW	1 42.2	1 51.7	2 6	2 12	3 30	0.5	...
		W—NS	.. ....	.. ....	.. ....	2 13.7	.. ....	0.2	...
		W—V	.. ....	.. ....	.. ....	2 13.7	.. ....	0.1	...
158	Aug. 3	B—EW	.. ....	.. ....	.. ....	14 13	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....	14 13	.. ....	...	...
		W—EW	.. ....	.. ....	.. ....	14 15	.. ....	0.1	...
159	Aug. 7	B—EW	.. ....	.. ....	.. ....	Traces	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....	15 42†	.. ....	...	...
		W—EW	.. ....	.. ....	.. ....	15 49±	.. ....	...	...
160	Sept. 7	B—EW	1 26.8	1 31.7	.. ....	.. ....	3 39	>40	...
		B—NS	1 26.9	1 31.6	1 35.8	.. ....	3 9	>50	...
		W—EW	1 26.9	1.31.6±	.. ....	1 37	3 27	21.4	3
		W—NS	1 26.9	1 31.6±	.. ....	.. ....	3 21	11.6	...
		W—V	.. ....	.. ....	1 36.1	1 39	1 43.3	4.4	...
161	Sept. 7	B—EW	.. ....	.. ....	4 22.0	4 24.0	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....	Trace	.. ....	...	4.?
		W—EW	.. ....	.. ....	4 22	4 23.1	.. ....	...	...
		W—NS	.. ....	4 17.3?	4 21.6	4 26	.. ....	...	...
162	Sept. 7	B—EW	4 33.9†	.. ....	4 45†	4 45.3	5 1	0.4	...
		B—NS	4 34.2?	4 39.8?	4 44.7	4 45.1	4 59	0.4	4.0
		W—EW	4 34.3?	.. ....	4 44.9	4 45.1	4 58	0.1	...
		W—NS	4 34.1	4 39.9?	4 44.6	4 45.0	5 1	0.2	...
163	Sept. 7	B—EW	5 8.8?	5 13.2?	5 18.7	.. ....	.. ....	...	...
		B—NS	.. ....	5 13.8†	5 19.7	5 21	.. ....	0.1—	...
		W—EW	.. ....	5 14.9†	.. ....	Trace	.. ....	...	3.8
		W—NS	5 9.4	5 14.7?	5 19.9	5 20.1	5 30	0.1	...

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NO.	DATE.	INST. COMP.	P	S	L	M	F	A	Δ
	1915		h m	h m	h m	h m	h m	mm.	mgm.
164	Sept. 7	B—EW	.. ....	.. ....	.. ....	13 9	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....	13 6–11	.. ....	...	...
		W—EW	.. ....	.. ....	.. ....	13 10	.. ....	...	...
		W—NS	.. ....	.. ....	.. ....	13 9	.. ....	...	...
165	Sept. 7	B—EW	.. ....	20 50.6	.. ....	20 58.3	21 13	0.1	...
		B—NS	20 46.6	20 50.4	20 56.7†	21 1.1	21 14	0.1	4
		W—EW	.. ....	20 52.0?	.. ....	.. ....	21 11	0.1	...
		W—NS	20 45.6?	.. ....	.. ....	21 1	21 12	0.1	...
166	Sept. 12	B—EW	20 50.5?	20 53.1?	21 10±	21 13	.. ....	0.2	...
		B—NS	.. ....	.. ....	.. ....	21 1–4	.. ....	...	...
167	Oct. 2	B—EW	23 52.6	23 55.2?	.. ....	.. ....	.. ....	...	...
		B—NS	23 53.4	.. ....	.. ....	.. ....	.. ....	0.5	...
		W—EW	23 53.4†	23 56.3	.. ....	.. ....	.. ....	0.2	1.8
		W—NS	23 53.5	23 56.9	.. ....	.. ....	.. ....	0.9	...
168	Oct. 3	B—EW	.. ....	.. ....	2 4.6	2 4.9	2 22	0.8	...
		B—NS	.. ....	.. ....	2 3.3?	2 3.5	2 21	0.7	4.5?
		W—EW	.. ....	1 57.8	2 4.3	2 4.6	.. ....	0.3	...
		W—NS	.. ....	.. ....	2 3.6±	2 4.6	.. ....	0.6	...
169	Oct. 3	B—NS	6 59.0	7 3.7	7 5.1	Off sheet	9 0	>67	...
		W—EW	6 59.0	7 3.8±	7 8.?	Double	9 2	28	3.0
		W—NS	6 59.0	7 3.7	7 7.2±	?	9 7	17.0	...
170	Oct. 11	B—EW	19 39.1	19 43.2	19 49.2	19 54.5	.. ....	1.3	...
		B—NS	19 39.3	19 43.5	19 49.5	19 52.1	.. ....	1.0	...
		W—EW	19 39.2	19 43.1?	19 50±	19 53.2	.. ....	0.2	...
		W—NS	19 39.1	19 43.3	19 50±	19 54.1	.. ....	0.3	...
171	Nov. 1	B—EW	.. ....	7 47.5	.. ....	8 23	.. ....	0.8	...
		B—NS	.. ....	7 48	.. ....†	8 22	.. ....	0.8	...
		W—EW	.. ....	7 48	.. ....	8 23	.. ....	0.5	...
		W—NS	.. ....	7 49 ±	.. ....	8 23	.. ....	0.8	...
172	Nov. 21	B—EW	0 23†	0 26.7	0 30.4	0 35	1 35	55.0	...
		B—NS	.. ....	0 27.1	29.8	Off sheet	1 25	>35.0	...
		W—EW	0 23†	0 27.4	31.0	34	1 35	8.5	2.7
		W—NS	.. ....	.. ....	30.7	32	1 10	15.9	...
		W—V	.. ....	.. ....	33.5	32.0	1 36.4	7.8	...
173	Nov. 26	B—EW	.. ....	.. ....	.. ....	Visible	.. ....	...	...
		B—NS	.. ....	.. ....	.. ....†	19 37	.. ....	0.1	...
		W—EW	.. ....	.. ....	19 31	?	.. ....	0.05	...
		W—NS	.. ....	.. ....	.. ....	Trace	.. ....	...	...
174	Dec. 12	B—EW	.. ....	.. ....	21 16.3	21 16.8	.. ....	1.4	...
		W—NS	21 9.1	21 13.6?	.. ....	21 19.3	21 27?	0.1	2.9
175	Dec. 29	B—NS	.. ....	.. ....	0 10.9	0 13.7	.. ....	0.15	...
176	Dec. 31	B—EW	12 26.4	12 31.6	12 39.4	12 39.8	13 16?	3.8	...
		B—NS	.. ....	.. ....†	12 36.4	12 40.0	13 20?	2.4	3.6?
		W—EW	12 26.3	12 31.6	12 39.1	12 39.7	13 19?	0.9	...
		W—NS	.. ....	.. ....	.. ....	12 39.8	.. ....	...	...

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126 and 127. The recorded L is the beginning of the disturbance, but later portions, if present, are very weak. However, since the period of the waves is short, 6-7 seconds, in both cases, it is possible that these times should be identified as S in both shocks.

July 13. W—NS shows a slight disturbance at 3^h17^m; possibly artificial.

129. Probably one or more phases have been misidentified since the values of Δ from different forms of the Laska formulae are very discordant.

130. The Jamaica earthquake.

131. B—NS record poor.

133. No record of shock on W—NS. Values of Δ discordant.

134. Some microseisms and probably a small shock.

136. Long waves from about 17^h34^m to 18^h. Trace on B—EW.

137. Ecuador earthquake?

138. B—EW shows short vibrations superposed on the first few long waves of the main portion; on W—EW they continue throughout the main portion.

139. A small disturbance, perhaps not a true earthquake. A trace on W—NS; a weaker trace on W—EW; and probably a trace on W—V.

140. Some flat slow waves, period 28 sec., visible on W horizontal.

141. Phases not well marked. Nothing seen on W—NS.

142. On B—NS microseisms are too strong to distinguish earthquake. Very small record on W—NS, the tail being strong, however, and apparently extending to 10^h16^m, though this is uncertain on account of microseisms.

143. Curious record; no well marked phases. A refers to initial throw of pen, which is large compared with the disturbances which follow.

146. Small irregular waves superposed on the large ones. Phases very uncertain.

Jan. 6, 1915. Slow flat waves on B—EW having a maximum at 0 hrs., period about 20 sec.

147. Slow flat waves lasting over 20 min. W—NS shows only a very slight trace. Preliminaries are not distinguishable among microseisms.

Feb. 25. A slight disturbance at 9^h25^m in the midst of sinusoidal waves.

Feb. 28. Some flat waves at 20 hrs., period about 20 sec.

148. An irregular disturbance beginning abruptly at times given as L, and lasting a few minutes.

March 28. A small disturbance at 20^h29^m lasting for several minutes in midst of microseisms. Period 10-12 sec.

April 7. Some slight disturbances about 16^h14^m.

149. If P and S are correctly identified the long waves (L) are not plain being apparently very feeble with short period vibrations superposed. On W—V P is markedly a single sudden small displacement.

April 25. A small disturbance (perhaps artificial) on W—V from $21^h29^m.8$ to $21^h30^m.4$. Nothing notable on other records.

150. No appreciable record on W—V.

151. Flat waves, period about 20 sec. Also some very weak waves about one hour earlier.

152. P may be wrongly identified as S.

153. Shock in the midst of slight microseisms on W. W shows short waves superposed on the long ones of the main portion.

154. Amplitudes of S.

B—EW	12.0
B—NS	2.5
W—EW	4.7

June 14. At 22^h6^m there is a small disturbance on W with a trace on B—perhaps artificial.

June 15. W—V shows a small irregular disturbance at $0^h59^m.6$ probably artificial.

156. In both shocks the long waves die away gradually disappearing in about 15 min.

July 22. B—NS shows a small shock at 4^h33^m.

July 25 B—NS and W—EW some very flat slow waves for several minutes beginning 21^h17-18^m. Amplitudes not over 0.1 mm. No well defined maxima.

Aug. 6. Probably some long waves, in the midst of microseisms, about 14^h6^m on W—EW; trace on B—EW; better shown on B—NS; continue feebly for 10-12 min.

Aug. 16. B—EW shows some very feeble waves for 15 minutes, beginning at 1^h30^m. Trace on B—NS and W—EW.

Sept. 6. B instruments show a very slight disturbance at 18^h51^m.

160. The W pens exhibit a peculiar jerky motion.

164. A very slight disturbance.

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165. Phases are very poorly marked and indistinguishable in many cases.

166. Phases poorly marked. Possibly S at $21^h\,0^m.7$. Traces on horizontal W. Record poor.

167. B—NS shows a peculiar disturbance beginning at $23^h\,53^m.4$ (= P?) visible for about 10 minutes having its maximum at $23^h\,55^m.6$. S is doubtful and L very weak if present.

169. B—EW not running. No action on W—V.

170. Phases misidentified?

171. F in microseisms.

Nov. 4. B—NS a few slow waves in the midst of microseisms at $19^h\,56^m$, period 18-20 sec., amplitude 0.2 mm.

172. F runs into microseisms. W—NS not very sensitive.

173. Slow flat shock in weak microseisms.

Dec. 7. A peculiar disturbance at $18^h\,45^m$ lasting a minute or two, consisting of very short vibrations superposed on longer rather irregular waves. Amplitudes:

B—EW	0.6
B—NS	0.7
W—EW	1.4
W—NS	0.5

174. B—NS trace but microseisms too strong. W—EW out or order.

Dec. 17. B—NS shows a few slow flat waves with M at $8^h\,5^m$.

175. B—EW microseisms mask. W—EW pen behaves badly.

176. W—NS insensitive. Time of L not accurate?

MICROSEISMS, 1914 AND 1915

1914.

Jan. 1-9.
Sinusoidal microseisms coming to a maximum on 3-4d. Best shown on EW components where they have a maximum amplitude of 0.2 mm. and a period of 5-6 seconds.

Jan. 9-11.
Waves slower and more irregular.

Jan. 11-12.
Tremors stronger.

INST.—COMP.	MAX. AMPLITUDE.	PERIOD.
	mm.	sec.
B—EW	0.25	5-6½
B—NS	0.3	5-6
W—EW	0.1	6
W—NS	?	...

Jan. 12-13.
Same. Period 5 seconds.

Jan. 13-15.
Weaker.

Jan. 16-18.
Tremors faint.

Jan. 18-19.
Stronger.

Jan. 19-20.
Considerably stronger.

INST.—COMP.	MAX. AMPLITUDE.	PERIOD.
	mm.	sec.
B—EW	0.4	6
B—NS	0.3	6
W—EW	0.1	6
W—NS	Very small.	

Jan. 20-21.
Tremors continue. Some well marked groups.

Jan. 21-23.
Weaker.

Jan. 23-24.
A few small tremors. Nearly continuous on B—EW.

Jan. 24-25.
Some slow irregular waves.

Jan. 25-26.
Weak, regular, sinusoidal waves, with occasional stronger groups on W--EW. Period 4 sec.

Jan. 26-29.
Tremors die away gradually.

Jan. 31-Feb. 5.
Slight irregular tremors.

Feb. 5-9.
Growing stronger, particularly on EW instruments where they are perfectly continuous on 7-9.

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Feb. 9-11.
Tremors grow weaker.

Feb. 11-13.
Irregular tremors stronger, particularly toward last of 12-13^d, being then very irregular and peculiar. Stronger on NS than before.

Feb. 13-14.
Much weaker, dying down during the day.

Feb. 14-15.
Weak tremors visible. Some regular sinusoidal waves.

Feb. 15-17.
Same, weaker.

Feb. 17-18.
Irregular tremors extremely feeble. Some regular sinusoidal waves on B—NS and more faintly on B—EW. Period about 6 seconds.

Feb. 18-19.
Slightly stronger. Visible on W—EW.

Feb. 19-20.
Weaker.

Feb. 20-21.
Regular sinusoidal waves in groups but usually connected by small tremors. Amplitude small, about 0.1 mm. Period about 6 seconds.

Feb. 21-25.
Weak sinusoidal waves which are more prominent on EW components.

Feb. 26-27.
Numerous small sinusoidal waves on all horizontal instruments.

March 1-3.
Regular sinusoidal waves nearly continuous. Best shown on NS records.

INST.—COMP.	MAX. AMPLITUDE.	PERIOD.
	mm.	sec.
B—NS	0.2	5
W—NS	0.05	5

March 3-5.
Traces of above.

March 9-15.
Very slight tremors, some being irregular.

March 16-20.
Irregular tremors, stronger during first part of day.

March 20-23.
Some feeble regular sinusoidal microseisms.

March 25-26.
Irregular tremors; stronger during the day than at night.

March 31-April 1.
Some slight regular sinusoidal waves on NS records.

April 1-2.
Same; stronger.

April 2-3.
Microseisms weaker and somewhat irregular.

April 4-5.
Some slow irregularities on NS records.

April 7-8.
Slight irregular motion toward latter part of record on B—NS.

April 8-12.
Same; visible also on B—EW.

April 19-20.
Fairly strong irregular microseisms on B—NS.

April 20-21.
Same; weaker.

April 24-25.
Many regular sinusoidal waves. period about 5½ sec.

April 25-26.
Some feeble tremors, slightly slower.

April 29-30.
Slight irregular tremors on B—EW.

May 13-14.
B—NS very slight irregular tremors, stronger toward morning of 14th.

May 22-23.
B—NS some slight microseisms.

May 29-30.
B—NS some slight microseisms occasionally.

June 5-6.
B—NS some very slight slow irregular tremors.

June 10-11.
B—NS very slight tremors.

June 13-16.
Same.

June 18-19.
Same.

June 29-July 2.
B—NS a few weak tremors.

July 7-10.
B—NS numerous very small tremors decrease in strength to 10th.

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July 11-12.
B—NS some very small temors of short period.

July 13-16.
Minute slow tremors.

July16-18.
Small tremors on B—NS.

July 24-25.
Some faint tremors on B—NS. It is not certain whether they continue after this day since the records of this instrument are poor during the next few days.

Aug. 11-12.
B—NS weak tremors.

Aug. 12-14.
Stronger; seen also on B—EW.

Aug. 14-16.
Weaker.

Aug. 17-18.
Some small tremors.

Aug. 25-27.
B—NS shows small tremors which are nearly continuous on 26-27.

Aug. 27-28.
Weaker.

Sept. 2-4.
Slight sinusoidal waves on B instruments.

Sept. 4-5.
Very slight tremors.

Sept. 5-6.
Stronger. ?

Sept. 6-8.
Stronger, particularly on EW. Trace on W.

Sept. 8-11.
Tremors die away gradually.

Sept. 15-26.
Weak regular sinusoidal waves; strongest on 17-18.

Sept. 26-27.
Very regular sinusoidal waves practically continuous.

Sept. 27-29.
Stronger. Traces on W.

Sept. 29-30.
Much weaker.

Sept. 30-Oct. 3.
Small sinusoidal waves.

Oct. 3-4.
Microseisms present but weaker.

Oct. 4-7.
Slight sinusoidal waves.

Oct. 11-12.
B—NS slow flat microseisms?

Oct. 12-13.
Slight regular sinusoidal waves.

Oct. 13-14.
Apparently some slight slow irregular oscillations on B—NS, but B—EW shows only regular sinusoidal waves.

Oct. 14-15.
Very weak regular sinusoidal waves.

Oct. 15-16.
The motion very feeble.

Oct. 16-21.
Rather strong sinusoidal pulsations on 16-17, and 18-19; die away slowly.

Oct. 23-28.
Sinusoidal.

Oct. 28-30.
Much stronger.

Oct. 30-31.
Not so strong.

Oct. 31-Nov. 8.
Sinusoidal, strongest on 1-3.

Nov. 8-9.
Numerous groups of sinusoidal waves,—trains?

Nov. 9-10.
Irregular tremors.

Nov. 10-11.
Same. Small sinusoidal waves on B—EW.

Nov. 11-12.
Regular sinusoidal waves, stronger.

Nov. 12-13.
Weaker, dying away.

Nov. 13.
B—NS irregular waves in the morning.

Nov. 15-16.
B—NS irregular waves stronger toward end of record. Very much smaller ones on B—EW.

Nov. 16-18.
Very strong irregular waves on B—NS; smaller ones on B—EW.

Nov. 18-24.
Microseisms, stronger on 20-21.

Nov. 24-25.
Slight irregular tremors.

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Dec. 2-7.
B—NS irregular microseisms, slightly stronger on 5-6.

Dec. 7-30.
Numerous microseisms every day.

Dec. 13-14.
Microseisms become considerably stronger during latter part of record.

Dec. 14-15.
B—NS very strong irregular tremors all day. Weaker and more regular on B—EW. Scarcely visible on W.

Dec. 15-21.
Above die down to weak tremors.

Dec. 21-22.
Tremors stronger on latter portion of B records.

Dec. 22-23.
Continued throughout record.

Dec. 23-25.
Weaker.

Dec. 25-27.
Tremors grow stronger.

Dec. 27-30.
Grow weaker.

1915

Dec. 31-Jan. 3.
Rather strong irregular microseisms, being weaker on 2-3.

Jan. 3-4.
Microseisms stronger and more regular.

Jan. 4-5.
Same; a few large slow irregular motions in first part of day.

Jan. 5-6.
Microseisms weaker.

Jan. 6-8.
Irregular tremors increase and die away.

Jan. 8-11.
Small tremors stronger on 10-11.

Jan. 11-13.
Sinusoidal waves in groups; better marked on EW.

Jan. 13-14.
All records, including W—V exhibit very striking sinusoidal waves, showing the group structure. The period on all instruments is about 5½ seconds.

Jan. 14-16.
Fade away.

Jan. 17-23.
Small microseisms, partly of sinusoidal nature on B—EW, more irregular on B—NS.

Jan. 23-24.
Tremors stronger, continuous B—NS.

Jan. 24-25.
W—EW some short sinusoidal waves, period 3-4 seconds.

Jan. 28-31.
Tremors as before, being more regular on EW.

Jan. 31-Feb. 1.
A few weak tremors.

Feb. 1-2.
Stronger toward latter part of day, sinusoidal.

Feb. 2-10.
Continue.

Feb. 14-17.
Few weak and slow tremors.

Feb. 17-18.
Slight quick sinusoidal waves, 15-20 per minute.

Feb. 18-20.
Grow less.

Feb. 24-25.
Some small sinusoidal waves.

Feb. 25-28.
Fairly strong irregular tremors, especially on EW; grow weaker.

Mar. 1-2.
Slight disturbances throughout day.

Mar. 3-5.
Slight microseisms.

Mar. 6-7.
Slight tremors.

Mar. 10-11.
Die down but become considerably stronger toward latter part of record.

Mar. 11-12.
Marked sinusoidal waves with strong groups.

Mar. 12-17.
Decrease in intensity.

Mar. 17-19.
Stronger and more continuous.

Mar. 19-24.
Decrease.

Mar. 24-25.
Somewhat stronger.

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Mar. 25-31.
Tremors much weaker at end of the period.

Mar. 31-Apr. 1.
Faint tremors most pronounced on W—NS where they appear as groups of sinusoidal waves.

Apr. 1-2.
Weaker.

Apr. 2-4.
Increase.

Apr. 4-8.
Die away.

Apr. 8-10.
Sinusoidal waves stronger again, especially on W.

Apr. 10-11.
Much weaker and in detached groups.

Apr. 11-12.
Slow irregular disturbances throughout day on B—NS; occasional groups of sinusoidal waves on W. Trains?

Apr. 12-17.
Microseisms, being strongest on 14-15.

Apr. 18-19.
Microseisms, more prominent on W.

Apr. 21-23.
Sinusoidal waves, stronger on 21-22.

Apr. 25-26.
Some sinusoidal waves.

Apr. 29-30.
Some irregular disturbances on B—NS.

May 6-7.
Traces of microseisms on W.

May 7-8.
Some sinusoidal waves, in groups.

May 14-15.
Some feeble sinusoidal waves.

May 27-31.
Same.

June.
First few days of month slight traces of sinusoidal waves almost continually on both components of EW, accentuated by trains.

July 1-2.
Some very slight sinusoidal waves.

July 6-7.
Slight and rather irregular microseisms.

July 15-16.
Slight microseisms.

July 20-21.
Slight sinusoidal waves.

July 23-24.
Occasional sinusoidal waves on W—EW.

July 26-27.
Sinusoidal waves on early part of W—EW record.

Aug. 5-6.
Microseisms, stronger toward end of record

Aug. 16-18.
Some very slight tremors on EW.

Aug. 18-20.
Stronger.

Aug. 20-22.
Die away.

Aug. 23-25.
Tremors as above.

Aug. 26-29.
Feeble tremors.

Sept. 1-5.
Slight sinusoidal waves, strongest on 3-4.

Sept. 8-9.
Slight traces of microseisms.

Sept. 13.
Weak microseisms, nearly continuous throughout remainder of the month, being stronger on Sept. 23-24, 27-29, and weak on Sept. 18-19, 25-27, 29-30.

Sept. 30-Oct. 2.
Slight microseisms.

Oct. 6-8.
Microseisms increase and wane.

Oct. 8-10.
Microseisms just visible.

Oct. 10-14.
Weak microseisms.

Oct. 15-22.
Very feeble microseisms.

Oct. 22-24.
Much stronger sinusoidal waves.

Oct. 24-27.
Weaker.

Oct. 27-28.
Practically disappear.

Oct. 28-29.
Some slight rather irregular motion.

Oct. 31-Nov. 1.
Fairly strong irregular microseisms all day, with an earthquake superposed on them.

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Nov. 1-2.
Fairly strong sinusoidal microseisms.

Nov. 2-4.
Continue, weaker.

Nov. 4-5.
Slightly stronger.

Nov. 5-7.
Much less prominent.

Nov. 7-9.
Somewhat stronger.

Nov. 9-11.
Feeble.

Nov. 11-13.
Somewhat stronger.

Nov. 13-14.
Weaker, not continuous.

Nov. 14-16.
Grow considerably stronger.

Nov. 16-17.
Slightly less.

Nov. 17-20.
Sinusoidal microseisms. Principally in groups.

Nov. 20-22.
Nearly continuous, rather irregular.

Nov. 22-26.
Grow weaker.

Nov. 28-29.
Irregular. Grow quite strong toward end of record, especially on B—NS.

Nov. 29-30.
Slow microseisms, with occasional groups of stronger waves. Best shown on B—NS.

Nov. 30-Dec. 1.
Microseisms visible.

Dec. 1-4.
More sinusoidal in character.

Dec. 4-10.
Weak microseisms.

Dec. 10-13.
Strong sinusoidal microseisms, slacking toward end.

Dec. 13-15.
Weaker but still conspicuous on B, especially B—NS.

Dec. 15-19.
Weaker.

Dec. 19-21.
Rather slow and irregular.

Dec. 21-24.
Sinusoidal microseisms, with emphasized groups.

Dec. 24-25.
Practically no microseisms.

Dec. 25-26.
Feeble microseisms toward end of record.

Dec. 26-27.
Sinusoidal microseisms mostly in groups.

Dec. 27-31.
Sinusoidal waves stronger.

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OBSERVATORY NOTES

ON A METHOD ADOPTED FOR THE DETERMINATION OF THE ELEMENTS OF SPECTROSCOPIC BINARIES WITH AN ABBREVIATION OF THE LEAST-SQUARES SOLUTION

By RALPH H. CURTISS

For several years the writer has presented in a course on spectroscopic binary orbits the various formulæ which have been proposed for the determination of the elements of these stellar systems from measures of radial velocity. In connection with this course all the geometrical methods, beginning with that of Lehmann-Filhés, have been tested and compared many times. Taking these tests and comparisons into account, a method has been adopted at this Observatory, which is short, determinate, and quite satisfactory in practice. The formulæ, which are obtained easily from those derived by H. C. Plummer on pages 214 and 215 of Volume 28 of the *Astrophysical Journal,* are given below.

In addition to the symbols more frequently employed in this connection, we define:

γ', the algebraic mean of the maximum and minimum ordinates of the velocity curve, referred to the zero axis.

t_1, t_2, t_3, t_4, the abscissae of the points of the velocity curve corresponding to the ends of chords of the orbit making angles of 45° with the line of apsides, (The ordinates of these points, beginning on the descending branch of the velocity curve are $\gamma' + 0.7071\,K$, $\gamma' - 0.7071\,K$, $\gamma' - 0.7071\,K$, and $\gamma' + 0.7071\,K$ respectively.)

η, the quantity, $\frac{1}{2}(E_2 - E_1)$,

η', the quantity, $\frac{1}{2}(E_4 - E_3)$,

τ, the quantity, $\frac{t_2 - t_1}{P}$,

τ', the quantity, $\frac{t_4 - t_3}{P}$,

The formulae are:

$$2\pi\tau = 2\eta - \sin 2\eta, \tag{1}$$

$$2\pi\tau' = 2\eta' - \sin 2\eta', \tag{2}$$

$$\tan(45° - \omega) = +\frac{\cot\eta}{\cot\eta'}, \tag{3}$$

$$\tan\phi = -\frac{\cot\eta}{\sin(45° - \omega)} = -\frac{\cot\eta'}{\cos(45° - \omega)} \tag{4}$$

$$v_1 = 45° - \omega, \tag{5}$$

$$v_3 = 90° + v_1, \tag{6}$$

$$T = t_1 - M_1/\mu = t_3 - M_3/\mu. \tag{7}$$

In applying this method the quantities, $\gamma' \pm 0.7071\,K$, are first computed and the corresponding abscissæ are read off the preliminary curve. The values of τ and τ' are formed and equations (1) and (2) are then solved by Table I below. ω and e follow from equations (3) and (4) and may be checked roughly by Tables II and III. v_1 and v_3 may then be written down from relations (5) and (6), M_1 and M_3, corresponding to v_1 and v_3, are taken from the *Tables for the True Anomaly,* given in the *Publications of the Allegheny Observatory,* Vol. 2, No. 17. Two values of T follow from formula (7), and a third more briefly by noting the value of t whose ordinate is $\gamma' + K\cos\omega$. The differences among these values will give some indication of the departure from ellipticity of the preliminary curve. The adopted value of T may be a straight mean of these values or a compromise suggested by a consideration of the conditions of the case.

An ephemeris is then computed at convenient intervals with the formula,

$$V = \gamma' + K\cos u,$$

using again the *Tables for the True Anomaly,* referred to above, to proceed from M to v. With the curve given by this ephemeris as a guide, it is a few minutes work to make a second, and usually final, determination of the elements if the first set is not satisfactory.

To facilitate the use of this method Table I is published below. This table contains values of η (or η') tabulated with τ (or τ') as argument and also first differences of the tabulated quantity. In form this table is a reversal of Schwarzschild's arrangement (*Astronomische Nachrichten,* Vol. 152, pages 69 to 72) of a tabulation of the same quantities. The values in Table I have all been computed and checked by the present writer. They correct certain discrepancies in

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TABLE I. VALUES OF η (OR η') WITH ARGUMENT τ (OR τ').

τ	0.0		0.1		0.2		0.3		0.4		τ
	η	DIFF.	η	DIFF.	η	DIFF.	η	DIFF.	η	DIFF.	
0	00°00		46°60		60°54		71°36		80°93		0
		20°88		1°66		1°17		0°99		0°92	
1	20.88		48.26		61.71		72.35		81.85		1
		5.57		1.58		1.15		0.99		0.91	
2	26.45		49.84		62.86		73.34		82.76		2
		3.96		1.51		1.12		0.97		0.91	
3	30.41		51.35		63.98		74.31		83.67		3
		3.21		1.45		1.11		0.97		0.91	
4	33.62		52.80		65.09		75.28		84.58		4
		2.73		1.39		1.08		0.96		0.91	
5	36.35		54.19		66.17		76.24		85.49		5
		2.42		1.34		1.07		0.95		0.90	
6	38.77		55.53		67.24		77.19		86.39		6
		2.21		1.30		1.05		0.94		0.91	
7	40.98		56.83		68.29		78.13		87.30		7
		2.00		1.27		1.04		0.94		0.90	
8	42.98		58.10		69.33		79.07		88.20		8
		1.87		1.24		1.02		0.93		0.90	
9	44.85		59.34		70.35		80.00		89.10		9
		1.75		1.20		1.01		0.93		0.90	
10	46.60		60.54		71.36		80.93		90.00		10

TABLE II. VALUES OF ω WITH ARGUMENTS, τ AND τ'.

τ \ τ'	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	τ' / τ
0.10	180.00	165.85	154.63	144.58	135.00	125.42	115.37	104.15	90.00	0.10
0.20	194.15	180.00	165.85	150.79	135.00	119.21	104.15	90.00	75.85	0.20
0.30	205.37	194.15	180.00	160.33	135.00	109.67	90.00	75.85	64.63	0.30
0.40	215.42	209.21	199.67	180.00	135.00	90.00	70.33	60.79	54.58	0.40
0.50	225.00	225.00	225.00	225.00	CIRCLE	45.00	45.00	45.00	45.00	0.50
0.60	234.58	240.79	250.33	270.00	315.00	0.00	19.67	29.21	35.42	0.60
0.70	244.63	255.85	270.00	289.67	315.00	340.33	0.00	14.15	25.37	0.70
0.80	255.85	270.00	284.15	299.21	315.00	330.79	345.85	0.00	14.15	0.80
0.90	270.00	284.15	295.37	305.42	315.00	324.58	334.63	345.85	0.00	0.90
τ / τ'	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	τ \ τ'

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Schwarzschild's numbers. In using this table, *when* τ (or τ') *is greater than 0.500 subtract from unity and take the supplement of the corresponding value of* η (*or* η').

Since the quantities, ω, e, M_1 and M_3, are functions of τ and τ', tables with τ and τ' as arguments may be constructed from which these quantities may be taken. But in view of the brevity of the process of computation of ω, e, M_1 and M_3 with the aid of Table I and the *Table for the True Anomalies* it is doubtful whether the publication of extensive tables of this kind would be warranted. Such tables would require cross interpolation and would not find very frequent application. Herewith are given brief tables which will serve in some cases to give preliminary values of ω and e and will furnish in all cases a rough check on the computed values.

In the least-square solution, the mean velocity, γ', may be used as an element instead of the velocity of the center of mass of the system. This substitution of γ' for γ removes from the equations of condition the three terms resulting from the differentiation of $Ke \cos \omega$ and shortens the computation appreciably, especially when several hypotheses are needed and the probable error of γ is not determined.

TABLE III. VALUES OF e WITH ARGUMENTS, τ AND τ'.

τ \ τ'	0.10	0.20	0.30	0.40	0.50	τ' / τ
0.10	0.801	0.740	0.709	0.692	0.687	0.10
0.20	0.740	0.624	0.550	0.506	0.492	0.20
0.30	0.709	0.550	0.431	0.350	0.320	0.30
0.40	0.692	0.506	0.350	0.220	0.158	0.40
0.50	0.687	0.492	0.320	0.158	0.000	0.50
τ / τ'	0.10	0.20	0.30	0.40	0.50	τ \ τ'

NOTE: If an argument is greater than 0.50 subtract it from unity before entering Table III.

Referring now to Schlesinger's adaptation (*Allegheny Observatory Publications*, Vol. I, No. 6) of the formulæ of Lehmann-Lilhés, we may introduce the element, γ', if in equations (4) of Schlesinger's paper we substitute for the third of the group with its four terms on the right hand side, the simple relation, $\Gamma' = \delta\gamma'$; and for Γ in equation (5), Γ'.

When the least-square solution is completed the center of mass velocity is derived from the equation,

$$\gamma = \gamma' - Ke \cos \omega.$$

Ann Arbor, Mich., January 11, 1916.

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PLATE K. THE THIRTY-SEVEN AND ONE-HALF INCH REFLECTING TELESCOPE

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DISCOVERY OF TWO BRIGHT-LINE STARS OF CLASS B

By PAUL W. MERRILL

ν GEMINORUM.

$\alpha = 6^h\,23^m.1$; $\delta = +20°\,17'$; Mag. = 4.1; Class B5.

A feeble bright Hα line is seen in the spectrum of this star on plates taken on the nights of November 12 and December 3, 1915, apparently being superposed on a broad, weak absorption line. The other hydrogen lines show no distinct bright portions. The star thus belongs to the Electra group[1] of bright-line stars of Class B.

The hydrogen lines other than Hα are rather wide, diffuse absorption lines, but several show a narrow central core, the effect, perhaps, of invisible bright components. Calcium K is present as an absorption line, possibly with bright borders. Several weak absorption lines of helium and other elements are also seen.

This star was found to have a variable radial velocity[2] by the Yerkes Observatory.

β CANIS MINORIS.

$\alpha = 7^h\,21^m.7$; $\delta = +8°\,29'$; Mag. = 3.1; Class B8.

Photographs taken on December 3 and December 10, 1915, show a bright Hα line in the spectrum of this star, superposed on weak absorption. As the other hydrogen lines are dark it should also be included in the Electra group. Several hydrogen lines show traces of structure similar to that described above for ν Geminorum. Aside from the hydrogen lines the spectrum is nearly continuous.

[1] *Lick Bul.*, 7, 176, 1913.

[2] *Ibid*, 6, 22, 1910.

OBSERVATIONS OF COMET TAYLOR, 1915 e

Made at Ann Arbor With the 12¼-inch Refractor

By BERNHARD H. DAWSON

1915 1916	ANN ARBOR M. T.	*	COMP.	Δα	Δδ	APP. α	APP. δ	LOG p.Δ FOR α	LOG p.Δ FOR δ
Dec. 10	$11^h\,57^m\,35^s$	1	10, 10	$-6^s.63$	$+7'\,53''.4$	$5^h\,20^m\,25^s.87$	$+0°\,49'\,25''.7$	8.1496n	0.7630
Jan. 7	9 51 7	2	8, 8	−27.93	−6 0.1	5 6 49.73	+10 14 31.7	8.2954n	0.6662
7	10 35 5	2	8, 8	−28.38	−5 12.4	5 6 49.28	+10 15 19.4	8.8130	0.6672
8	10 22 0	3	., 5	.....	−12 0.0	. . ..	+10 40 34.1	.....	0.6614
8	10 37 12	3	7, .	+31.63	.....	5 6 42.35	.. ..	8.8858	.....

MEAN PLACES OF COMPARISON STARS.

*	α 1915.0	RED. TO APP. PL.	δ 1915.0	RED. TO APP. PL.	AUTHORITY.
1	$5^h\,20^m\,27^s.48$ 1916.0	$+5^s.02$	$+0°\,41'\,21''.0$ 1916.0	$+11''.2$	A. G. Nicolajew, 1300.
2	5 7 15.52	+2.14	+10 20 25.7	+ 6.1	A. G. Leipzig I, 1547.
3	5 6 8.55	+2.17	+10 52 27.9	+ 6.2	BD + 10°726, connected with *4.
4	5 5 20.14	+2.17	+10 47 12.2	+ 6.2	A. G. Leipzig I, 1535.

*2 was connected with A. G. Leipzig I, 1553, $\Delta\alpha = -40^s.97$, $\Delta\delta = +1'\,58''.6$.

Ann Arbor, January, 1916.

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NOTE ON REPRODUCTIONS OF SPECTRA IN THIS VOLUME

By RALPH H. CURTISS

The reproductions of spectra in this volume have been made by the writer with a commercial enlarging camera of the usual pattern to which has been attached in place of the original plate holder a special apparatus for widening spectra, designed by the writer and constructed in the Observatory instrument shop by Mr. E. J. Colliau.

This special apparatus for widening spectra employs the clepsydra principle, the fluid element being a high grade of machine oil. A cylinder of commercial brass tubing is mounted with its axis vertical immediately below the plate holder. A brass piston ringed with rawhide fits tightly in the cylinder. Two tubes, leading from the lower to the upper end of the cylinder, form outside connections around the piston, one with a stop-cock for fast motion and the other with a screw valve permitting very slow motion capable of accurate regulation. The piston rod extends upward from the cylinder to the lower side of the plate holder. As the plate holder moves vertically in V-groove guides its weight is carried solely by the piston rod.

The source of light is a pair of nitrogen tungsten lamps with a combined candle power of about four hundred. Two parallel ground glass plates diffuse this light properly before it reaches the subject to be enlarged. The spectrogram is set with the direction of dispersion horizontal, *i. e.*, with the spectrum lines vertical. Near the image plane adjustable diaphragms limit as desired the region to be photographed; and slides of different apertures in the plate holder determine the length and width of the photograph and make it possible to expose star and comparison spectra side by side on the same plate.

When an exposure is to be made, the diaphragms and slides in front of the photographic plate having been properly adjusted, the clepsydra stop-cock is opened and the piston rod with the plate holder resting upon it is raised to its initial position. The stop-cock is closed; the light is turned on and the plate holder is allowed to fall by its own weight and that of the piston rod and piston against the resistance of the clepsydra fluid flowing through the slow motion valve, which must be set in advance for the length of exposure desired. When the exposure is started the attention of the operator is no longer required as the exposure is stopped automatically when the plate holder falls to a certain point.

The apparatus in its present condition, as used in the preparation of the reproductions of spectra in this volume, is avowedly in preliminary form. Antifriction bearings have not been used and the cylinder of ordinary commercial tubing has not been bored. In this state the instrument could not be expected to give perfect results but its performance is satisfactory for many purposes.

The reproductions in the present volume have been enlarged from six to eight fold in length and from 35 to 50 fold in width. The original copies have been made on Seed 23 or Process plates. Negatives were then made on Process plates and from these the prints were made. The spectra of each set were brought to the same scale by altering the image distance in the enlarging camera, though this reduction to identical scale was not accurately possible for two spectra, the one made with the first prism used in our spectrograph and the other made with the prism now in use. Also interesting difficulties were encountered in bringing spectra made with the same prism into exact linear correspondence because of scale variations arising in the same or different papers after printing.

PLATE C.

The spectra of γ Cassiopeiæ and β Monocerotis with titanium comparison show interesting types of emission lines both of hydrogen and of the metals. The central reversals of the hydro-

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gen lines develop strongly in β Monocerotis whereas in γ Cassiopeiæ the broad absorption borders of these lines are relatively strong. The variation of these features from line to line may be observed in accordance with published descriptions except in the case of the Hβ line in γ Cassiopeiæ's spectrum. This line is abnormally weak because the original was made on a lantern slide plate which was not sensitive in this region. Absorption lines, especially of helium, are brought out.

PLATE D.

This plate reproduces five spectrograms of f^1 Cygni exhibiting the progressive increase in the intensity of hydrogen absorption with the lapse of time, together with a change of structure most clearly seen in the case of Hβ. The other absorption lines, mainly due to helium, show little or no changes in intensity. In the fourth photograph two relatively sharp absorption lines near Hδ are easily recognized as spurious features due to pin holes in the original negative.

PLATE E.

These two spectra of H. R. 985, the one made in 1912 and the other in 1916, bring out changes in the structure of the hydrogen lines in the interval of four years. The emission is stronger to the right of the central absorption of Hδ, Hγ and Hβ in 1912 and to the left in 1916. No certain change is noted in the helium lines, which are very broad and diffuse as in f^1 Cygni.

PLATE F.

In order to permit of a scale of enlargement not too small and to adapt the exposure time to the different densities in its different parts the spectrum of *o* Ceti has been reproduced in two overlapping sections. The second section is not strongly exposed in the original and hence strong contrast could not be expected in the copy. The weakness of G, the strength of g and the absence of Hϵ in the series of bright hydrogen lines are notable features. The spectra of S Hydræe and W Lyræ were enlarged without widening and were reproduced in parallel in order to show the range of displacement of Hδ and Hγ emission lines in two different stars.

PLATES G AND H.

These plates include our best photographs of spectra of Class R compared with the spectrum of the Sun (Class G) and α Arietis (Class K2) on the one side and with several Class N spectra on the other. The spectrum of U Hydræ at the end of the series is known as the standard of Class N. The series has been arranged as nearly as could be judged in the order of development from the solar type to Type IV. In this series the spectra of Class R supply graded steps in the transition from Class G to Class N with some overlapping of late R and early N spectra. The arrangement in order of development is based largely on the increasing absorption in the region of shorter wave-length partly due to increased general absorption in this region and partly to strengthening of the absorption of Cyanogen Group III beginning at λ 4216 and partly to Group V of the Swan spectrum. In the copies it was intended to preserve the intensity ratios of the same regions on the different original plates and to bring all plates to a similar degree of contrast. Obviously the actual intensity curve of an original plate is an accidental effect due to the combined influence of many factors aside from the real distribution of energy in the star's light before it enters the earth's atmosphere. But comparison among different plates made with the same apparatus is of the greatest value. On the different negatives in the first half of the series the strength of the region bordering the carbon band near λ 4700 on the side of greater wave-length was kept at nearly uniform density but in the second half this region was made less dark in order that the interesting region on either side might not be printed out.

The average strengthening of the carbon bands at and near λ 4700 down the series was a criterion also employed in arranging these spectra in order of probable evolution though notable variations from this rule are evident.

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The weakening of some lines and the strengthening of others down the series were also kept in mind. These changes, which are brought out quantitatively in Doctor Rufus's tables, are very interesting and in some cases very conspicuous. Among such changes may be mentioned the strengthening in Class R of a line near λ 4224 very close to the g line on the side of shorter wave-length, and the weakening of Hδ, Hγ and Hβ. Doctor Rufus refers especially to the variations in the group of lines near Hβ on the side of greater wave-length. These variations may be followed down the entire series. Of these lines λ 4958 is designated on Plates G and H. The lines used by Adams and Kohlschütter to determine spectral class by comparison with Hγ and Hβ may be identified easily on the reproductions though in every case they are blended more or less with close lines. λ 4326 and λ 4352 are the first well defined lines on either side of Hγ. λ 4872 is the first line visible to the right of Hβ. λ 4405 and λ 4958 are indicated. In the sun Hγ is blended with a close line of shorter wave-length from which it is resolved in α Arietis. Of the lines which Adams and Kohlschütter found strong in high luminosity stars of Class K, λ 4216 is at or near the head of a cyanogen band in Class R, λ 4395 is the second line to the left of λ 4405 and λ 4408 is the first line to the right of λ 4405. λ's 4395 and 4408 are not especially strong in spectra of Class R.

The interesting divergence of the spectrum of B. D. — 10° 5057 from the general type of other members of Class R is obvious in Plate G. Hγ, Hβ and G are absent or weak in this spectrum.

PLATE I

This plate is intended to bring out the important steps in the transition from the solar spectrum to that of a standard Class N star. For this purpose selections have been made from the spectra in Plates G and H.

PLATE J

This plate furnishes a comparison of the solar spectrum with the "earliest" Class R spectrum in Plates G and I. Both of these copies have been shaded to show the spectral features to the H line and have been reproduced on a scale somewhat greater than that of Plates G and H. The spectrum of the titanium spark at the top establishes wave-lengths for the identification of faint lines. The spectrum of an "early" Class N star is added for comparison at the bottom.

The spectra in Plates G, H, I and J support very clearly Doctor Rufus' conclusion that spectra of Class R form the connecting links between Class N and the solar spectrum.

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